Evolutionary Strategies of Adaptive Plan System with Genetic Algorithm*

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Abstract

A new method of Adaptive Plan system with Genetic Algorithm called APGA is proposed to reduce a large amount of calculation cost and to improve a stability in convergence to an optimal solution for multi-peak optimization problems with multi-dimensions. This is an approach that combines the global search ability of Genetic Algorithm (GA) and the local search ability of Adaptive Plan (AP). The APGA differs from GAs in handling design variable vectors (DVs). GAs generally encode DVs into genes and handle them through GA operators. However, the APGA encodes control variable vectors (CVs) of AP, which searches for local optimum, into its genes. CVs determine the global behavior of AP, and DVs are handled by AP in the optimization process of APGA. In this paper, we introduce some strategies using APGA to solve a huge scale of optimization problem and to improve the convergence towards the optimal solution. These methodologies are applied to several benchmark functions with multi-dimensions to evaluate its performance. We confirmed satisfactory performance through various benchmark tests.

Key words : Adaptive System, Genetic Algorithms, Global-Local Search, Memetic Algorithms, Multi-Peak Problems

1. Introduction

Product design is becoming more and more complex because of various requirements from customers and their claims. As a consequence, the problems associated with product design seem to be multi-peak problem with multi-dimensions. It is necessary to solve the multi-peak optimization problems. Genetic Algorithm (GA)^{(1),(2)} is the most prominent emergent computing method that has been applied to various multi-peak optimization problems. The validity of this method has been reported by many researchers. Digalakis and Margaritis presented a review and experimental results on major benchmark functions which are used for performance and control of GA⁽³⁾. Sakuma and Kobayashi have proposed a real-coded GA using the combination of two crossovers, UNDX-m and EDX⁽⁴⁾, where the effect is reported through benchmark functions. Li and Kirley have introduced fine-grained parallel GA, which is derived from percolation theory⁽⁵⁾. Author group has reported the efficiency of GA-based approaches through various studies, including topology optimization for a mechanical structure^{(6)–(8)}, the development of a General-purpose Optimization Engine (GOE), and the application of the GOE to various optimal designs^{(9),(10)}. However, emergent computing methods require a huge computational cost to obtain stability in the convergence to an optimal solution.

To reduce the cost and to improve stability, a strategy that combines global and local search methods becomes necessary. As for this strategy, current research has proposed various methods. The popular approach is a combination of global search ability of GAs with local search ability of Simulated Annealing (SA)⁽¹¹⁾. As a pioneering research, Mahfoud and Goldberg have proposed Parallel Recombinative Simulated Annealing (PRSA) that applied SA to a selection of GA⁽¹²⁾. Later, Uehara et al. have introduced metropolis loop process of SA to an

elite strategy in GA process^{(13),(14)}. Hiroyasu et al. have proposed Parallel SA using Genetic crossover (PSA/ANGA)^{(15),(16)}. These hybrid methods have been applied to major benchmark functions and have been reported to be valid. They are believed to be both locally and globally efficient. However, the major multi-peak benchmark functions for multi-dimensions, i.e., 20 dimensional or more Rastrigin (RA) and Griewank (GR) functions, require about 10^6 function calls for arriving at an optimal solution. Moreover, when the optimal problem exhibits a dependence on design variable vectors (DVs) and the steepness of the objective function is small in the feasible space of DVs, it is difficult to obtain an optimal solution⁽¹⁷⁾.

Various optimization methodologies are proposed to overcome these difficulties^{(17)–(25)}. In Memetic Algorithms (MAs)^{(18)–(23)}, for instance, Ong and Keane has proposed meta-Lamarckian learning⁽¹⁸⁾ that improves the search ability for multi-peak functions with multi-dimensions by introducing a human expert judgment, where local search methods are used. Additionally, Fast Adaptive Memetic Algorithm (FAMA) has been proposed in Ref. (22). In the FAMA, coordination and choosing of local search method are dynamically controlled by means of a measurement of fitness diversity over the individuals of the population. On the other hand, Hasegawa et al. have proposed a hybrid meta-heuristic method (HMH) by reflecting recognition of dependence relations among design variables automatically, and have reported the effectiveness of this method^{(17),(24)}. The HMH needs to switch from the SA to the intuitive method, direct search using the learning result of the dependency of a DV, just before convergence to improve the local search ability of the optimal solution environs. These methodologies need to choose suitably a best local search method from various local search methods for combining with a global search method within the optimization process. Furthermore, since genetic operators are employed for a global search method within these algorithms, DVs which are renewed via a local search are encoded into its genes many times at its GA process. These certainly have the potential to break its improved chromosomes via gene manipulation by GA operators, even if these approaches choose a proper survival strategy.

Hasegawa has proposed a new Evolutionary Algorithm (EA) called an Adaptive Plan System with Genetic Algorithm (APGA)⁽²⁵⁾ to solve these problems and maintain the stability of the convergence to an optimal solution for multi-peak optimization problems with multi-dimensions. In this paper, we introduce some strategies using APGA to dramatically improve the stability up to the optimal solution.

This paper is organized in the following manner. The algorithm of proposed strategy, APGA is described in Section 2, the concept of Variable Neighborhood range Control (VNC) is in Section 3. Section 4 explains the hybrid neighborhood control method of APGA (H-APGA), and Section 5 discusses about the convergence to the optimal solution of multi-peak benchmark functions. Finally, Section 6 includes some brief conclusions and future works.

