PSO/GA Hybrid Method and Its Application to Supersonic-Transport Wing Design

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Abstract
The hybrid method between multi-objective particle swarm optimization and adaptive range multi-objective genetic algorithm has been developed and its performance has been measured by using three test functions with noise. Moreover, it was applied to a large-scale and real-world engineering design problem. The performance measurement was carried out under the conditions of a small number of population size and generations to apply the practical problem which it needed large computational time for the evaluation. The convergence metric and the cover rate were employed as the measurement manners. Consequently, it revealed that the present hybrid method had similar performance for a simple three-dimensional test problem compared with genetic algorithm in a small number of generations. Moreover, it had the best performance for the test functions with noise. Therefore, the present hybrid method was applied to the wing design of the silent supersonic technology demonstrator. As a result, the efficient design exploration was performed and obtained 75 non-dominated solutions revealed the beneficial knowledge to decide a compromise solution.

Key words: Evolutionary Algorithms, Multi-Objectives, Multidisciplinary Design Optimization, Aerodynamics, Composite Structures, Boom Noise

1. Introduction

Large-scale and real-world engineering design problems often have many design objectives with conflicting requirements. These problems are typically formulated as a multi-objective (MO) optimization problem. As an MO optimization problem has tradeoffs among multiple optimization targets, an MO optimization should be performed to identify tradeoffs efficiently. MO evolutionary algorithms (MOEAs) are applied to MO optimizations to sample multiple non-dominated solutions because MOEAs seek optimum solutions in parallel using a population of design candidates. Since MOEAs share the information of individuals, global searching is achieved. MOEAs search from multiple points in design space simultaneously and stochastically, instead of moving from a single point deterministically like gradient-based methods. MOEAs can search for optimal solutions in non-smooth design space because MOEAs do not require any sensitivity derivatives. It also means that MOEAs are not prone to premature failure even when the design space is noisy. Various evaluation tool such as computational fluid dynamics (CFD) and computational structural dynamics (CSD) are easily coupled with MOEAs, because MOEAs only require the objective values.

As MOEAs require a large number of evaluations, Kriging-based response surface surrogate model is recently prone to employ practical engineering design problems to restrain evaluation time [1]. However, surrogate models require the many initial sample points for the accuracy of response surface when problems with many design variables are considered [2]. The number of initial sample points generally require 10 times of that of design variables. When the same number of individuals is evaluated, MOEAs can evolve generations. Hence, an MOEA is required which has high performance under the conditions of a small number of
population size and generations.

The objective of this study is the development of the optimizer with high performance under the conditions of a small number of population size and generations. Real-coded hybrid method between MO particle swarm optimization (PSO) [3, 4] and adaptive range MO genetic algorithm (ARMOGA) [5] has been developed for the above purpose and it was applied to a large-scale and real-world multidisciplinary design optimization (MDO) problem. In the present study, the performance of PSO/GA hybrid method is firstly studied using three different MO analytical test problems included in the functions with noise. The performance of the hybrid method was compared with those of simple MOPSO and ARMOGA. Moreover, the hybrid method was applied to the wing design of a silent supersonic technology demonstrator (S3TD) as a practical design problem.

2. Particle Swarm Optimization / Genetic Algorithm Hybrid Method

A hybrid method between MOPSO [3, 4] and ARMOGA [5] was developed. Recent optimization works often use a kriging-based response surface surrogate model to restrain evaluation time [1, 2, 6]. However, when the problem with many design variables is considered, many initial sample points are required to maintain the accuracy of response surface. In this study, response surface surrogate model was not selected to avoid the huge initial evaluation time due to a large number of design variables.

GAs generally have not for a capability to search for local optima but a faculty of global searching. On the other hand, PSO is efficient to search for local optima because it deals with the coordinates of design variables directly. The hybridization between them is expected to produce both capabilities. As PSO and GA use mutation (called as perturbation in PSO) for the maintenance of solution diversity and the prevention of convergence to local optima, the convergence to Pareto solutions becomes worse. The hybrid method is expected to improve diversity and to enrich the quality of the obtained non-dominated solutions.

The present hybrid method firstly sets on the ratio of the number of individuals in a generation which PSO and GA serve. Then, each optimizer independently operates the selection of parents (the selection of the personal best $P_{\text{best}}$ and the global best $G_{\text{best}}$ in PSO), crossover (update in PSO), and mutation (perturbation in PSO). Two optimizers have evaluations and obtained archive of non-dominated solutions in common.

