Heat Transfer Equation and Finite Element Analysis for Phase Change Condition in Frozen Ground

Jae Mo Kang, Jangguen Lee, Hak Seung Kim, YoungSeok Kim, Seung Seo Hong

1. INTRODUCTION

This study presents an efficient and simple method of overcoming numerical problems associated with sudden jump of heat capacity due to the phase change from water to ice in the pore space. This paper proposes heat transfer equation and finite element method when the saturated soils or porous rocks are subjected to freezing. Numerical analyses using the proposed method agree well with the known closed form solution.

2. Numerical Modeling of Phase Changes and Freezing Pipe

The latent heat required for the phase change of water or saturated earth materials from the unfrozen to the frozen condition can be represented by a sudden jump in heat capacity at the freezing point. To avoid the numerical difficulties associated with this large jump, Comini et al. (1974) have introduced a heat capacity averaging scheme based on the spatial distribution of the enthalpy gradient with respect to the temperature in an element:

$$\rho C = \frac{1}{2} \left( \frac{\partial H}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial H}{\partial y} \frac{\partial T}{\partial y} \right)$$

Where, $\rho$ is mass density, $C$ is Heat Conductivity, and $H$ is the enthalpy defined as the integral of heat capacity with respect to temperature.

Though equation (1) can be used successfully under small temperature changes within a time step, a simpler and better way is to compute average heat capacity based on the time history of the enthalpy gradient with respect to the temperature:

$$\rho C = \frac{H_t - H_{t-\Delta t}}{T_t - T_{t-\Delta t}}$$

Where, $H_t$ and $T_t$ are the enthalpy and temperature, respectively, at time $t$.

3. Finite Element Analysis for Phase Change

Eq. (2) has been implemented in the Finite Element Method (FEM) code, and numerical results were compared with simple analytical solutions. Comparison study is based on 1-dimensional water column with infinite length. Initial temperature of water column is assumed to be 10°C and top boundary is defined as -20°C. Transient temperature changes are presented in Fig. 1, and numerical results show excellent agreement with analytical solutions. Therefore, phase change from water to ice can be simulated without severe errors caused by a sudden jump in heat capacity at the freezing point.

ACKNOWLEDGEMENT

The authors gratefully acknowledge financial support from Korea Institute of Construction and Technology (KICT).

REFERENCE


*1 韓國建設技術研究院 Geotechnical Engineering & Tunnelling Research Division, Korea Institute of Construction Technology

©2011 雪工研究大会