2. The Proposed Strategy – APGA

2.1. Formulation of Optimization Problem

The optimization problem is formulated in this section. Design variable vectors (DVs), objective function and range of DVs are defined as follow:

$$\text{Design variable : } X = [x_1, \dots, x_n] \quad (1)$$

$$\text{Objective function : } -f(X) \rightarrow \text{Max} \quad (2)$$

$$\text{Range of DVs : } X^{LB} \leq X \leq X^{UB} \quad (3)$$

Where $X^{LB} = [x_1^{LB}, \dots, x_n^{LB}]$, $X^{UB} = [x_1^{UB}, \dots, x_n^{UB}]$, and n denote the lower boundary condition vectors, the upper boundary condition vectors, and the number of DVs, respectively. A number of DVs significant figure is defined, and DV is rounded off to its decimal places within optimization process.

Fig. 1 APGA conceptual process

Fig. 2 APGA Algorithm

2.2. APGA

Natural and artificial systems adapt their behavior to changes in global and social environments over generations. Holland has defined this type of system as an Adaptive System (AS)⁽¹⁾, and the AS has an Adaptive Plan (AP) that determines its behaviors through responses to environments.

The APGA concept was introduced as a new EA strategy for multi-peak optimization problems. Its concept differs in handling DVs from general EAs based on GAs. EAs generally encode DVs into the genes of a chromosome, and handle them through GA operators. However, APGA completely separates DVs of global search and local search methods. It encodes Control variable vectors (CVs) of AP, which searches for local minima, into its genes on AS. Moreover, this separation strategy for DVs and chromosomes can solve MA problem of breaking chromosomes.

The conceptual process of APGA is shown in Fig. 1. In the APGA, the design space of the optimization problems, DVs, and response value vectors (RVs) are defined as the environment of AS, the point of the design space and response values from its environment, respectively. The CVs steer the individuals global behavior of AP for a global search, and are renewed via genetic operations by estimating fitness value which is a part of RVs. For a local search, AP generates next values of DVs by using CVs, RVs, and current values of DVs in the APGA optimization process. The DV generation process generates a new DV from the current search point via AP according to the formula:

$$X_{t+1} = X_t + NR_t \cdot AP(C_t, R_t) \quad (4)$$

Where NR , $AP()$, X , C , R , and t denote neighborhood ratio, a function of AP, DVs, CVs, RVs and generation, respectively.

The APGA's algorithm is described by the pseudocode given in Fig. 2. In addition, for a verification of APGA search process, refer to Ref. (17).

2.3. Adaptive Plan - Sensitivity Plan

It is necessary that the AP realizes a local search process by applying various heuristics rules. In this paper, the plan introduces a DV generation formula using a sensitivity analy-

Fig. 3 Encoding into genes of a chromosome

sis that is effective in the convex function problem as a heuristic rule, because a multi-peak problem is combined of convex functions. This plan uses the following equation:

$$AP(C_t, R_t) = -Scale \cdot SP \cdot sign(\nabla R_t) \quad (5)$$

$$SP = 2C_t - 1 \quad (6)$$

Where $Scale$, and ∇R denote scale factor and sensitivity of RVs, respectively.

A step size SP is defined by CVs for controlling a global behavior to prevent it falling into the local optimum. $C = [c_{i,j}, \dots, c_{i,p}]$; ($0.0 \leq c_{i,j} \leq 1.0$) is used by eq. (6) so that it can change the direction to improve or worsen the objective function, and C is encoded into a chromosome by 10 bit strings (shown in Fig. 3). In addition, i , j and p are the individual number, design variable number and its size, respectively.

2.3.1. Handling of Significant Figures In the optimal design of the product design, dimensions of products can be mainly dealt with as DVs. There are always assigned dimensional accuracies on a mechanical drawing. Therefore, a value of DVs is done well to use a number of significant figures of assigned dimensional accuracy in its drawing in the optimal process. In APGA, a number of significant figures of DVs are defined, and DVs are truncated to it within optimal process.

2.3.2. Handling out of Range DVs DVs are renewed by the AP, and when their values exceed their range, the APGA returns them to their range by using equation:

$$\begin{cases} X_t = 2X^{LB} - X_t, & X_t < X^{LB} \\ X_t = 2X^{UB} - X_t, & X_t > X^{UB} \end{cases} \quad (7)$$

2.3.3. Coding into Chromosome for CVs CVs are individually coded into a string to form a chromosome. This 10 bit string with two values (0 and 1) represents a real value of CVs by using the procedure shown in Fig. 3. In addition, this figure shows both DVs and CVs of 2 dimensions cases.

2.4. Control of Neighborhood Range

In case, the optimal problem of benchmark function exhibits a dependency on DVs and the steepness of this function is small, to solve its multiple dimensional optimization problem is difficulty.

For overcoming this difficulty, we propose the control of neighborhood range. Applying a neighborhood range control method is a common approach for an SA, and the distance for a search point can be changed by controlling the neighborhood range. The neighborhood range is decided by neighborhood ratio, according to the formula, as in:

$$NR_t = (t_{\max} - t)/(t_{\max} - 1) \quad (8)$$

Where $t_{\max}$ denote terminal generation. Equation (8) is a linear formula, and the relationship between the neighborhood ratio and the number of generation is shown in Fig. 4.

The neighborhood range of the early generations is adjusted to wide range by the control using this relationship. Moreover, it is modified to small range at the late generations. As results, we believe that search process is varied according to this control schedule roughly or delicately toward the good search direction. To increase search distance of early generations, scale factors are used in eq. (5).

