Tetrahedral Mesh Reduction Technique*

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Abstract
Mesh simplification (decimation) refers to reducing the number of vertices and elements in an initial mesh while preserving the appearance of the dataset. In either case, some degree of simplification may be required for various reasons, for example, reducing visualization and calculation times or reducing storage requirements. The goal of the algorithm discussed in the paper is to reduce the total number of tetrahedral elements in the volume meshes for acceleration of calculation processes and to test the feasibility of proposed solution. One of the decimation criterions proposed in this approach is a bending energy as a decimation metrics to select tetrahedral elements as the candidates for collapsing. The final step of our algorithm is improving a quality of the simplified mesh. For improving we apply interpolation approach based on radial basis functions. To interpolate the overall displacement, we use a volume spline and employ an approach that uses displacement of \(N\) control points as a difference between the original and deformed geometric forms of tetrahedral mesh elements.

Key words: Tetrahedral Mesh, Meshes Simplification, Mesh Reduction, Mesh Improvement, Bending Energy

1. Introduction

Finite element analysis (FEA), medical imaging, range scanning, large-scale visualization volumetric models are very common in the research area as computational vector field simulation. Commonly used mesh types of homogeneous polyhedral meshes are the tetrahedral meshes. It is a type of unstructured triangular volumetric data where the elements are tetrahedrals. Tetrahedron as the basic 3D primitive is easy to deform and to merge or subdivide. However, the initial models usually have too many tetrahedrals and the shape of many is poor, making physical simulation impossible. Simplification refers to reducing the number of vertices and tetrahedral elements while preserving the appearance of the dataset as much as possible. In either case, some degree of simplification may be required for various reasons, for example, reducing visualization times or reducing storage requirements. There are three commonly used simplification algorithms: vertex clustering, decimation, edge and tetrahedron contraction. Clustering algorithms are based on replacing a cluster of vertices in a mesh by a single representative vertex, and the mesh topology is updated. Decimation algorithms iteratively remove vertices or higher dimensional unit from a mesh. This method has been used mostly on triangular surfaces but for volumetric meshes it may not work effectively. Contraction algorithms collapse edges, faces or cells of a mesh to vertices, thereby eliminating a number of cells at a single operation. Tetrahedron contraction is the merging of the four vertices of the tetrahedron into a single vertex. The most basic simplification technique is edge collapse. Much work has been done for
simplifying triangular surface meshes while maintaining surface fidelity [2, 4]. There are some techniques for volumetric data simplification [5, 6, 7, 8]. A variety of local transformations can be used to improve the quality of individual elements of the mesh [9, 10, 11, 12]. FEA is conveniently performed on tetrahedral meshes [13]. Generally, the running time of a physical simulation is polynomial in the number of elements so reducing number of the mesh elements is the target of using simplification technique [14].

The final goal of the method discussed here is to reduce the total number of elements in the tetrahedral meshes by using the bending energy as the decimation criteria. Further post-processing will improve a tetrahedral mesh quality by using the transformation function.

The paper is organized as follows. In section 2 we introduce the overview of our method. Section 3 presents the simplification algorithm. In section 4 we give a short account of a space transformation and a volume spline. Applying the space mapping to mesh quality improvement is given also. Section 5 is devoted to experimental results of mesh simplification. In section 6 we discuss the mesh quality improvement of the simplified models.

2. Overview of the method

We apply the decimation approach, which takes all the four vertices of a tetrahedron and fuses them onto the barycenter $c$ of the tetrahedron [5]. In Figure 1, the illustration of the tetrahedron collapse technique is demonstrated. Tetrahedron contraction technique (TetFuse) presented in [5] is simple, intuitive and a lower bound on the decimation ratio per step is higher than in an edge collapse technique.

![Fig.1 The tetrahedron collapse technique. The initial configuration of neighborhood elements (left side). The grey color element is the tetrahedron for collapse. The same configuration after collapse of the tetrahedron, where $c$ is it's barycenter.](image)

In our approach, we use decimation criterions for mesh reduction (simplification) instead of using the initial and limiting values of control parameters (a scalar attribute error metrics and a scalar error tolerance, which are defined by the user) as proposed in [5]. Simplification process should be preserved to avoid possible dangerous situations:
- tetrahedral elements inversion;
- generation of poor shaped elements;
- self-intersection of the mesh boundary.

In [7] various cost functions to drive the edge-collapsing process are discussed and a technique to check and prevent the occurrence of intersections and inversions of the tetrahedral elements involved in a collapse process is presented.