2.1. Particle Swarm Optimization

PSO method evolved from a simple simulation model of the movement of social groups such as birds and fishes, in which it was observed that local interactions underlie the group behavior and individual members of the group can profit from the discoveries and experiences of other members. PSO is the algorithm using plural points such as GA. PSO learns $P_{\text{best}}$ for exploiting the best results found so far by each of the particles, and $G_{\text{best}}$ found so far by the whole swarm for encouraging further exploration and information sharing between the particles. In PSO, each solution (particle) $x_n$ in the swarm of $N$ particles is endowed using the following equations.

$$
\begin{align*}
    x_n^{(t+1)} &= x_n^{(t)} + \chi u_n^{(t)} + \epsilon^{(t)} \\
    v_n^{(t+1)} &= w v_n^{(t)} + c_1 r_1 (P_{\text{best}_n} - x_n^{(t)}) + c_2 r_2 (G_{\text{best}_n} - x_n^{(t)})
\end{align*}
$$

where $\chi \in [0, 1]$ is a constriction factor which controls the magnitude of velocity. $w$, $c_1$, and $c_2$ are parameters. $r_1$ and $r_2$ are two uniformly distributed random numbers in the range $[0, 1]$. $t$ denotes the searching cycle which is similar to the generation in GA. $n$ denotes each individual. $x$ is the vector for design variable, and $v$ denotes the velocity vector. $\epsilon$ denotes perturbation. Although the original PSO introduces a normal perturbation [3], a Laplacian density $p(\epsilon) \propto \exp(-|\epsilon|/0.1)$ is used in the present study. The Laplacian distribution yields occasional large perturbations enabling wider exploration.
2.2. Multi-Objective PSO

MOPSO was developed by using the method proposed by Alvarez-Banitez et al [4]. The determination manner of \( P_{\text{best}} \) and \( G_{\text{best}} \) is essential, because the result of MO problem has plural optimum solutions. Especially, the determination of \( G_{\text{best}} \) is important. In this study, the manner is employed which determines the \( G_{\text{best}} \) based on the Pareto dominance concepts. The non-dominated solutions are held as the archive \( A \). \( X_a \) and \( A_x \) define as

\[
X_a \equiv \{ x \in X \mid a \prec x \}
\]

and

\[
A_x \equiv \{ a \in A \mid a \prec x \},
\]

respectively. \( G_{\text{best}} \) is determined by using \( X_a \) as follows;

\[
G_{\text{best}}^n = \begin{cases} 
    a \in A & \text{with probability } \propto |X_a|^{-1} \quad \text{if} \quad x \in A \\
    a \in A_x & \text{with probability } \propto |X_a|^{-1} \quad \text{otherwise} 
\end{cases}
\]

(2)

\( P_{\text{best}} \) is updated unless the present position is dominated by the position of last generation.

2.3. Operators for GA

The real-coded ARMOGA was used in this study because the value of design variables was directly employed for the chromosome of individual. Regarding crossover, the blended crossover method (BLX-\( \alpha \)) [7] and the principal component analysis-BLX-\( \alpha \) method (PCA-BLX-\( \alpha \)) [8] were used, and then a half of the population size was assigned to each crossover method. PSO and GA served a half of population size each other. When the mutation rate is high, an EA search is close to a random search and results in slow convergence. Therefore, the mutation rate was defined by using the inverse of the number of design variable.

3. Evaluation of Performance Using Test Functions

In this study, the practical engineering application with large evaluation time is assumed (for example, in the present application, it takes roughly seven days for a generation). Therefore, the optimizer which efficiently explores in a small number of generations is needed. The performance among PSO, GA, and their hybrid method is compared to select for practical engineering applications. Hence, the performance until 20th generation at most is evaluated, although the performance at a large number of generations is generally estimated. The qualitative performance is evaluated for the consideration of mathematical test functions with noise to select a method which is adequate for large-scale and real-world applications.

3.1. Test Functions

Three famous test functions were employed in this study. The first function is DTLZ1 [9] which describes three dimensional Pareto front. The second function is ZDT1 [10] with noise described by using normal distribution, which is employed to investigate the performance under the condition with noise for a simple two-dimensional problem. The final function is TNK [11] as a constraint test function with noise, which is employed to investigate the performance under the condition with noise for a constraint two-dimensional problem. When an optimizer is applied to practical problems, experiment and computation are usually employed for the evaluation of objective functions. Experiment has error due to the flow quality in wind tunnel. Computation similarly has error due to mesh and various schemes etc. That is, as noise is occurred for the evaluated value under an identical condition, the consideration of noise is important to investigate the performance of optimizer.