Fig. 4 Relationship between neighborhood ratio and generation

Fig. 5 Elite strategy

2.5. GA Operators

2.5.1. Selection Selection is performed using a tournament strategy to maintain the diverseness of individuals with a goal of keeping off an early convergence. A tournament size of 2 is used.

2.5.2. Elite Strategy An elite strategy, where the best individual survives in the next generation, is adopted during each generation process.

It is necessary to assume that the best individual, i.e., as for the elite individual, generates two behaviors of AP by updating DVs with AP, not GA operators. Therefore, its strategy replicates the best individual to two elite individuals, and keeps them to next generation. As shown in Fig. 5, DVs of one of them (Δ symbol) is renewed by AP, and its CVs which are coded into chromosome are not changed by GA operators. Another one ($\circ$ symbol) is that both DVs and CVs are not renewed, and are kept to next generation as an elite individual at the same search point.

2.5.3. Crossover and Mutation In order to pick up the best values of each CV, a single point crossover is used for the string of each CV. This can be considered to be a uniform crossover for the string of the chromosome as shown in Fig. 6(a).

Mutation are performed for each string at mutation ratio on each generation, and set to maintain the strings diverse as shown in Fig. 6(b).

2.5.4. Recombination of Genes At following conditions, the genetic information on chromosome of individual is recombined by uniform random function.

- (1) One fitness value occupies 80% of the fitness of all individuals
- (2) One chromosome occupies 80% of the population

If this manipulation is applied to general GAs, an improved chromosome into which DVs have been encoded is broken down. However, in the APGA, the genetic information is only CVs used to make a decision for the AP behavior. Therefore, to prevent from falling into a local optimum, and to get out from the condition of being converged with a local optimum, a new AP behavior is provided by recombining the genes of the CVs into a chromosome. And the optimal search process starts to re-explore by a new one. This strategy is believed to make behavior like the re-annealing of an SA.

3. Variable Neighborhood Range Control – VNC

In the multi-point search of the APGA, individuals move from their various points to new search points in the design space of DVs. For example, as shown in Fig. 7, individual

Fig. 6 Crossover and Mutation

Fig. 7 Example of individual situations

A requires a slight change to the values of the DVs to obtain the global optimum solution. On the other hand, individual B cannot reach a global optimum solution without a significant change. In addition, individual C has landed at a local optimum solution. Such a situation, in which the individuals are intermingled, can generally occur at any time in search process. Therefore, it is necessary to find a suitable DV generation process for the situation of each individual in the design space.

To improve the multi-point search capability of the APGA, we propose a Variable Neighborhood Range Control (VNC) method, introduced to the AS. This method automatically adapts the neighborhood range to obtain DV generation accuracy for the situation of each individual. As a result, we believe that it will steadily provide a global optimum solution and reduce the calculation cost.

3.1. Assignment of Variable Neighborhood Range

In the VNC method, APGA process generates a new DV via AP using a sensitivity analysis by eq. (4).

The variable neighborhood range is determined by the NR , and the assignment step for the NR is shown in Fig. 8. The VNC method sorts all of the individuals by estimating their fitness, then ranking them by results. A ranked individual is labeled with the rank number and is assigned an NR that corresponds with this number. In this method, the formula for the NR by sigmoid function is as follows:

$$NR_t = \frac{1}{1 + \exp\left(\alpha \cdot \frac{\text{rank} - (\text{invidual}/2)}{\text{invidual}}\right)} \quad (9)$$

Fig. 8 Assignment step for neighborhood range

Fig. 9 Variable Neighborhood Control by APGA/VNC ($|\alpha| = 10.0$). Note that, if the absolute value of gain α is increased, the gradient of the sigmoid function becomes steep

$$0.0 \leq NR_t \leq 1.0 \quad (10)$$

Where α , $rank$, and $individual$ denote the gain of the sigmoid function, rank number and number of individual, respectively.

The search process is varied according to these models, roughly or delicately, toward the best search direction for all of individuals.

3.2. APGA/VNC Method

The APGA/VNC method defines two models by the sign of the gain α as follows:

$$\begin{cases} APGA/VNC1, & \alpha < 0 \\ APGA/VNC2, & \alpha > 0 \end{cases} \quad (11)$$

3.2.1. APGA/VNC1 In APGA/VNC1, as shown in Fig. 9, the individuals with high fitness values (good individuals) are allocated small NR values to perform a local search efficiently. On the other hand, the individuals with low fitness values (low individuals) are allocated large NR values to search the global area in the design space of the DVs.

3.2.2. APGA/VNC2 As for the APGA/VNC1, if a good individuals position belongs to a local optimum neighborhood area, the good individual searches its own neighborhood area without performing a global search. Therefore, it cannot escape from the local optimum solution. As a counter-plan for this problem, APGA/VNC2 employs the inverse model of APGA/VNC1 (shown in Fig. 9). This model assigns a large NR to the good individuals. As a result, they cannot search their own neighborhood areas. Moreover, as for the low individuals, this model assigns a small NR . Just before converging to the global optimum solution, to allow individuals to gather in the neighborhood area of the elite individual, the low individuals perform a local search.

4. Hybrid Neighborhood Control Method – H-APGA

To improve the multi-point search capability of APGA, we propose a Hybrid neighborhood control method introduce into APGA (H-APGA). In H-APGA method, the neighborhood ratio is controlled by mixing between linear function and sigmoid function.