Cases of element-boundary intersection can occur when an interior tetrahedron stretches through and over a concave boundary region. The inversion of tetrahedron can be implemented by calculating the volume of all tetrahedral elements in the set of...
neighborhood elements before and after simplification process. If volume of one of the element becomes negative, tetrahedral inversion occurs. After the collapse operation, it is possible for some tetrahedral elements that the moving vertex goes to the other side of the unchanged triangle. In this case the tetrahedron-candidate for decimation fails the consistency test.

In our algorithm, a simple ray-shooting method is implemented to prevent element-boundary intersection and checking the inversion of tetrahedrons is included in decimation process.

A key ingredient of our approach is to use the bending energy as the decimation criteria, nevertheless, various and different decimation criterions for simplification process and error metrics were developed and realized in our method. Let us note that the various combinations of decimation criterions can be also used. At this place, we introduce terms that are used in our algorithm.

2.1 Decimation criterions

Decimation criterions are used to decide if the tetrahedral element, which we call the element-candidate, can be deleted. Tetrahedral elements, which will be modified while decimation process, are called the non-collapsed element. In the original model, we consider a 1-ring neighborhood of the tetrahedral elements with a central element, which is called a ball (Figure 2(b)), and determine the element-candidate in it according to the decimation criteria. Three decimation criterions can be used in our algorithm.

**Minimal volume term.** Tetrahedron with the minimal volume can be selected as a candidate for decimation.

**Minimal aspect ratio term.** Tetrahedron with the aspect ratio (AR) closed to 0 can be selected as a candidate for decimation. In our case the AR is defined as a ratio between an inscribed sphere radius multiplied by 3 and a circumsphere radius of a cell.

**Minimal deformation term.** We propose to use the bending energy as an error/quality cost to select element-candidates for collapsing (see section 4 for details).

2.2 Error metrics

We use three measurements for the mesh quality estimation of elements after collapsing the tetrahedral element-candidate: stretch ratio (SR), aspect ratio factor (ARF), ball volume (BV). If there exists at least one non-collapsed tetrahedral element, which does not satisfy a given threshold value, then simplification is not proceeded. Below we give a short description of these metrics.

SR is the maximum allowed value of the ratio between the distance from centroid of the base triangle to barycenter of the non-collapsed tetrahedron in original mesh and the same distance in this tetrahedron after collapsing the tetrahedral element-candidate. If the value of this ratio overcomes some threshold value, we do not produce contraction.

ARF is the ratio between the AR of the non-collapsed element before decimation and the AR of this element after collapsing the tetrahedral element - candidate. If at least one non-collapsed element after collapsing the element-candidate has the value of the ARF higher than given threshold value, decimation operation is not produced for the chosen candidate.

BV, as a volumetric error, is defined as a ratio of the volume of the each element of the ball to the total volume of the ball elements after collapse process. It is defined as $\frac{\text{volume}(k)}{\sum_{i} \text{volume}(k_i)}$, where $\text{volume}(k)$ is the volume of each element in the ball after simplification process. If the error is less than the measure of tolerance defined by the end
user, we execute decimation.

3. Basic steps of simplification algorithm

In this section we present the main simplification algorithm steps. In Figure 2(a) the sketch of our algorithm is demonstrated. The choice of the tetrahedral elements for further collapsing is done as follows:

1. Boundary points extraction.
   Any boundary tetrahedrons can not be collapsed and interior tetrahedrons that affected boundary tetrahedrons can not be selected as collapsed ones.

2. Estimation the inner elements.
   - consider the ball for each tetrahedron \( i \) as the center;
   - neighbor elements of tetrahedron \( i \) in the ball are checking according to decimation criterions;
   - determine a contractible (element-candidate) tetrahedron \( j \), which satisfies decimation criterions:
     - the minimal deformation, minimal volume or minimal aspect ratio.

3. Topological reconnection.
   - contracting tetrahedron \( j \);
   - determining the non-collapsed tetrahedrons \( k \) - the neighbors of tetrahedron \( j \). Testing tetrahedrons \( k \) in the relation to error metrics SR, ARF, and BV. If the values of these metrics overcome threshold values the element contraction is not produced. Testing tetrahedral element inversion and element- boundary intersection.
   - updating the position of tetrahedrons \( k \).
   - creating the new data structure.