3.1.1. DTLZ1

As a simple three-dimensional test problem, the three-dimensional test function with a linear Pareto-optimal front was considered as follows:

Minimize: \( f_1(x) = \frac{1}{2} x_1 x_2 (1 + g(x)) \)

Minimize: \( f_2(x) = \frac{1}{2} x_1 (1 - x_2) (1 + g(x)) \)

Minimize: \( f_3(x) = \frac{1}{2} (1 - x_1) (1 + g(x)) \)

subject to:

\[
g(x) = 100 \left[ |x| + \sum_{k=3}^{K} \left( (x_k - 0.5)^2 - \cos (20 \pi (x_k - 0.5)) \right) \right] \geq 0,
\]

\( 0 \leq x_k \leq 1, \quad k = 1, 2, \cdots, K, \quad K = 10. \)
The Pareto-optimum solution corresponds to $x_i = 0.5$ (for all $x_i \in x$) and the objective function values lie on the linear hyper-plane: $\sum_{m=1}^{3} f_m = 0.5$. Since the Pareto front of this test function is linear, the fundamental characteristics of the present optimization method emerge.

### 3.1.2. ZDT1

As a test problem with noise, the following two-dimensional test function was considered:

Minimize: $f_1(x) = x_1$

Minimize: $f_2(x) = g(x) \left( 1 - \sqrt{\frac{f_1(x)}{g(x)}} \right)$

subject to: $g(x) = 1 + 9 \cdot \frac{1}{K-1} \sum_{k=2}^{K} x_k \geq 0$, $0 \leq x_k \leq 1, \quad k = 1, 2, \cdots, K, \quad K = 30$.

The Pareto-optimum front is formed with $g(x) = 1$. As noise is appended to this test function, the performance for noise occurred in practical problems is confirmed.

### 3.1.3. TNK

As a test problem with noise, the following two-dimensional test function was considered:

Minimize: $f_1(x) = x_1$

Minimize: $f_2(x) = x_2$

subject to: $c_1(x) = x_1^2 + x_2^2 - 1 - 0.1 \cos \left( 16 \arctan \frac{x_2}{x_1} \right) \geq 0$ (5)

$c_2(x) = (x_1 - 0.5)^2 + (x_2 - 0.5)^2 \leq 0.5$

$0 \leq x_i \leq \pi, \quad i = 1, 2$.

This is a two real-valued variable constrained test problem. Since the function is simple and the objective-function space corresponds to the design-variable space, the Pareto front is determined by the constraints. As this function is a minimization problem, the discontinuous region which is not dominated by the other region in the curve described by $c_1(x) = 0$. The ratio which the feasible region accounts is approximately 5% of the whole region. The Pareto front of this test function is non-convex surface. Therefore, this test function with noise reveals the performance for intricate practical problems.

### 3.2. Performance Measurement Manners

Several performance measurement manners for evaluating the efficiency of MOEAs were suggested [4, 12]. In this study, two metrics were used. The first metric is Convergence metric $\Upsilon$ [13]. It measures the distance between the obtained non-dominated front $Q$ and the set $P^*$ of Pareto-optimum solutions as follows:

$\Upsilon = \frac{1}{|Q|} \sum_{i=1}^{\|Q\|} d_i$, (6)

where $d_i$ denotes the Euclidean distance in the objective-function space between the solution $i \in Q$ and the nearest member of $P^*$. The value near zero means better performance.

The second metric is Cover rate $R_c$ [14]. $R_c$ evaluates the width and closeness of non-dominated solutions compared with Pareto-optimum front. The design space closed by the objective values from minimum to maximum is taken discretization. This metric describes the degree that non-dominated solutions cover discrete region. In this study, two/three-dimensional test functions are evaluated. The objective-function space is separated by squares and cubes. The cover rate $R_c$ is the following equation:

$R_c = \frac{NNDS}{NPareto}$, (7)

where $NNDS$ denotes the number of the cubes included in the derived non-dominated solutions. $NPareto$ denotes the number of the cubes intersected by the Pareto front. The maximum value of $R_c$ gives one and the minimum value of $R_c$ gives zero, and then the value near one means better performance.
3.3. Results

The performance was described by the average of 20 trials using different initial populations. The population size was set on 16 individuals, and the population was evolved until 200th generation. Here, the performance until 20th generation would be discussed due to the application of practical engineering problems. Three optimizers such as PSO/GA hybrid method, GA, and PSO were tried for each test function. It is notable that the representation as “GA” means ARMOGA and “PSO” is MOPSO in this study.