Fig. 10 Neighborhood range control by H-APGA ($\alpha = 10.0$)

4.1. Linear Function

The neighborhood ratio is determined by NR , and the formula for the NR by linear function is as follows:

$$NR_t = 1 - \frac{1}{2} \left(\frac{inv}{individual/2} \right) \quad (12)$$

Where inv , $individual$ denote the current individual and number of individual.

To calculate NR , this method uses individual number only. Therefore, NR is distributed to an individual at random. It is adjusted to wire range with the first half of individual number and it is modified to small range with the last haft of one (as shown in Fig. 10).

4.2. H-APGA Method

4.2.1. H-APGA1 In H-APGA1, as shown in Fig. 10, the good individuals (high fitness values) are allocated large NR values to search global area in the design space of the DVs following eq. (12). On the other hand, the low individuals (low fitness values) are allocated small NR values to perform a local search efficiently following eq. (9).

4.2.2. H-APGA2 H-APGA2 employs the inverse method of H-APGA1 (as shown in Fig. 10). H-APGA2 assigns a large NR to the good individuals following eq. (9). Moreover, as for the low individuals, this method assigns a small NR following eq. (12). Before converging to the global optimum, the low individuals perform a local search.

5. Numerical Experiments

In this section, the numerical experiments are performed to compare among methods. Next, these methods are compared with other methodologies for the robustness of the optimization process. These experiments are performed 20 trials for each function. The initial seed number is randomly varied during every trial. In each experiment, the GA parameters used in solving benchmark functions are set as follows, selection ratio, crossover ratio and mutation ratio are 1.0, 0.8 and 0.01 respectively. The population size is 50 individuals and the terminal generation is 5000th generation.

5.1. Benchmark Functions

For the APGAs, we estimate the stability of the convergence to the optimal solution by using five benchmark functions with multiple dimensions Rastrigin (RA), Ridge (RI), Griewank (GR), Rosenbrock (RO) and Ackley (AC) function. These functions are given as follows:

$$RA : f_1 = 10n + \sum_{i=1}^n \{x_i^2 - 10\cos(2\pi x_i)\} \quad (13)$$

$$RI : f_2 = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2 \quad (14)$$

Table 1 Characteristics of benchmark functions

Function	Epistasis	Multi-peaks	Steepness
RA	No	Yes	Average
RI	Yes	No	Average
GR	Yes	Yes	Small
RO	Yes	No	Big
AC	No	Yes	Average

Table 2 Range, digits of DVs, and scale factor for normalizing benchmark functions

Function	Design range	Number of digits	Scale factor
RA	$-5.12 \leq X \leq 5.12$	2	10.0
RI	$-51.2 \leq X \leq 51.2$	1	100.0
GR	$-51.2 \leq X \leq 51.2$	1	100.0
RO	$-2.048 \leq X \leq 2.048$	3	4.0
AC	$-5.12 \leq X \leq 5.12$	2	10.0

$$GR : f_3 = 1 + \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) \quad (15)$$

$$RO : f_4 = \sum_{i=1}^n [100(x_{i+1} + 1 - (x_i + 1)^2)^2 + x_i^2] \quad (16)$$

$$AC : f_5 = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e \quad (17)$$

Table 1 shows their characteristics, and the terms epistasis, multi-peaks, steepness denote the dependence relation of the DVs, presence of multi-peak and level of steepness, respectively. In Table 2, their design range, the digits of DVs, and the sensitivity plan parameters in eq. (5) are summarized. Note that, it is difficult to search for their optimal solutions by applying one optimization strategy only, because each function has specific complex characteristics.

All functions are minimized to zero, when optimal DVs $X = 0$ are obtained. The search process is terminated when the search point attains an optimal solution or a current generation process reaches the termination generation.

5.2. Experiment Results

The experiment results are shown in Table 3-7. When the success rate of optimal solution is not 100%, "-" is described, and the improvement rate denotes a rate of a calculation cost of APGA/VNC method and H-APGA method compared with simple APGA. The improvement rate is average value of improvement rate of five benchmark functions expect for functions with which the optimal solution was not reached.

The solutions of all benchmark functions with 20 dimensions reach their global optimum solution. However, there are some differences among methods. APGA/VNC1 could not reach a global optimum solution as for some trials of RI and RO function. On the other hand, APGA/VNC2 could arrive at a global optimum solution in all trials. Moreover, H-APGA method converged faster than APGA/VNC method, especially H-APGA2 could arrive at a global optimum at a high probability with every function.

As a result, we assign the best trial that is found by the maximum improvement rate. Its best gain value is trial 2 with APGA/VNC1, trial 9 with APGA/VNC2, trial 19 with H-APGA1 and trial 17 with H-APGA2. Moreover, the results using the best trial of these methods are compared. From this comparison, we can confirm that H-APGA2 is the best solution.

Next, Fig. 11-15 shows trend diagrams for the average fitness of an elite individual until these methods reach global optimum solutions, in the numerical experiment again to confirm above mentioned result.

In summary, its validity confirms that this strategy can reduce the computation cost and improve the stability of the convergence to the optimal solution.

