Deformation Modes for Axial Crushing of Cylindrical Tubes Considering the Edge Effect

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Abstract
In this paper, the elastoplastic deformation behaviors of cylindrical tubes subjected to statically axial compression are studied by using finite element method (FEM). Specifically, the effects of tube geometries and strain hardening are investigated. Although it is generally recognized that the deformation in the circumferential direction is dependent on the ratio of the radius to thickness \( R/t \), the deformation is also greatly dependent on the edge constraint. In this study, we used flanges as an edge constraint. The deformation mode in the circumferential direction also affects the deformation in the axial direction. A method to control the deformation mode, such as adding a disk in the tube center, is proposed to maintain the deformation in the axisymmetric mode.

Key words: Plasticity, Axial Collapse, Cylindrical Tube, Buckling, Deformation Mode, Finite Element Method

1. Introduction

Cylindrical tubes are used as energy-absorbing components in automobiles. In order to evaluate the energy-absorption characteristics, it is vital to understand the deformation behaviors of cylindrical tubes subjected to axial compression. Such deformation behaviors have been studied for a long time(1)∼(8), including the landmark study by Jones. According to these studies, when a relatively short cylindrical tube is subjected to axial compression, it will progressively fold into either axisymmetric concertina shaped folds or non-axisymmetric shaped folds depending on the ratio of the radius to wall thickness \( R/t \). Generally, relatively thin tubes \( (R/t > 40) \) deform into non-axisymmetric shaped folds and thick tubes \( (R/t < 25) \) deform into axisymmetric folds. Some tubes first deform into axisymmetric folds but then revert to non-axisymmetric shaped folds as collapse progresses(8). In addition, when a long cylindrical tube is subjected to axial compression, it will globally bend in a strut-like manner. Andrews(4) presented the results of an experimental investigation of the axial crushing modes of quasi-statically compressed aluminum alloy tubes and classified into four deformation modes which occur due to the tube geometry:

(a) Concertina type (axisymmetric)
(b) Diamond type (non-axisymmetric)
(c) Euler type (global bending)
(d) Mixed type (mix of concertina type and diamond type)

On the contrary, Abramowicz(7) classified deformation modes in the axial direction into the bending type and progressive type, and showed these deformation modes as functions of...
$L/D$ (ratio of cylinder length to diameter) and $D/t$ (ratio of diameter to thickness). By using the classifications of deformation modes provided in both of these studies, one can predict the deformation modes for axial crushing of arbitrary cylindrical tubes due to the geometrical properties of the tube.

Other studies have examined the influence of the material properties. The axisymmetric type of deformation occurs more often than the non-axisymmetric type when the work-hardening coefficient is small\(^9\). In the case of a long cylinder (large $L/D$), Euler type buckling (global bending) occurs more often when the work-hardening coefficient is small\(^7\).

However, the origin of the mixed type described by Andrews\(^4\) and the transition from the progressive folding type to bending type, which Abramowicz\(^7\) described, have not yet been revealed. In this study, the effects of the material and geometrical properties of a cylindrical tube subjected to static axial compression are investigated by an elastoplastic numerical analysis using the finite element method (FEM). Moreover, the influence of a flange attached to both ends of a cylinder is investigated to establish guidelines for the development of energy-absorbing components in automobiles using cast aluminum alloy. Therefore, by examining the influences that control the deformation mode, this study contributes to future guidelines.

### 2. Analytical method

The commercial FEM analysis package MSC.Marc\(^10\) is used in this study to analyze large elastoplastic deformations of the cylindrical tubes shown in Fig. 1. Cylinders were formed with flanges at both ends, and one flange end was completely fixed to a rigid floor. Axial compression was applied from above by modeling a rigid body moving downward under displacement control. The effects of various geometric parameters, such as tube length $L$, tube thickness $t$, and tube radius $R$, on the deformation mode were investigated. The value of flange thickness $t_f$ was set to five times $t$ as the basic geometry. The coefficient of friction $\mu$ between the tubes and the rigid body was set to 0.03, because friction at the contact boundary was not considered.

The cylinder material used in the analysis was assumed to be homogeneous and isotropic elastoplastic material that conforms to von Mises yield conditions. In addition, the material could be approximated by the bilinear hardening law described by the following stress-strain relation:

$$\sigma = \sigma_y + E_h \left( \varepsilon - \frac{\sigma_y}{E} \right) = \frac{E}{E - E_h} \sigma_y + E_h \varepsilon$$

where $E$ is Young’s modulus, $E_h$ is the work-hardening coefficient, and $\sigma_y$ is the yield strength.