Figure 1 shows the histories of the convergence metric for DTLZ1 using each optimizer. This figure reveals that the PSO/GA and GA give similar performance under the condition of low generation. On the other hand, PSO has low performance compared with their performance. Figure 2 shows the histories of the cover rate for DTLZ1. Although the performance of GA is the best after 20th generation, there make little predominance under the condition of low generation. The PSO/GA gives the best performance for ZDT1 with noise, while GA and the PSO/GA give similar performance for DTLZ1. Figure 3 shows the histories of the convergence metric for ZDT1 with noise. This figure reveals that the PSO/GA gives the best performance under the condition of low generations. Figure 4 shows the history of the
cover rate for ZDT1 with noise. Although this figure shows that the PSO/GA is the best performance under the condition of low generations, the range that the cover rate fluctuates is negligible small. Figure 4 mentions no predominance of the optimizers’ performance. Figure 5 shows the histories of the convergence metric for TNK with noise. This figure shows that the PSO/GA is the best performance, whose location is similar to Fig. 3. Therefore, PSO/GA gives the best performance regarding the convergence under the condition of low generations for the test problems with noise. As the non-dominated solutions can be efficiently explored by the hybrid method with a small number of generations and can be shared in archive due to the faculties of each method for local and global searchings, the hybrid method gives better performance. Since evaluation values often include error due to evaluation manners and a small number of population size and generations can be evolved in practical engineering problems, PSO/GA has advantage under the condition of low generations.

4. Application to Practical MDO Problem

Since the flight experiment of the non-powered supersonic experimental scaled airplane NEXST-1 was succeeded in October 2005 [15], the silent supersonic technology demonstrator (S3TD) then has been researching and developing as a next step in Japan Aerospace Exploration Agency (JAXA) [16]. The initial 0th shape was already designed to focus on low boom and low drag. However, its shape has insufficient performance regarding lift at low speed. Therefore, the second shape with a primary purpose of lift-performance improvement would be re-designed to keep low boom intensity (the first shape was for minor change to re-design low-boom geometry). One of the views of this demonstrator is the design using multidisciplinary design optimization (MDO) system due to the efficient design making the best use of computer technology. Hence, the hybrid method would be applied for the present MDO problem [17].

4.1. Problem Definition of MDO

4.1.1. Objective Functions

The following five objective functions were defined. 1) The minimization of the pressure drag at supersonic cruising condition: \( S \cdot C_{Dp} \) (Mach number of 1.6, altitude of 16km, and target \( C_L \) of 0.132 for the 0th shape. \( S \cdot C_L = \text{const.} \) \( S \) denotes the one-sided wing reference area. 2) The minimization of the friction drag at supersonic condition: \( S \cdot C_{Df} \). In this study, since a simple equation was used for \( C_{Df} \) evaluation, the each fidelity of \( C_{Dp} \) and \( C_{Df} \) was different. Therefore, the objective functions were separated to avoid disappearing one influence for the inconsistency. 3) The maximization of the lift at subsonic condition: \( S \cdot C_L \) (Mach number of 0.2 and angle of attack of 10.0deg). 4) The minimization of sonic boom intensity \( I_{\text{boom}} \) at supersonic condition. This objective function value was defined as \(|\Delta P_{\text{max}}| + |\Delta P_{\text{min}}|\) at the location with largest peak of sonic-boom signature across boom carpet. 5) The minimization of the composite structural weight \( Wc \) for wing using fiber angle of ply and a number of ply with the fulfillment of the strength and vibration requirements. When an individual could not be satisfied with the requirements, the penalty was given to the rank in the optimizer.