Table 3 Experiment results, averaged over 20 runs by APGA (20 dimensions; population size 50). "Mean" indicates average of optimum values obtained and "Std Dev" stands for standard deviation

Function	Num. of. Gen.	Mean (Std Dev)
RA	376	0.00E+00 (0.00E+00)
RI	2030	0.00E+00 (0.00E+00)
GR	468	0.00E+00 (0.00E+00)
RO	-	-4.78E+02 (5.14E+02)
AC	378	-4.44E-16 (0.00E+00)

Table 4 Experiment results by APGA/VNC1. Number of generations required to reach the global optimum (20 dimensions; population size 50)

Trial	Function					Improvement Rate
	RA	RI	GR	RO	AC	
1	242	-	418	1321	275	
2	251	1607	322	1277	279	72%
3	257	1412	349	1605	275	71%
4	237	-	299	-	279	
5	234	1892	309	-	249	
6	226	1844	278	-	280	
7	237	-	274	-	255	
8	223	-	251	-	255	
9	218	1610	214	-	253	
10	220	1538	211	-	248	
11	218	1438	230	-	263	
12	207	1365	232	-	239	
13	222	1239	260	-	211	
14	209	1325	238	-	214	
15	237	1607	277	-	230	
16	248	-	274	-	232	
17	255	-	302	-	241	
18	280	-	324	-	282	
19	269	-	371	-	292	
20	275	-	371	-	285	
Average generation	238	-	290	-	257	54%

5.3. Discussion for Assigning Gain α and Method

We discuss the convergence of the optimization process using the trend of benchmark function with the difference from the calculation cost among models. As shown in Fig. 11-15, APGA/VNC1 converged earlier than other models in early generations, that because this model updated the elite individual with certainly. However, after that, APGA/VNC1 could not update it efficiently, and finally other models reach a global optimum solution faster than this one.

H-APGA1 was really good with RI function but it could not get high probability with RO function, compared with H-APGA2. H-APGA2 could reach a global optimum at a high probability with every function.

We made various tests using the values of gain α that ranges from 0 to 20 of RO function with 20 dimensions by H-APGA method. From this comparison as shown in Fig. 16, we confirmed that H-APGA2 converged faster than H-APGA1, and the best value of gain α is 10. That is the reason why we assigned this value within all experiments.

To sum up, from these results via optimization experiments, we employed APGA/VNC2 for the APGA/VNC, and H-APGA2 for H-APGA. As shown in Fig. 17, H-APGA converged faster than APGA/VNC with high improvement rate.

Table 5 Experiment results by APGA/VNC2. Number of generations required to reach the global optimum (20 dimensions; population size 50)

Trial	Function					Improvement Rate
	RA	RI	GR	RO	AC	
1	261	1264	361	1229	216	66%
2	246	1269	354	1025	216	65%
3	242	1620	292	962	260	69%
4	243	1481	302	1229	245	67%
5	228	1190	289	1025	243	61%
6	235	1366	288	962	254	65%
7	255	1090	295	827	226	59%
8	240	1071	281	997	257	61%
9	216	929	268	898	253	57%
10	226	1155	302	939	230	61%
11	227	1111	262	844	251	59%
12	241	1224	290	1000	238	62%
13	233	1306	314	1302	247	65%
14	229	1136	291	1092	250	61%
15	247	1096	310	906	278	65%
16	246	1153	340	1018	279	67%
17	251	1321	332	1008	294	70%
18	274	1151	379	1052	297	72%
19	269	1267	361	1175	307	73%
20	269	1476	426	1302	300	79%
Average generation	242	1234	317	1040	257	65%

Table 6 Experiment results by H-APGA1. Number of generations required to reach the global optimum (20 dimensions; population size 50)

Trial	Function					Improvement Rate
	RA	RI	GR	RO	AC	
1	219	533	384	1792	84	47%
2	220	461	429	1456	169	54%
3	206	648	429	1779	107	52%
4	166	1218	377	1466	174	58%
5	218	469	305	1863	125	45%
6	216	1209	387	1664	104	57%
7	194	617	421	1693	98	49%
8	217	633	448	1479	100	53%
9	202	602	402	1800	175	54%
10	205	270	471	1072	119	50%
11	217	983	301	1825	116	50%
12	193	661	472	1851	93	52%
13	212	672	459	1118	107	54%
14	213	637	360	1574	124	49%
15	218	237	352	1873	113	44%
16	195	738	409	1612	151	54%
17	207	472	403	1810	155	51%
18	210	705	403	1303	139	53%
19	205	362	232	1878	121	38%
20	217	273	337	1443	138	45%
Average generation	208	620	390	1618	126	51%

Table 7 Experiment results by H-APGA2. Number of generations required to reach the global optimum (20 dimensions; population size 50)

Trial	Function					Improvement Rate
	RA	RI	GR	RO	AC	
1	209	1236	276	1049	193	57%
2	172	943	307	1108	182	51%
3	212	822	327	1162	231	57%
4	207	1189	274	835	225	58%
5	180	1759	305	1057	244	66%
6	200	1850	298	1184	190	65%
7	214	738	299	1134	232	55%
8	167	831	311	1171	227	53%
9	211	858	319	1016	195	55%
10	205	657	275	1212	267	54%
11	190	1256	323	1076	175	57%
12	207	1844	239	1061	217	64%
13	178	611	305	1045	194	48%
14	206	847	316	976	279	59%
15	201	605	311	1022	231	53%
16	205	1042	327	1191	172	55%
17	125	750	306	1156	141	43%
18	210	755	294	1116	262	56%
19	201	1664	242	1190	206	60%
20	208	1057	293	999	196	55%
Average generation	196	1066	298	1088	213	56%

Fig. 11 Elite individual's fitness of RA function with 20 dimensions

Fig. 12 Elite individual's fitness of RI function with 20 dimensions

Fig. 13 Elite individual's fitness of GR function with 20 dimensions

Fig. 14 Elite individual's fitness of RO function with 20 dimensions

Fig. 15 Elite individual's fitness of AC function with 20 dimensions

5.4. Comparison for Robustness

All new evolutionary strategies of APGA (APGAs) - simple APGA⁽²⁵⁾, APGA/VNC⁽²⁶⁾, and H-APGA⁽²⁷⁾ were compared with other methodologies - Parallel SA with Selection (PSAS) in Ref. (14), Hybrid meta-heuristics (HMH) in Ref. (17), (24). The comparison is given in Table 8.