In this study, it was assumed that Poisson’s ratio $\nu = 0.3$, $E = 72.4$ GPa, and $\sigma_y = 0.001E$. The influence of the material properties on the deformation mode of the cylinder was investigated in terms of $E_h$.

In this study, the updated Lagrange method was used to formulate the geometric nonlinear behavior, and the algorithm based on the Newton-Raphson method and the return-mapping method were used to solve the nonlinear equation. The cylinders were modeled using four-node quadrilateral thickness shell elements (Element type 75). The elements were 2.5 mm long in the axial direction, and divided the circumference into 45 sublengths.

Compressive stress $\sigma_x$, which is used for following discussion of the analysis result, is an axial compressive load $P_x$ divided by the actual cross section $A$ of each cylinder, i.e.

$$\sigma_x = \frac{P_x}{A}$$

### 3. Results and discussion

In this study, cylindrical tubes with various geometric properties and material properties were studied by using FEM numerical analysis. As a result, we confirmed the various deformation shapes investigated by other researchers\(^1\)–\(^9\) and further classified the deformation modes.
The deformation modes can be firstly classified from the viewpoint of the axial and circumferential direction. Figure 2 shows typical deformation modes, classified from the viewpoint of the axial direction. As shown in Fig. 2, the deformation modes for axial crushing of cylindrical tubes are classified according to whether the cylindrical tube maintains an upright shape and crushed with progressive local buckling (wrinkling) along entire length. The deformation shapes which can be crushed along entire length (Fig. 2(a)) are called the stable type here, and the deformation shapes which cannot be crushed along entire length because of bending (Figs. 2(b) and (c)) are called the unstable type here.

By extending the classifications of deformation modes in the circumferential direction proposed in other studies(2)∼(8), the stable type can be further classified into the axisymmetric mode (see Fig. 3(a)), non-axisymmetric mode (see Fig. 3(b)), and mixed mode, which changes from the axisymmetric to non-axisymmetric mode (see Fig. 3(c)).

In this study, Figure 4 shows that the cross-sectional shapes of cylindrical tubes in non-axisymmetric deformation become multi-angular shapes, in which \( n = 2, 3, 4, 5, 6 \) (where \( n \) is the number of corners in the circumferential direction).

Figures 5(a) and (b) show the cross-sectional shapes for various tube thicknesses \( t \), given two hardening coefficients \( E_h/E \) of 0.01 and 0.05, respectively. In Figs. 5(a) and (b), the fr...
mark represents the axisymmetric shapes. Besides, the ||, △, ◆, ★ and ♠ marks represent the non-axisymmetric shapes of \( n = 2, 3, 4, 5 \) and 6, respectively. The logarithmic abscissa axis in Figs. 5(a) and (b) shows relative thickness \( t/R \), and the vertical axis shows nondimensional axial distance \( s/R \) (where \( s \) is the distance from the position of each wrinkle to the cylinder end of initial deformation). Figures 5(a) and (b) confirm that, as indicated in another study\(^5\), the shape of the wrinkle greatly depends on the ratio \( t/R \); as \( t/R \) decreases, the number of corners \( n \) in the cross-section of the non-axisymmetric shapes, increases. It is apparent from the previous study\(^5\) that an axisymmetric wrinkle occurs when the relative thickness is large (\( t/R > 0.04 \)). However, in our results, instead of an axisymmetric wrinkle, a non-axisymmetric wrinkle of \( n = 2 \) occurs when the relative thickness \( t/R \) is large (\( t/R > 0.048 \) for \( E_h/E = 0.05; t/R > 0.084 \) for \( E_h/E = 0.01 \), see Fig.5). This is because the deformation mode depends on the flanges at both ends of the cylinder. As can be seen from Figs. 5(a) and (b), even if relative thickness \( t/R \) is the same, as the distance from the cylinder end \( s/R \) increases, namely, as the location of wrinkling moves farther from the flange, the deformation shape changes from an axisymmetric wrinkle to a non-axisymmetric wrinkle. It is assumed that this is because the flanges at both ends of the cylinder constrain the displacement in the circumferential direction. Moreover, the cylindrical tube with \( t/R = 0.072 \) (\( R = 25 \text{ mm}, t = 1.8 \text{ mm} \)) in Fig. 5(b) shows the axisymmetric mode due to the flanges at both ends. The value of the flange thickness \( t_f \) that corresponds to the numerical analyses shown in Fig. 5(b) for \( t/R = 0.072 \) is 9 mm, and Fig. 6(a) shows the deformation shape. Figure 6(b) shows the deformation shape when the flange thickness \( t_f \) is set to be 3.6 mm. As can be seen from the figure, a non-axisymmetric wrinkle of \( n = 2 \) occurs after four axisymmetric wrinkles (three wrinkles from the upper end and one wrinkle from the bottom end). This result means that it is difficult to generate an axisymmetric wrinkle without the constraint of the flanges.