![Fig.2. (a) Sketch of the algorithm. (b) Pair of tetrahedrons in the ball. The white element is an element-candidate \( j \), the dark grey is an arbitrarily chosen element \( i \).](image)

For each tetrahedron \( j \) in the current mesh the number of neighbors for the final vertex position is determined on whether tetrahedron \( j \) has a primitive (vertex, edge or face) on the
boundary of the mesh. If tetrahedron \( j \) has a single face (triangle) or even an edge on the boundary, then problem becomes one of surface simplification. In our present implementation we do not consider this case.

When tetrahedron \( j \) is contracted to a final vertex (barycenter) than tetrahedron \( j \) disappears. All faces and edges of the neighbors of tetrahedron \( j \) are degenerated. Positions of the vertices of non-collapsed elements are modified. The initial mesh topology is changed and some tetrahedrons, which previously were uncontractible, may become contractible and so on.

4. Minimal deformation

In our implementation, we apply a specific decimation metrics. We propose using minimization of the bending energy \( h^T A^{-1} h \), where \( A^{-1} \), the bending energy matrix, is the inverse \( N \times N \) upper left submatrix of \( T \) (see below), which is used as an error/quality cost to select candidates for collapsing. \( h \) is the difference between the coordinates of the initial and possible point placements. The formula for the bending energy – the formula whose value is proportional to that integral of summed squared second derivatives – is a quadric form (usually written \( A^{-1} \)) determined by the coordinates of the landmarks of the reference form. That is, if \( h \) is a vector describing the heights of a plane above a set of landmarks, then bending energy is \( h^T A^{-1} h \). In morphometrics, the bending energy of a general transformation is the sum \( x^T A^{-1} x + y^T A^{-1} y \) of the bending energy of its horizontal \( x \)-component, modeled as a “vertical” plate, and the bending energy of its vertical \( y \)-component, modeled similarly as a “vertical” plate.

The centroid of a non-collapsed tetrahedron is considered as a point that can slide to a new position after a decimation step. The selection of the tetrahedral element-candidate is made according to the bending energy. We exploit a simple idea that the more smoothly we transform the points to the new positions, fewer residuals there will be between the original and subsequent meshes. In this step, we form a list of cell elements to be contracted; this list contains a number of candidate cells. In the contraction step we eliminate processing of cells that can be contracted twice or more.

4.1 Space mapping and the volume spline

Here, we shall give a short account of calculation of matrix \( T \) elements and a space transformation (space mapping) method used in the applications considered in this section (for further references, see [3]). To interpolate the overall displacement, we use the volume spline based on radial basis functions (RBFs) and employ an approach that uses displacements of \( N \) control points as the difference between the original and deformed geometric forms.

Space mapping in \( R^n \) defines a relationship between each pair of points in the original and deformed objects. Let an \( n \)-dimensional region \( \Omega \subset R^n \) of an arbitrary configuration be given, and let \( \Omega \) contains a set of arbitrary control points \( \{q_i = (q_{i1}, q_{i2}, \ldots, q_{in}) : i = 1, 2, \ldots, N \} \) for a non-deformed object, and \( \{d_i = (d_{i1}, d_{i2}, \ldots, d_{in}) : i = 1, 2, \ldots, N \} \) for the deformed object. \( N = l + m \), where \( l \) is the number of elements in the ball, \( m \) is the number of the boundary nodes of the ball. It is assumed that the points \( q_i \) and \( d_i \) are distinct and given. \( q_i \) and \( d_i \) are the centroids of the non-collapsed tetrahedral elements before and after collapse operation. The goal of the construction of the deformed object is to find a smooth mapping function that approximately describes the spatial transformation applied to the
ball. In the case of mesh simplification, we consider the deformed object as the non-collapsed tetrahedron after a decimation step. Figure 3 illustrates the construction of the deformed object in simplification algorithm.

Fig. 3 An illustration of construction of the deformed object. 2 joint elements \( i \) and \( j \) in the ball. \( q \) is the starting centroid of a non-collapsed tetrahedron \( i \), \( d \) is the centroid of the tetrahedron \( i \) after collapsing the tetrahedron \( j \).

The inverse mapping function that is needed to produce the transformation is given in the form:

\[
q_i = f(d_i) + d_i, \quad (1)
\]

where the components of the vector \( f(d_i) \) are volume splines interpolating displacements of initial points \( q_i \).