4.1.2. Geometry Definition

The design variables were related to planform, airfoil shape, wing twist, and position relative to the fixed fuselage on the reference configuration. As the fuselage of the reference configuration was carried out low-boom and low-drag design, it was fixed. A wing planform with a kink was determined by seven design variables as shown in Fig. 6. Airfoil shapes were defined at the wing root, kink, and tip using thickness distributions and camber lines. The thickness distributions were described by Bézier curves using nine control points with 10 design variables, and linearly interpolated in the spanwise direction. The camber line distributions were parameterized using Bézier curves with four control points with four design variables, and incorporated linearly in the spanwise direction. Wing twist was represented by using B-splines using six control points with six design variables. The twist position was 80% chordwise location so that the straight hinge line for aileron was secured.
The position of the wing root relative to the fuselage was parameterized by \( z \) coordinate of the leading edge, angle of attack, and dihedral. The entire computational geometry was thus defined by using 58 design variables. Although the \( \text{S}^3\text{TD} \) has the components of fuselage, main wing, engine, and tail wing, the wing-body configuration was considered. In the present study, a robust surface mesh was automatically generated in the following steps: a) generation of the wing geometry, b) extraction of the intersection line between the body and wing, c) deletion of the wing geometry which is inside the body, and they are united, d) generation of the mesh point distributions along created ridges, and e) generation of unstructured surface mesh using advancing front method [18].

**4.1.3. Constraints** The geometrical constraints were considered for wing shape definition. The ridge line between wing and fuselage should be extracted. Moreover, the chord length \( c \) should satisfy \( c_{\text{root}} > c_{\text{kink}} > c_{\text{tip}} \). In addition, the constraint of maximum thickness at each spanwise location was considered. When an individual corresponded to a constraint, another individual was generated because the individual could not take form as an airplane. This operation was run over until the provided population size was set. In addition, there was no geometrical constraint regarding wing volume because it was indirectly appreciated in the evaluation of the sonic boom intensity using equivalent area distribution.

In this study, there was no constraint for the objective functions because they were selected to give tradeoffs. For this reason, the aerodynamic evaluation was performed for all configurations with and without the satisfaction of the structural requirements. It might be loss of cost. However, the optimum wing shape for aerodynamics and sonic-boom noise might have a large sweep-back angle and a large aspect ratio. Therefore, more practical geometry can be explored to simultaneously optimize aerodynamics sonic-boom noise, and structures.

**4.2. Evaluation Method** The present MDO system prepared two evaluation phases for aerodynamics, structures, and boom noise. As the weak aerodynamic and structure coupling was carried out, these phases could not be parallelized. The master processing element (PE) managed PSO/GA, while the slave PEs computed aerostructural evaluation processes. Slave processes did not require synchronization. It took roughly seven days at least to evaluate one generation. It is notable that the accuracy of each evaluation tool for aerodynamics, structures, and boom noise was validated through NEXST-1 design [19, 20] and the conceptual design of \( \text{S}^3\text{TD} \) initial configuration [21].

**4.2.1. Aerodynamic Evaluation** In the present study, TAS-Code, parallelized unstructured Euler/Navier-Stokes solver using domain decompositions and message-passing interface (MPI) library, was employed to evaluate \( S \cdot C_D \) and \( S \cdot C_L \). The three-dimensional Euler equations were solved with a finite-volume cell-vertex scheme on the unstructured mesh [18]. The Harten-Lax-van Leer-Einfeldt-Wada Riemann solver [22] was used for the numerical flux computations. The Venkatakrishnan’s limiter [23] was applied when reconstructing the second order accuracy. The lower-upper symmetric-Gauss-Seidel implicit scheme [24] was applied for time integration.
Euler computations were performed under subsonic and supersonic flight conditions, respectively. Taking advantage of the parallel search in PSO/GA, the present optimization was parallelized. Moreover, the CFD computation was also parallelized on the scalar machine.

The following Prandtl-Hoerner’s equation was used for the estimation of $D_{sf}^{sup}$ to avoid huge computational time due to Navier-Stokes computation. This empirical equation is often employed for practical designs in business.

$$C_{D_f} = C_f(Re, M) \cdot \frac{S_{wet}}{S} = \frac{0.455}{(\log_{10} Re)^{2.58}} \cdot (1 + 0.15 M^2)^{-0.58} \cdot \frac{S_{wet}}{S}$$

### 4.2.2. Structural Evaluation

In this MDO system, structural optimization of a wing stacking sequences of laminated composites was performed to realize minimum $W_c$ with constraints of strength and vibration requirements. Given the wing outer mold line for each individual, the finite element model (FEM) was automatically generated from aerodynamic evaluation result of supersonic cruising condition, such as the coordinates, the pressure coefficient, and the normal vector ($x$, $y$, $z$, $C_p$, $x_{normal}$, $y_{normal}$, and $z_{normal}$). The strength and vibration characteristics were evaluated by using the commercial software MSC. NASTRAN™ [25].