By means of the comparison lists, APGAs could certainly achieve optimal solution with low calculation cost. The convergence of the optimal solution could be improved more significantly in APGAs than that in EAs for the same calculation cost.

H-APGA had better convergence than APGA/VNC with four benchmark functions RA, RI, GR and AC. However, H-APGA did not gain a high probability with RO function as APGA/VNC.

The comparison among all strategies of APGA is also shown in Fig. 18. APGA/VNC and H-APGA were better than simple APGA in all benchmark functions. Therefore, it is desirable to introduce these models into APGA.

Fig. 16 Gain α by H-APGA method of RO function with 20 dimensions

Fig. 17 The improvement rate through trials

Table 8 Average generations over 20 independent trials by PSAS, HMH, APGA, APGA/VNC and H-APGA (20 dimensions; population size 50)

Function	Dim.		PSAS Ref. (14)	HMH Ref. (17), (24)	APGA Ref. (25)	APGA/VNC Ref. (26)	H-APGA Ref. (27)
RA	20	Generation	1,500	3,630	376	242	196
		Func. call	150,000	145,184	18,800	12,100	9,800
RI	20	Generation	5,200	17,686	2,030	1234	1066
		Func. call	520,000	707,440	101,500	61,700	53,300
GR	20	Generation	4,100	9,003	468	317	298
		Func. call	410,000	360,132	23,400	15,850	14,900
RO	20	Generation	-	-	-	1,040	1088
		Func. call	-	-	-	52,000	54,400
AC	20	Generation	1,500	3,309	378	257	213
		Func. call	150,000	132,360	18,900	12,850	10,650

Fig. 18 Comparison of APGA, APGA/VNC and H-APGA

To evaluate the performance of APGAs, we compared to other EAs such as GA, Particle Swarm optimization (PSO), Particle Swarm Inspired Evolutionary Algorithm (PS-EA) in Ref. (28) and Artificial Bee Colony (ABC)⁽²⁹⁾, Differential Evolution (DE)⁽³⁰⁾ and Self-Adaptive DE (SaDE)⁽³¹⁾, and Advanced DE (ADE)⁽³²⁾. Maximum number of generation and the population size, i.e. 100, 125, as in the study presented in Ref. (28), (31). The mean and the standard deviations of the function values obtained by these methods are given in Table 9 and Table 10.

In particular, it confirmed that the calculation cost with these models could be reduced for benchmark functions. And it showed that the convergence to the optimal solution could be significantly improved.

The advantage of the APGAs over other EAs seems to be limited because the APGAs specify small number of digits of design variables in the numerical experiments.

Overall, APGAs was capable of attaining robustness, high quality, low calculation cost and efficient performance on many benchmark problems. We confirmed satisfactory performance through various benchmark tests.

Table 9 The mean and the standard deviations obtained by GA, PSO, PS-EA, ABC and APGAs (30 dimensions; population size 125)

Function	Dim.	Gen.	GA ⁽²⁸⁾ Mean (Std Dev)	PSO ⁽²⁸⁾ Mean (Std Dev)	PS-EA ⁽²⁸⁾ Mean (Std Dev)	ABC ⁽²⁹⁾ Mean (Std Dev)	APGAs Mean (Std Dev)
RA	30	1000	10.4388 (2.6386)	32.476 (6.9521)	3.0527 (0.9985)	0.033874 (0.181557)	0 (0)
RI	30	1000	–	–	–	–	0 (0)
GR	30	1000	1.2342 (0.11045)	0.011151 (0.014209)	0.8211 (0.1394)	2.87E-09 (8.45E-10)	0 (0)
RO	30	1000	166.283 (59.5102)	402.54 (633.65)	98.407 (35.5791)	0.219626 (0.152742)	-1.29E-02 (1.03E-02)
AC	30	1000	1.0989 (0.24956)	1.49E-06 (1.86E-06)	0.3771 (0.098762)	3.00E-12 (5.00E-12)	-4.44E-16 (0.00E+00)

Table 10 The mean and the standard deviations obtained by DE, SaDE, ADE and APGAs (30 dimensions; population size 100)

Function	Dim.	Gen.	DE ⁽³⁰⁾ Mean (Std Dev)	SaDE ⁽³¹⁾ Mean (Std Dev)	ADE ⁽³²⁾ Mean (Std Dev)	APGAs Mean (Std Dev)
RA	30	1500	173.405 (13.841)	1.50E-15 (4.80E-15)	0 (0)	0 (0)
RI	30	1500	1.63086 (0.886153)	0.090075 (0.080178)	–	0 (0)
GR	30	1500	2.90E-13 (4.20E-13)	0 (0)	0 (0)	0 (0)
RO	30	1500	7.80E-09 (5.80E-09)	3.10E-15 (8.30E-15)	3.57E-05 (8.90E-05)	0 (0)
AC	30	1500	9.70E-08 (4.20E-08)	7.70E-15 (1.40E-15)	2.66E-15 (0.00E+00)	-4.44E-16 (0.00E+00)

5.5. APGAs with 100 Dimensions and Different Population

We applied to benchmark tests with 100 dimensions by 20 independent runs and 10,000th terminal generation. APGA/VNC and H-APGA could reach global optimum solution with all benchmark functions, and the success rate of optimal solution is 100%. The experiment results, average generations required to reach the global optimum are given in Table 11.