In 2D case a mapping function is the thin plate interpolation [1], which minimizes the bending energy

\[
\iiint \left( \frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 \, dx \, dy.
\]

In 3D case the volume spline \( f(P) \) having values \( h_i \) at \( N \) points \( P_i \) is the function written as

\[
f(P) = \sum_{j=1}^{N} \lambda_j \phi(|P - P_j|) + p(P), \quad (2)
\]

where \( p = v_0 + v_1x + v_2y + v_3z \) is a degree-one polynomial (coefficients \( v \) correspond to the affine part of the transformation), \( N \) is the number of data points, \( \lambda \)'s are the interpolation coefficients to be determined and \( \phi(P-P_j) = \phi(r) = r^2 \) are RBFs, \( r \) is the Euclidean distance between two points. \( \lambda \) and \( v \) are the coefficients that satisfy the linear system \( Tx = b \).

\[
T = \begin{bmatrix} A & B^T \\ B & D \end{bmatrix}, \quad x = [\lambda_1, \lambda_2, ..., \lambda_N]^T, \quad b = [h_1, h_2, ..., h_N, 0,0,0,...0]^T.
\]

To solve for the weights \( \lambda_j \) we have to satisfy the constraints \( h_i \) by substituting the right part of Equation (2), which gives

\[
h_i = \sum_{j=1}^{N} \lambda_j \phi(|P_i - P_j|) + p(P_i), \quad (3)
\]

Solving for the weights \( \lambda_j \) and \( v_{0,1,2,3} \) it follows that in the most common case there is a double bordered matrix \( T \), which consists of three blocks, square sub-matrices \( A = [\phi(P_i - P_j)] \) and \( D = 0 \) of size \( N \times N \) and \( 4 \times 4 \) respectively, and \( B = [p(P_i)] \), which is not necessarily square and has the size \( N \times 4 \). Notice that, once we have the Gauss \( L' \) (or Householder...
QR) decomposition of $T$, we solve with three right-hand sides to obtain $h$-values for $x, y, z$-directional transformations. It is important to point out that to prevent a rigid motion of the ball, boundary conditions as zero displacements of nodes adjustment to the contracted cell are used.

4.2 Applying the space mapping to mesh quality improvement

There is no room to discuss details of the mesh improvement algorithm. Interpolation of $(x,y,z)$ points is implemented in $R^3$ and defines a relationship between the coordinates of points in the original and deformed objects according to formula (1). In mesh improvement case, we consider the deformed (potential) object as the tetrahedron that is close to being equilateral. $q_i$ and $d_i$ are the centroids of the original and potential forms of tetrahedral elements.

In the original model, we consider a star, which is defined as a set of elements sharing a node/center, whose boundary forms a polygonal shell.

The basic steps of the algorithm for improving tetrahedral elements are as follows:

1. Constructing the potential forms (close to the ideal shape) of the elements of the star as shown in Figure 4.
2. Calculating the coordinates of the new center of the star and move the initial center $o$ to the new location obtained by the transformation function (1).

Fig. 4 An illustration of construction of the deformed object. 2 joint elements in the star. $o$ is the initial center of the star. Vectors show starting tetrahedron centroids $q_i$ and potential (destination) centroids $d_i$.

5. Experimental results

In Figure 5, we show the visual results of the simplification of the “Cube” model (Figure 5(a)), for various decimation criterions: the minimal deformation (Figure 5 (b, c)) and the minimal AR (Figure 5 (d)). We investigated their influence on the simplification process with the different values of the SR. The number of tetrahedrals in the initial mesh is 9436. In Figure 6, the AR histograms (the “Cube” model) are shown for 4 different decimation criterions after one decimation step. In the cases of the minimal deformation, the minimal volume, the minimal AR, the combination of the minimal AR and the minimal volume criterions 29%, 28%, 27%, and 34% of tetrahedral elements were deleted, respectively. From the mesh quality point of view the case with the minimal volume criterion is preferable for the “Cube” model because it has the best AR distribution in the AR range from the minimum value 0.0 to the maximum value 1.0. In Figure 7, we demonstrate the “Tetra” model (Figure 7(a)) and the results after one decimation step with using 4 decimation criterions. For this model the cases of the minimal AR and the combination of the minimal AR and the minimal volume are preferable according to the AR histograms of distribution (Figure 7(b)). In the cases of: the minimal deformation – 13%, the minimal
volume – 18%, the minimal AR – 13%, the combination of the minimal AR and the minimal volume – 19% of tetrahedral elements were deleted. For the “Tetra” model in the case of the combination of the minimal AR and the minimal volume criteria the experimental results show that the number of elements with the minimal AR is equal to 0. If the minimal deformation or the minimal AR criteria is chosen the number of elements with the AR closed to 1 are more than in other cases. In this case calculation time for the model in Figure 5 is about 6 sec. and for the model in Figure 7 is 0.04 sec. Figure 8 presents the “Mechanical part” model and the results after one decimation step with the control over self-intersection of the mesh boundary and four decimation criterions. The number of tetrahedral elements is 44862. The histograms of element distribution show that in the case of the minimal deformation, the quality of the mesh elements is higher than in other cases while 29% of the elements were deleted.