Wing had inner and outer boards, and then inboard wing was made up of beams structure from frame, rib, and spar, and outboard wing compounded from a full-depth honeycomb sandwich structure.

The design variables were six, such as stacking sequences (fiber angle of a ply and number of ply) of the skin in an outboard wing, the skin in an inboard wing, and the beams in an inner wing. Fiber angle of a ply was defined as a symmetrical stacking $[0/\theta/90]^n$. Note that $\forall n \in N \leq 25$. When $n$ was greater than 25, the individual was considered not to fulfill the structural requirements. $\theta$ was set on 15, 30, 45, 60, and 75 degs. As this optimization of stacking sequence was a combination one, a manner like as a sensitivity analysis was used instead of the PSO/GA hybrid method.

First, strength analysis was carried out until six design variables fulfill the strength requirement at each node of FEM mesh on each stacking sequence. Then, vibration analysis was performed using the combinations of the design variables satisfied with the strength requirement until they fulfill the vibration requirements (greater than 8Hz for bending-first-mode and also greater than 50Hz for twist-first-mode). The computation condition was set on the symmetrical maneuver $+6G$ and the margin of safety was set on 1.25. The speed of sound and the air density was set under the condition of altitude of 16km.

### 4.2.3. Sonic Boom Evaluation

The CAD-based Automatic Panel Analysis System (CAPAS) [21] was used to evaluate $I_{boom}$. CAPAS was a conceptual aerodynamic design tool in JAXA. This tool comprised four design processes as follows; 1) geometry definition of airplane component, 2) combination of all components in an airplane configuration using CAD, 3) generation of panel and aerodynamic analysis using panel method, 4) sonic-boom analysis using a modified linear theory. As an aerodynamic evaluation module in CAPAS was a low-fidelity, the aerodynamic performance was used only to evaluate $I_{boom}$.

### 5. Optimization Results

The population size was set on 16 taking the evaluation time for one individual into consideration. It took roughly six hours of CPU time of JAXA’s super computer system 20 PEs for an Euler computation. Although another individual was re-generated due to the dissatisfaction of a geometrical constraint, a fulfilled individual could not rarely generate because of a local search in PSO. When a population size was not set, GA served all population. Simple PSO is rarely insufficient for a constraint-handling optimization because of the maintenance of local searching. The total evolutionary computation of 12 generations was performed, and 75 non-dominated solutions were obtained. The evolution might not converge yet. However, the result was satisfactory, because several non-dominated solutions achieved improvements
Table 1  Geometrical characteristic values of the extreme solutions.

<table>
<thead>
<tr>
<th>Individual</th>
<th>$C_L_{design}$</th>
<th>$\alpha_{cruise}$[deg]</th>
<th>AR</th>
<th>$S_{wetted_{wing}}$[m$^2$]</th>
<th>$N_{wetted_{skl}}^{ply}$/$N_{wetted_{skl}}^{ply}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDS-A</td>
<td>0.1300</td>
<td>3.94</td>
<td>2.92</td>
<td>10.81</td>
<td>16/88/24</td>
</tr>
<tr>
<td>NDS-B</td>
<td>0.3103</td>
<td>6.85</td>
<td>2.86</td>
<td>4.15</td>
<td>8/24/8</td>
</tr>
<tr>
<td>NDS-C</td>
<td>0.0632</td>
<td>1.76</td>
<td>3.08</td>
<td>22.52</td>
<td>56/48/56</td>
</tr>
</tbody>
</table>

over the 0th shape. Furthermore, sufficient number of solutions has been searched so that the
sensitivity of the design space can be analyzed. This can provide useful knowledge for
designers.

Figure 7 shows the non-dominated solutions projected on two-dimensional plane be-
tween two objectives. Note that the orange curves denote non-dominated fronts for each
plane. This figure indicates the following tradeoff information. There are tradeoffs between
$S \cdot C_L$ and $W_c$. Especially, there are severe tradeoffs between $S \cdot C_{D_f}$ and $S \cdot C_L$, and $S \cdot C_{D_f}$ and $S \cdot C_L$. Whereas, there is no tradeoff between $S \cdot C_{D_f}$ and $S \cdot C_{D_p}$. The relations are unclear between the other combinations of the objective functions. This figure shows that the
0th shape is one of the non-dominated solutions, and it is on the edge of the objective-function
space. Because two values of design variables to describe 0th shape, such as the span length
and sweepback angle of inboard wing, are near the edge of the defined design-variable range.