From this table, it can be concluded that as the population size increases, the performance of the APGAs significantly improves. Additionally, the results show that the proposed APGAs is effective in all benchmarks for various population sizes. The performance of the APGAs show relatively improvement with the growth of the population size, which suggests that the APGAs is more stable and robust on population size.

Table 11 Experimental results, averaged over 20 runs by APGAs of 100 dimesions with different population size. "Mean" indicates average of optimum values obtained and "Std Dev" stands for standard deviation

Population size 50			
Function	APGA/VNC	H-APGA	Mean (Std Dev)
RA	737	729	0.00E+00 (0.00E+00)
RI	2736	3777	0.00E+00 (0.00E+00)
GR	1464	1230	0.00E+00 (0.00E+00)
RO	5430	5514	0.00E+00 (0.00E+00)
AC	828	644	-4.44E-16 (0.00E+00)
Population size 100			
Function	APGA/VNC	H-APGA	Mean (Std Dev)
RA	592	540	0.00E+00 (0.00E+00)
RI	1145	1579	0.00E+00 (0.00E+00)
GR	980	904	0.00E+00 (0.00E+00)
RO	2819	2802	0.00E+00 (0.00E+00)
AC	432	371	-4.44E-16 (0.00E+00)

6. Conclusion

In this paper, to overcome the weak point of EAs, and to achieve the global search for the solution space of multi-peak optimization problems with multi-dimensions, we proposed new evolution strategies of Adaptive Plan System with Genetic Algorithm, APGAs. Then, we verified the effectiveness of APGAs by the numerical optimization experiments performed five benchmark functions. The obtained points are shown below

The search ability of APGAs with multi-dimensions optimization problems is very effective, compared with that of EAs. Nevertheless, the number of digits of design variables in the numerical experiments is insufficient to discuss about the stability of convergence.

All the strategies of APGA achieved at the global optimum solution. However, these models have strengths and weaknesses. About the optimal solution such as minimum time and maximum reliability, it is a future work.

Finally, this study plans to do a comparison with the sensitivity plan of AP by applying other optimization methods into AP and optimizing benchmark tests, constrained real-parameters, futher real life applications.

References

- (1) Holland, J., *Adaptation in Natural and Artificial Systems*, The University of Michigan 1975, MIT Press, (1992).
- (2) Goldberg, D.E., *Genetic Algorithms in Search Optimization and Machine Learning*, (1989), Addison- Wesley.
- (3) Digalakis, J.D. and Margaritis, K.G., An experimental study of benchmarking functions for Genetic Algorithms, *Proceedings of IEEE Conference on Transactions*, Vol.5, (2000), pp.3810-3815.
- (4) Sakuma, J. and Kobayashi, S., Extrapolation-Directed Crossover for Real-coded GA: Overcoming Deceptive Phenomena by Extrapolative Search, *Proceedings of Congress on Evolutionary Computation (CEC 2001)*, (2001), pp.685-662.
- (5) Li, X. and Kirley, M., The Effects of Varying Population Density in a Fine-grained Parallel Genetic Algorithm, *Proceedings of Congress on Evolutionary Computation (CEC 2002)*, No.2, (2002), pp.1709-1714.
- (6) Hasegawa, H. and Kawamo, K., A Method for the Shape and Topology Optimization of Mechanical Structures by Using Genetic Algorithm (Layout Optimization Method by Adoption of Removal and Addition parameters of Elements as Chromosomes), *Transactions of the Japan Society of Mechanical Engineers, Series A*, Vol.61, No.581 (1995), pp.183-190 (in Japanese).
- (7) Tsuruta, Y., Hasegawa, H. and Kawamo, K., A Method for the Shape and Topology Optimization of Mechanical Structures Using Genetic Algorithm (2nd Report, On Con-