It is concluded from Fig. 7 that the flanges affect the cross-sectional shapes. Figure 7 shows how the deformation shape changes when the flange thickness \( t_f \) is 5 mm or 2 mm, given hardening coefficient \( E_h/E \) of 0.05, cylindrical radius \( R \) of 25 mm, and cylindrical thickness \( t \) of 1 mm. When the axial compressive displacement \( U_x \) imposed by the rigid body is 35 mm, the deformation mode of \( t_f = 5 \text{ mm} \) is \( n = 3 \), and the deformation mode of \( t_f = 2 \text{ mm} \) is \( n = 2 \). Both Figs. 6 and 7 indicate that weakening the constraint in the circumferential direction at edges easily generates a wrinkle of \( n = 2 \). Moreover, when the axial compressive displacement \( U_x \) imposed by the rigid body is 90 mm, the deformation mode of \( t_f = 5 \text{ mm} \) is the stable type, and the deformation mode of \( t_f = 2 \text{ mm} \) is the transition type. Therefore, the constraint of the cylinder end also affects the deformation mode in the axial direction.
Fig. 5 Location of deformation for tubes with $L = 200$ mm and (a) $E_h/E = 0.05$; (b) $E_h/E = 0.01$ ($\bigcirc$: axisymmetric mode, $\parallel$: $n = 2$, $\triangle$: $n = 3$, $\diamond$: $n = 4$, $\star$: $n = 5$, $\ast$: $n = 6$)

Fig. 6 Dependence of flange thickness on the deformation mode with $L = 200$ mm, $R = 25$ mm, $t = 1.8$ mm, $E_h/E = 0.01$ and (a) $t_f = 9$ mm; (b) $t_f = 3.6$ mm
Fig. 7 Dependence of the flange thickness on the deformation mode with $L = 150$ mm, $R = 25$ mm, $t = 1$ mm, $E_h/E = 0.05$ and (a) $t_f = 5$ mm; (b) $t_f = 2$ mm

It is found from the comparison of Fig. 8(a) and Fig. 8(b) that as the hardening coefficient $E_h/E$ increases, the deformation mode tends to be non-axisymmetric. Figures 8(a) and (b) show the wrinkle shape seen in the axial direction, given the same cylinder geometry ($L = 200$ mm, $R = 25$ mm, $t = 1$ mm) and two hardening coefficients, $E_h/E$ of 0.01, 0.05, respectively. As can be seen from Fig. 8(a), in the case of the large hardening coefficient $E_h/E = 0.05$ at axial compressive displacement $U_x = 30$ mm, a non-axisymmetric wrinkle of $n = 3$ (equilateral triangle) occurs, and the triangle progressively folds by rotating 60 degrees in the circumferential direction. In contrast, in the case of the small hardening coefficient $E_h/E = 0.01$ at axial compressive displacement $U_x = 30$ mm, an axisymmetric wrinkle occurs, and at axial compressive displacement $U_x = 80$ mm, a non-axisymmetric wrinkle of $n = 3$ occurs for the first time.

According to the above explanation, the non-axisymmetric mode is more likely to occur when the cylinder length is long because the constraint of the flange is weaker in the middle of the cylinder. Therefore, in the mixed mode, which changes from axisymmetric to non-axisymmetric, an axisymmetric wrinkle is generated at the beginning because of the constraint of the flange, even if the generation of an axisymmetric wrinkle is difficult due to the cylinder geometries and material properties; then, a non-axisymmetric wrinkle is naturally generated because the constraint of the flange is weaker with the spread of the wrinkle.