5.1. Comparison among the Extreme Solutions regarding the Objective Functions

The extreme solutions are three as the non-dominated solution A (NDS-A) for the mini-
mization of $S \cdot C_{D_f}$ and $I_{boom}$, NDS-B for the minimization of $S \cdot C_{D_f}$ and $W_c$, and NDS-C for the maximization of $S \cdot C_L$. It is notable that the individuals without fulfillment of structural
requirements was excepted from the candidates. The planform shapes are shown in Fig. 8,
and their geometrical characteristic values are summarized in Table 1. AR, $S_{wetted}$ denote the aspect ratio, wetted area of one-side wing, and number of ply (for the skin of out-
board wing, the skin of inboard wing, and the rib of inboard wing), respectively. AR defines $b^2/(S \cdot 2)$, $b$ denotes a full span length.

NDS-A shows that large swept angle of leading edge holds the front-edge boom and the
shock wave. Therefore, the minimizations of $S \cdot C_{D_f}$ and $I_{boom}$ are simultaneously achieved. But, because eigenvalue of twist first mode becomes low, the number of ply should be stacked for the skin of inboard wing. NDS-B reveals that the smallest wing area achieves the mini-
mization of $S \cdot C_{D_f}$, and the smallest wing area and the low swept angle of leading edge realize
the minimization of $W_c$. But, as the angle of attack at supersonic cruising condition must be
high to secure the target $C_L$, the separation might be triggered. Furthermore, high landing
speed must be also secured, because of the small wing area. NDS-C shows that the large wing
area achieves the maximization of lift at subsonic condition. But, the drags and $W_c$ become
high due to the large wing area. Consequently, as these extreme solutions cannot design prac-
tically, a compromise solution would be decided using the knowledge in the design space.
But, as these three extreme solutions show the physically optimum planform geometries,
the hybrid method efficiently explores the design space using a small number of the individuals
and the generations.

5.2. Comparison between 0th Shape and Selected Compromise Solution

The 75 non-dominated solutions are narrowed down to determine the compromise solu-
tion for the prototype of the 2nd shape. The applicable solutions to the following conditions
are firstly excluded from derived 75 non-dominated solutions; 1) The structural requirements
are not fulfilled, 2) $S \cdot C_L$ is low, or wing area is low (this means the constraint for the land-
ing speed.), 3) $S \cdot C_{D_f}$ and $S \cdot C_{D_p}$ are impractically large. As a result of this operation, 24
non-dominated solutions as the practical designs are sorted. The compromise solution was
determined from these individuals taking the balance of the five objective functions and the
low-boom competence as the primary objective of $S^3$TD into consideration.
Fig. 7 Derived non-dominated solutions on two dimensional planes between the objective functions. The orange arrow denotes the optimum direction.

(a) $S \cdot C_{Dy}$ vs. $S \cdot C_{Df}$
(b) $S \cdot C_{Dy}$ vs. $S \cdot C_L$
(c) $S \cdot C_{Dy}$ vs. $I_{boom}$
(d) $S \cdot C_{Df}$ vs. $S \cdot C_L$
(e) $S \cdot C_{Df}$ vs. $I_{boom}$
(f) $S \cdot C_L$ vs. $I_{boom}$
(g) $S \cdot C_L$ vs. $W_c$
(h) $I_{boom}$ vs. $W_c$

Fig. 8 Comparison among the wing planform of the extreme solutions colored by $C_p$ distribution on CFD and displacement on CSD.