Fig.5 The “Cube” model. The initial mesh (a); The mesh after simplification: (b) Minimal deformation, number of elements: 4540 (48%), SR = 2.0; (c) Minimal deformation, SR =4.0, number of elements: 4520; (d) Minimal AR, SR = 2.0, number of elements: 4729 (50%). Courtesy of J. Dompierre et al.

Fig.6 The “Cube” model (9436 elements). 1 step of decimation. The AR histograms of distribution of tetrahedral elements for 4 decimation criteria cases. ((x) axis – the indexes of the intervals, where the AR values belong to range X = [0, 1]; (y) axis – the number of the mesh elements).
6. Improving the simplified mesh for modal analysis

Preparing volumetric models for simulation is a difficult task. The initial models can have huge number of tetrahedrons, and the shape of many of them is poor, as a result, physical simulation is not possible. NVH (Noise, Vibration and Harshness) analysis requires much time computing than stress analysis. Thus a key point in gaining efficiency in NVH analysis is to reduce the quality of the mesh while maintaining the correct mass and stiffness distributions and do not deviate from the original computer aided design (CAD) geometry. Detailed survey of mesh generation, simplification, improvement and refinement techniques can be found in [15]. Most of mesh simplification techniques do not necessarily improve the quality of the mesh elements. A new type of finite element p-element has gained acceptance in general FEA programs. p-element can directly represents curvature, resulting in simpler meshes containing fewer elements. Accuracy of the analysis is controlled by the p-level (polynomial) order assigned to each element. The p-element tends to be very resources intensive and computationally time consuming than h-element (traditional finite element)
within the condition of using the same number of elements. The mesh simplification with further improvement can be used for retrieving the accurate modal analysis results within an acceptable calculation time. Simulations on uniform density meshes, which elements have similar volume and nearly equilateral shapes, are accurate and stable, but slow, due to the high number of elements. After the mesh simplification the most elements of the mesh tend to have higher length of the edges. As a result value of AR is much worse than before process. For improving mesh quality and decreasing consuming time of the FEA it is necessary to use the mesh improving after simplification step. At this place, we demonstrate the results after mesh simplification with the minimal deformation criteria described above, which is combined with mesh improvement (see subsection 4.2) to increase the quality of tetrahedral elements for physical simulation such as the finite element method (FEM). In Figure 9, the simplified “Torus” model and the AR histograms of distribution before and after improving are presented. For this model, statistics of the simplified mesh (1) and for the improved mesh (2) are presented below: 

1. The maximal value of the AR: 85.174, M (the average AR) = 0.666, the number of tetrahedral elements: 7891 (30% reduction from initial number of elements: 11050), the minimal value of the AR=3.17e-006. 
2. The maximal value of the AR: 64.109, M=0.711, the number of the tetrahedral elements: 7891 (30% reduction), the minimal value of the AR=0.0016.

We can conclude that M after improvement is increased by 7% and the difference between the maximal and the minimal values of the AR is much smaller than for the simplified mesh. The AR histograms of the distribution elements ((x) axis - the indexes of the intervals, where the AR values belong to range X = [0, 1]; (y) axis – the number of the mesh elements) according to their quality presented in Figure 9 (a, b) show that after improving the distribution of the mesh quality parameter (AR) is sufficiently smooth. Our next
example of combined technique is connected to testing the simplified “Bed” model (see Figure 10) after 5 improvement steps. The number of tetrahedral elements: 153434 (30% reduction). In Table 1, we demonstrate the results of the tetrahedral element verification of the simplified “Bed” model after 5 improvement steps. For this model the experimental results show that in the case of the improved mesh the number of elements with better mesh quality parameters is increased about by 30%.