However, it is possible to control the deformation mode by adding a disk or a ring in the middle position of the cylinder. Figure 9 compares the deformation shapes for (a) the cylinder without a disk and (b) the cylinder with a disk (thickness: 4 mm) in the middle for a cylinder with $L = 200$ mm, $R = 25$ mm, $t = 1$ mm, $E_h/E = 0.01$. As can be seen from the figure, although the non-axisymmetric mode occurs at a location far from the flange in Fig. 9(a), the non-axisymmetric mode does not occur in Fig. 9(b).
According to previous studies, the average compressive stress $\sigma_{x \text{ ave}}$ in the crushing process, which represents the energy-absorbing properties for axial crushing, depends on the deformation mode. Therefore, some approximations for the axisymmetric or non-axisymmetric deformation modes were separately proposed \cite{2,9-13}. In contrast, another report \cite{5} states that the average compressive stress is only a function of the ratio of the thickness and radius $t/R$, independent of the deformation mode. In all of these studies, the average compressive stress was investigated by considering the various deformations corresponding to the cylinder geometry. Despite the studies’ detailed arguments of the relation of the average compressive stress and the deformation mode being cut through because of the paper length, the change of the average compressive stress in the case of the mixed mode, which changes from axisymmetric to non-axisymmetric, is examined in Fig. 10. Figure 10 shows the relationship between axial compressive stress $\sigma_x$ and axial displacement $U_x$, and the deformation shapes in the crushing process for mixed mode with $L = 200$ mm, $R = 25$ mm, $t = 1$ mm, $E_h/E = 0.01$. It is found from the value of the average compression stress (dotted lines in the figure) in each region of the axisymmetric mode and non-axisymmetric mode that the average compression stress decreases when the deformation mode changes from axisymmetric to non-axisymmetric. With respect to this explanation, the mixed mode changing from axisymmetric to non-axisymmetric occurs, but the deformation mode changing from non-axisymmetric to axisymmetric does not occur.

Figure 11 compares the average compression stress between the axisymmetric mode and mixed mode. As can be seen from this figure, the average compression stress for both hardening coefficients $E_h/E = 0.05$ and $E_h/E = 0.01$ is always higher in the axisymmetric mode.
than in the mixed mode. This result means that it is difficult to generate an axisymmetric wrinkle without the constraint.

![Load-deflection curve for tube collapsed under mixed mode](image)

**Fig. 10** Load-deflection curve for tube collapsed under mixed mode

![Comparison of average crush stress between the axisymmetric mode and mixed mode](image)

**Fig. 11** Comparison of average crush stress between the axisymmetric mode and mixed mode

Next, as shown in Figs. 2(b) and (c), the unstable type can be further classified into two different modes: the bending type (as referred to in this paper), which does not generate a local wrinkle but instead causes a sideways bend at the beginning, and the transition type (as referred to in this paper), which causes a sideways bend after generating some local buckling wrinkles. Abramowicz(7) showed the existence of a transition type like this when he examined the dynamic cylindrical deformation mode for axial crushing. Moreover, he showed that the cause of its generation was the decrease of the inertia force with the progress of dynamic axial compression. However, there is no influence by the inertia force in this study because a quasi-static analysis was conducted. It is considered that the transition type in quasi-static axial compression occurs by the change of the boundary condition due to the folding of a wrinkle at edges. In other words, the transition type necessarily generates some non-axisymmetric wrinkles before bending. A slight displacement in the horizontal direction occurs between the rigid body and the cylinder edge while generating a non-axisymmetric wrinkle. In this study, it was not confirmed that a cylinder bends by generation of only axisymmetric wrinkles.

Figure 12 shows the amount of horizontal displacement $U_y$ in the center of the cylinder...
end, given cylindrical thickness $t = 1$ mm, cylindrical radius $R = 25$ mm, hardening coefficient $E_h/E = 0.05$, and three cylindrical lengths $L = 150, 200, 400$ mm. These cylinders generate non-axisymmetric wrinkles of $n = 3$ after generating two or three axisymmetric wrinkles, and the amount of displacement $U_y$ gradually increases.

The deformation type of the cylinders with $L = 150$ mm and $L = 200$ mm becomes the stable type because of crushing along entire length before the amount of displacement $U_y$ becomes sufficiently large. In contrast, the deformation type of the cylinders with $L = 400$ mm becomes the transition type because the amount of displacement $U_y$ becomes sufficiently large before finishing the crushing. In these analyses, horizontal displacement of the upper end (as in the deformation mode for $L = 400$ mm and $U_x \approx 130$ mm in Fig. 12) is possible because the coefficient of friction $\mu$ was defined as 0.03 to disregard friction between the upper end and the rigid body moving downward. The cylinder of the unstable type for $L = 400$ mm in Fig. 12 was analyzed again by setting the coefficient of friction $\mu$ from 0.03 to 0.3 in order to restrain the horizontal displacement of the upper end. Figure 13 compares the deformation shapes for $\mu = 0.03$ and $\mu = 0.3$. It is seen in Fig. 13 that the deformation type for $\mu = 0.3$ becomes the stable type because of the restraint of the horizontal displacement of the upper end, in spite of the generation of the non-axisymmetric wrinkles as well as the case $\mu = 0.03$. From these results, it is concluded that the bending type is buckling of the Euler type when the boundary condition is fixed at both ends, and the transition type does not generate buckling of the Euler type because the cylindrical length is too short to generate Euler buckling when the boundary condition is fixed at both ends. In the transition type, the upper end part bends because of the shifting center of axial load by the generation of non-axisymmetric wrinkles.