(a) NDS-A
(b) NDS-B
(c) NDS-C
The comparison of the planform between the 0th shape and the selected compromise solution (called as ‘compromise’) is shown in Fig. 9. Also, their airfoil shapes of 0th and compromise near the junction relative to the fuselage, kink, and tip are shown. It is notable that 0th shape has no twist and its airfoil is described by NACA64A series. The installed angle of wing is 
\( \theta \) deg relative to the fuselage. Their characteristics and performance are summarized in Tables 2 and 3. As \( S \cdot C_L \) is the maximization objective, compromise has a larger wing area than that of 0th shape. And, inner wing area of compromise becomes large to secure the structural strength. The sweepback angle has more gentle not to give the effects on \( I_{boom} \) so that the wing area and strength are also secured. But, the chord length near kink becomes short to achieve low \( W \) and \( S \cdot C_D \). Therefore, the number of ply increases to augment the eigen frequency. Compromise has the supersonic leading edge near root to reduce the effect on \( I_{boom} \) of the front boom. Also, compromise has the blunt leading edge near kink to improve the strength, eigen frequency, and subsonic aerodynamic performance. The sharp leading edge near tip gives effect on \( I_{boom} \).

The location in the design space for 0th shape and compromise is shown in Fig 7. 0th shape has low \( S \cdot C_D \) and \( S \cdot C_L \) due to a small wing area, and it locates on the edge in the design space. COMPROMISE locates low \( I_{boom} \) region as well as compromises of the other objectives. These location in Fig. 7 shows the aerodynamic performance strictly depends on the wing area. The airfoil shapes of the compromise solution in Fig. 9 show that twist angle on outer wing is large. Outboard is not worked as wing, and down force occurs. \( C_D \) becomes still larger due to inverted camber line near the kink. As the design variables regarding twist down and inverted camber line give no effects on the objective functions, the re-design of these design variables can improve the aerodynamic performance without corrupting the other objectives. The design knowledge regarding the airfoil shape is insufficient. The primary reason is that its effect is weak to the aerodynamic performance compared with the wing planform. The secondary reason is that only a small number of the generations was evolved.

### Table 2 Comparison of geometrical characteristic values between 0th shape and compromise solution.

<table>
<thead>
<tr>
<th>Individual</th>
<th>( C_L )design</th>
<th>( \alpha )cruise [deg]</th>
<th>AR</th>
<th>( S^0 )wing [m(^2)]</th>
<th>( N^0 )out/skin( / N^0 )in/skin( / N^0 )rib</th>
</tr>
</thead>
<tbody>
<tr>
<td>0th shape</td>
<td>0.132</td>
<td>2.33</td>
<td>3.81</td>
<td>10.41</td>
<td>8/72/24</td>
</tr>
<tr>
<td>compromise</td>
<td>0.0898</td>
<td>3.61</td>
<td>3.18</td>
<td>15.68</td>
<td>56/88/24</td>
</tr>
</tbody>
</table>

### Table 3 Comparison of the objective-function values between 0th shape and compromise solution.

<table>
<thead>
<tr>
<th>Individual</th>
<th>( S \cdot C_D ) (( C_D^0 ))</th>
<th>( S \cdot C_D )</th>
<th>( S \cdot C_L ) (( C_L^0 ))</th>
<th>( I_{boom} )</th>
<th>( W_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0th shape</td>
<td>0.0656 (0.0118)</td>
<td>0.0482</td>
<td>3.896 (0.7029)</td>
<td>0.800</td>
<td>88.74</td>
</tr>
<tr>
<td>compromise</td>
<td>0.1777 (0.0218)</td>
<td>0.0584</td>
<td>4.719 (0.5794)</td>
<td>0.676</td>
<td>214.58</td>
</tr>
</tbody>
</table>

Fig. 9 Comparison of wing shape colored by \( C_D \) and displacement distributions, and \( C_D \) distributions (red) and airfoil shapes (green) near the junction relative to the fuselage, kink, and tip.
6. Conclusions

The hybrid method between multi-objective particle swarm optimization and adaptive range multi-objective genetic algorithm has been developed and its performance has been measured by using three test functions, and it has been applied to a practical engineering design problem. The performance of each optimizer was measured by the convergence metric and the cover rate for the test functions such as three-dimensional DTLZ1, ZDT1 with noise, and TNK with noise. As a result, the hybrid method gives the best performance for the test functions with noise under the condition of a small number of population size and generations.

The hybrid method has been applied to a silent supersonic technology demonstrator wing design as a large-scale and real-world multidisciplinary design optimization problem. Consequently, 75 non-dominated solutions were efficiently obtained. Since the efficient exploration of the design space using the hybrid method gives the extreme solutions for each objective function, tradeoffs were revealed. Moreover, a compromise solution was acquired through the designers’ selection using design knowledge. The compromise solution improves in the intensity of sonic boom and lift performance through the efficient exploration of the design space using the hybrid method.

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References