Table 1. The "Bed" model. MSC.PATRAN

<table>
<thead>
<tr>
<th>Mesh quality parameters</th>
<th>#of el. Simpl.mesh</th>
<th>#of el. Impr. mesh</th>
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<tr>
<td>AR&gt;10</td>
<td>1301</td>
<td>816</td>
</tr>
<tr>
<td>AR&gt;20</td>
<td>482</td>
<td>240</td>
</tr>
<tr>
<td>AR&gt;50</td>
<td>100</td>
<td>53</td>
</tr>
<tr>
<td>AR&gt;90</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Edge angle&gt;90 deg.</td>
<td>1403</td>
<td>897</td>
</tr>
<tr>
<td>Edge angle&gt;75 deg.</td>
<td>3936</td>
<td>2932</td>
</tr>
<tr>
<td>Edge skew&gt;75 deg.</td>
<td>517</td>
<td>267</td>
</tr>
<tr>
<td>Edge skew&gt;60 deg.</td>
<td>5233</td>
<td>4295</td>
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<tr>
<td>Collapse</td>
<td>154</td>
<td>83</td>
</tr>
</tbody>
</table>

In Table 2, we demonstrate the calculation time of the eigenvalues analysis of the initial “Bed” model, the initial “Bed” model after improving, and the simplified “Bed” model after improving. In the initial “Bed” model some elements have been detected to have zero volume. These elements are then removed in order to complete the analysis (topologically modified initial model is used). The improved model can be used for numerical calculations without any modifications. The simplified “Bed” model after improving and the initial model after improving show almost the same eigenvalues (are not introduced here), however, the calculation time is decreased by above 30%. The decreasing the calculation time means increasing the number of analysis cycles which can be completed within the same amount of the time.

Table 2. The "Bed" model

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Model</th>
<th>Calculation time (s.)</th>
<th>#of el.</th>
</tr>
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<tr>
<td>1</td>
<td>Initial “Bed”</td>
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<td>205564</td>
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<td>2</td>
<td>Improved “Bed”</td>
<td>635</td>
<td>205564</td>
</tr>
<tr>
<td>3</td>
<td>Simplified “Bed” after improvement</td>
<td>456</td>
<td>153434</td>
</tr>
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</table>

Conclusions

In this paper, we present the mesh reduction algorithm for tetrahedral meshes to reduce the large amount of 3D datasets. After investigation of some algorithms based on edge collapse decimation methods we have applied the mesh decimation approach, which was introduced in the paper [5]. The idea of this approach is simple (all the four vertices of a tetrahedron are fusing onto the its barycenter) and the approach is better suited for 3D volumetric meshes than edge collapse-based methods because of a symmetrical distribution of the volume of all the decimated tetrahedra amongst its neighbors. Now we do not have a
ready-made software for testing existing algorithms so we can not demonstrate in this paper the comparison results with other algorithms.

Current algorithm is basically realized for the case of minimal volume, minimum aspect ratio, and minimal deformation criterions, however, various combinations with decimation metrics, which are not discussed here, were realized. Some well-known problems of inversion and self-intersection for concave cases were solved. Deleted elements by simplification technique are replaced by one node – barycenter, thus the adjacent elements is lengthening symmetry along the model. This is certainly better for the FEM calculation. Three constrains are considered: stretch factor, volume factor and aspect ratio factor. We investigate an influence of the different constrains, but presently we can state that the constrains used together are very severe and it is better to select each of them for a given model. However, our experiments with different models clearly show that results of decimation (quality of the mesh) depend on the original model. Thus, we can not state beforehand which decimation metrics is preferable for the given model but approach based on implementation of the minimal deformation criteria is effective and provides fewer residuals between the initial and final meshes. For considered examples the method can produce reduction of tetrahedral model by 30%. The experimental results amply show the applicability of the approach. So far we implement improvement steps after decimation steps. We plan to extend our approach to incorporate improvement steps in decimation algorithm. The software that includes the mesh simplification algorithm with the minimal deformation criteria and the mesh improvement algorithm using RBFs - based interpolation approach can be an effective tool in mesh processing applications. The combined technique (p-element mesh refinement, tetrahedral mesh simplification, and mesh improvement) allows attaining meshes suitable for FEM with less calculation time (about 30%) than in the case of h-refinement within similar accuracy.

Acknowledgements

We appreciate Grant-in-Aid for Scientific Research (S) (No.20226006) of The Ministry of Education, Culture, Sports, Science and Technology, titled "Research on the Manufacturing Method and Creating the Function of New Lightweight Core Structure by Fusion of the Origami Engineering and the Computational Mechanics".

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