![Fig. 12](image_url) Displacement $U_y$ of upper edge in horizontal direction
Fig. 13 Effect of the friction coefficient on the deformation mode

Fig. 14 Deformation map for tubes with (a) $E_h/E = 0.05$; (b) $E_h/E = 0.01$
It is found from the above investigation that cylindrical deformation modes for axial compression are strongly affected by not only the cylinder dimension (thickness, radius and length) but also the end conditions (flange thickness and frictional force).

Figures 14(a) and (b) show that the mode classification chart of the axial direction and the circumferential direction correspond to various combinations of $R/t$ and $L/R$, given the two hardening coefficients $E_h/E = 0.05, 0.01$ and two flange thicknesses $t_f/t = 5, 2$. The deformation modes of the axial direction are shown by the heavy line in the graph, and the region of the axisymmetric mode is shown by the thin line in the graph. As can be seen from the figures, as non-dimensional length $L/R$ increases, the deformation tends to become the unstable type, and as non-dimensional length $L/R$ decreases, the deformation tends to become the stable type. In addition, as non-dimensional radius $R/t$ increases, the deformation tends to become the transition type, and as non-dimensional radius $R/t$ decreases, the deformation tends to become the bending type. It is also found from the figures that the length of the stable type is the longest when $R/t$ is almost 23 for $t_f/t = 5, E_h/E = 0.01$. With respect to the influence of the hardening coefficient $E_h/E$, when the hardening coefficient $E_h/E$ is small ($E_h/E = 0.01$ in comparison with $E_h/E = 0.05$), the region of the stable type and axisymmetric mode is large. Figure 14 also shows that when the flange thickness is small ($t_f/t = 2$ in comparison with $t_f/t = 5$), the border of each deformation mode shifts from the solid line to the dashed line. Also, as can be seen in the figure, the flanges in the cylinder ends do not influence the bending type, but the flanges do influence the region of the stable type and axisymmetric mode because of the cross-sectional shape wrinkle. The region of the stable type and axisymmetric mode decreases with the decrease of the flange thickness.

4. Conclusions

In this paper, the investigation of the effects of the material and geometrical properties of a cylindrical tube subjected to static axial compression by using elastoplastic numerical analysis of the finite element method was presented. The following results became apparent during this investigation.

(1) Although it is usually assumed that the cross-sectional shape of a wrinkle with cylindrical axial compression depends on the ratio of the thickness and radius $t/R$ and an axisymmetric wrinkle occurs when $t/R$ is large, we showed the following different two points: (i) the cross-sectional shape of the wrinkle greatly depends on the constraint of the flanges in the cylinder ends as well as the ratio of the thickness and radius $t/R$; (ii) it is difficult to generate an axisymmetric wrinkle without the constraint of the flanges.

(2) In the mixed mode, an axisymmetric wrinkle is generated at the beginning because of the constraint of the flanges, and a non-axisymmetric wrinkle easily occurs because the constraint of the flange weakens with the spread of the wrinkle. Therefore, it is possible to control the deformation mode by adding a disk equivalent to a flange in the middle of the cylinder.

(3) It is considered that the transition type is generated by displacement at the ends in the horizontal direction with the generation of a non-axisymmetric wrinkle. Therefore, the transition type does not occur without a transition to a non-axisymmetric wrinkle after an axisymmetric wrinkle initially occurs.

(4) Concerning the influence of $E_h/E$, an axisymmetric wrinkle easily occurs when $E_h/E$ is small. In addition, the region of the transition type is small when $E_h/E$ is small.

(5) Flanges in the cylinder ends do not influence the bending type, and flanges in the cylinder ends influence the region of the stable type and axisymmetric mode through the influence of the cross-sectional shape of wrinkle. The region decreases with decreasing flange thickness.

References