- vergence of Solutions of Our Method by Adoption of Removal and Addition parameters of Elements as Chromosomes), *Transactions of the Japan Society of Mechanical Engineers, Series A*, Vol.63, No.605, (1997), pp.170-177 (in Japanese).
- (8) Tsuruta, Y., Suzuki, Y., Hasegawa, H. and Kawamo, K., A Method for the Shape and Topology Optimization of Mechanical Structures Using Genetic Algorithm, *JSME Centennial Grand Congress by the JSME International Symposium on OPID97*, (1997), Paper 137.
 - (9) Nishi, H., Okamoto, T. and Kawamo, K., Study on the Development of General-purpose Genetic Algorithm Engine, *Extend Abstracts of WCSMO-4*, (2001), pp.428-429.
 - (10) Kawada, H., Uehara, H. and Kawamo, K., Development of Generalpurposed Optimization Engine and Test Run, *Journal of Japan Society for Simulation Technology*, Vol.21, No.4, (2003), pp.280-288 (in Japanese).
 - (11) Rosen, B.E. and Nakano, R., Simulated annealing - basics and recent topics on simulated annealing, *Journal of Japanese Society for Artificial Intelligence*, Vol.9, No.3, (1994) (in Japanese).
 - (12) Mahfoud, S.W. and Goldberg, D.E., A genetic algorithm for parallel simulated annealing, *Parallel Problem Solving from Nature*, No.2, (1992), pp.301-310.
 - (13) Uehara, H., Kawada, H. and Kawamo, K., Numerical Experiments on Optimal Points Searching Using Hybrid Method of Genetic Algorithm and Simulated Annealing, *52nd Japan National Congress for Theoretical and Applied Mechanics*, (2003), pp.117-118 (in Japanese).
 - (14) Uehara, H., Study on Development of General-purposed Optimization Engine and its Performance Evaluation - The proposal of Parallel Simulated Annealing with Selection, *M of Eng thesis, Graduate School of Engineering, Shibaura Institute of Technology*, (2004) (in Japanese).
 - (15) Hiroyasu, T., Miki, M. and Ogura, M., Parallel Simulated Annealing using Genetic Crossover, *Proceedings of the ISCA 13th International Conference on PDCS-2000*, (2000).
 - (16) Miki, M., Hiroyasu, T. and Fushimi, T., Parallel Simulated Annealing with Adaptive Neighborhood Determined by GA, *IEEE International Conference on System, Man and Cybernetics*, (2003).
 - (17) Hasegawa, H., Yoshikawa, M., Uehara, H. and Kawamo, K., The Hybrid Meta-heuristics by Reflecting Recognition of Dependence Relation among Design Variables for Integer Optimization of Multi-peak Problems, *Journal of Japan Society for Simulation Technology*, Vol.25, No.2, (2006), pp.144-155 (in Japanese).
 - (18) Ong, Y.S. and Keane, A.J., Meta-Lamarckian Learning in Memetic Algorithms, *IEEE transactions on evolutionary computation*, Vol.8, No.2, (2004), pp.99-110.
 - (19) Hart, W.E., Krasnogor, N. and Smith, J.E., *Recent Advances in Memetic Algorithms*, (2005), Springer.
 - (20) Ong, Y.S., Lim, M.H., Zhu, N. and Wong, K.W., Classification of Adaptive Memetic Algorithms: A Comparative Study, *IEEE transactions on Systems, Man and Cybernetics Part B*, Vol.36, No.1, (2006), pp.141-152.
 - (21) Neri, F., Tirronen, V., Kärkkäinen, T. and Rossi, T., Fitness Diversity Based Adaptation in Multimeme Algorithms: A Comparative Study, *IEEE Congress on Evolutionary Computation (CEC 2007)*, (2007), pp.2374-2381.
 - (22) Caponio, A., Cascella, G.L., Neri, F., Salvatore, N. and Sumner, M., A Fast Adaptive Memetic Algorithm for Online and Offline Control Design of PMSM Drives, *IEEE transactions on Systems, Man and Cybernetics Part B, Special Issue on Memetic Algorithms*, Vol.37, No.1, (2007), pp.28-41.
 - (23) Tirronen, V., Neri, F., Kärkkäinen, T., Majava, K. and Rossi, T., An Enhanced Memetic Differential Evolution in Filter Design for Defect Detection in Paper Production, *Evolutionary Computation Journal, MIT Press*, Vol.16, No.4, (2008), pp.529-555.

- (24) Hasegawa, H., Sasaki, H., Uehara, H. and Kawamo, K., The optimisation of spot-weld positions for vehicle design by using hybrid metaheuristics, *International Journal of Vehicle Design*, Vol.43, No.1-4, (2007), pp.151-172.
- (25) Hasegawa, H., Adaptive Plan System with Genetic Algorithm based on Synthesis of Local and Global Search Method for Multi-Peak Optimization Problems, *Proceedings of the 6th EUROSIM Congress on Modelling and Simulation*, (2007).
- (26) Tooyama, S. and Hasegawa, H., Adaptive Plan System with Genetic Algorithm using the Variable Neighborhood Range Control, *IEEE Congress on Evolutionary Computation (CEC 2009)*, (2009), pp.846-853.
- (27) Pham, N.H., and Hasegawa, H., Hybrid neighborhood control method of adaptive plan system with genetic algorithm, *Proceeding of the 7th EUROSIM Congress on Modelling and Simulation*, (2010).
- (28) Srinivasan, D., and Seow, T.H., Evolutionary Computation, *IEEE Congress on Evolutionary Computation (CEC 2003)*, (2003), pp.2292-2297.
- (29) Karaboga, D., and Basturk, B., A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm, *Journal Global Optimization*, Vol.39, (2006), pp.459-471.
- (30) Storn, R., and Price, K., Differential Evolution - a simple and efficient Heuristic for global optimization over continuous spaces, *Journal Global Optimization*, Vol.11, No.4, (1997), pp.341-357.
- (31) Brest, J., Greiner, S., Boskovic, B., Mernik, M., and Zumer, V., Self-adapting control parameters in differential evolution: A comparative study on numerical benchmark problems, *IEEE Transactions on Evolutionary Computation*, Vol.10, No.6, (2006), pp.646-657.
- (32) Wagdy Mohamed, A., Sabry, H.Z., Farhat, A., Advanced Differential Evolution algorithm for global numerical optimization, *IEEE International Conference on Computer Applications and Industrial Electronics (ICCAIE)*, (2011), pp.156-161.