Monte Carlo Simulation of Dynamic Problem Using Model Order Reduction Technique Highlighting on Tail Probability*

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Abstract
This paper aims at practical Monte Carlo (MC) simulation by finite element method for a dynamic problem where the load condition includes uncertainty factors. A new sampling scheme is proposed to predict an extreme case with high stress by unexpected combination of load parameters, which is involved in the tail probability, to be used in the fatigue life estimation of structures. The proposed scheme, named as stepwise limited sampling (SLS), can provide the expected value of the stress with moderate accuracy and provide reliable results with extremely high stress. In the demonstrative example, the proposed method was computationally cost-effective than usual MC simulation. In addition, a model order reduction (MOR) technique is employed to reduce both the computational cost and the memory requirement. The latter contributed to the fast MC simulation by parallel processing.

Key words: Uncertainty, Sampling Scheme, Tail Probability, Dynamic Analysis, Monte Carlo Simulation, Model Order Reduction

1. Introduction

After the publication of verification and validation (V&V) guideline for finite element modeling and simulation in 2006(1) by American Society of Mechanical Engineers (ASME) as well as the standard for V&V in fluid dynamics and heat transfer in 2009(2), the quality management of engineering simulation is a matter of concern. ASME V&V guideline requires quantitative evaluation of uncertainties(3). There can be many uncertainty factors in computational solid mechanics, such as the geometrical modeling, material properties and boundary conditions. Since there are many fields where experiments for real objects are impossible such as aerospace, nuclear plant and human organ in vivo, the uncertainty modeling is an important issue in finite element method (FEM). However, the numerical scheme for uncertainty quantification has not been widely spread in industries.

A Monte Carlo (MC) method is the most popular scheme to consider the probabilities of variables. Here, the sampling scheme is the key to provide good accuracy and reliability. Gibbs sampling, Metropolis-Hastings algorithm and Latin hypercube sampling are well known. A resampling method is one of the generation techniques of representative sampling points to approximate the distribution of interest in more than one step, such as weighted
resampling method. A generation algorithm of random number is also important to express the probability density function of a variable. The Mersenne twister (MT) is known to provide most reliable random numbers, which is used in this study.

MC simulation can provide the probability density function of a quantity of interest (QoI) together with its expected value and standard deviation. However, at least 10,000 sampling points are needed in MC simulation, which is a hurdle for its practical use.

More than that, even with 10,000 sampling points generated by MT, the reliability of the tail probability has not yet been clarified. That is, if one wants to predict the case that can happen with very low probability, the necessary number of sampling points is unknown. Hence, the efficient sampling scheme to predict such an extreme case that may appear in the tail probability is studied. A stepwise limited sampling (SLS) is proposed in 2.2.

A dynamic problem is analyzed in this study, where uncertainty parameters are involved in load condition. Since the fluctuation of load can lead to fatigue fracture, it is important to consider the uncertainty factors in the load condition. In such a problem, it is more important not to miss a case that has high stress value than to calculate the average and standard deviation of stress. This study aims at predicting a case that results in high stress values by unexpected combination of load parameters. In ref. (4), a target probability of failure is described in the discussion of uncertainty in fatigue damage accumulation. While this paper demonstrates how to predict a case with much lower probability.

The computational efficiency is important to be widely used in industries. The proposed SLS does not increase the computational cost than conventional MC. In addition, a model order reduction (MOR) technique is employed for fast dynamic analysis.

Various algorithms for MOR have been reported so far, and the comparison of merits and demerits has been summarized in detail in refs. (5),(6) among balanced truncation approximation, singular perturbation approximation, Hankel norm approximation, Guyan reduction and Krylov subspace methods such as Lanczos algorithm and Arnoldi process. Also there are many research works recently on proper orthogonal decomposition (7) or proper generalized decomposition. It should be noted that Korvink and his colleagues (5),(8) have pointed out the merit of Arnoldi process beyond other methods and have demonstrated its effectiveness. Its extension to nonlinear analysis (5),(8) and parametric model order reduction (5),(9) to preserve parameters in the system matrices has been presented. The application of Arnoldi process to global-local problems has been shown in ref. (10) and the authors (11) have also presented the usefulness of Arnoldi type MOR applied to multiscale finite element mesh superposition analysis. In this paper, therefore, to solve a problem with uncertainty parameters in the definition of load condition, a block second order Arnoldi (BSOAR) method (12),(13) is adopted. When BSOAR type MOR is used in MC simulation, it was found to be cost-effective in the analysis of typical frame structure (14).

2. Computational Methods

2.1 Model order reduction technique

This paper employs Arnoldi process among various model order reduction (MOR) techniques as described in the introduction. It calculates \( n \) basis vectors in Krylov subspace, which converts the system with \( N \) d.o.f., Eq. (1), into a reduced system with \( n \) d.o.f., where \( n \ll N \).

\[
[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = \{F(t)\}
\]

This works well if both coefficient matrices and load vector are constant. However, the parametric model order reduction technique (5),(9) allows to change the matrices in the repeated analyses for mechanical design. To consider the uncertainty factors in the load condition, the load vector is defined in this paper by Eq. (2) for 2 dimensional problems.

Figure 1 illustrates the definition of load parameters.

\[
\{F(t)\} = \alpha(t)\{F\} = \alpha(t)\left(\{F_x\}\sin \theta + \{F_y\}\cos \theta\right)
\]
This definition allows to apply the Arnoldi process to unit vectors $\{F_x\}$ and $\{F_y\}$. Then, the reduced system can be expressed by Eq. (3).

$$[m]\{\dot{z}\} + [d]\{\ddot{z}\} + [k]\{z\} = \alpha(t)\{f\}$$ (3)

The detail of the numerical algorithm of this second order Arnoldi process (SOAR) is written in the authors’ previous paper(11). To calculate the basis vectors for two unit vectors in 2 dimensional problems, a block second order Arnoldi (BSOAR)(12),(13),(15) can produce them simultaneously without increasing the computational cost. The detailed algorithm of BSOAR is not rewritten in this paper, but please see the authors’ previous paper(15).

A question arises for any kind of MOR that how to determine the necessary number of basis vectors(16). Especially under the quality management of the simulation, it is mandatory to check the sensitivity of the number of basis vectors on the simulation results. That is, the results by using different numbers of basis vectors should be compared in the same way with the mesh convergence check. If this is costly, we can find no merit to use MOR. When BSOAR type MOR is used in Monte Carlo (MC) simulation, however, the overhead time for this verification becomes negligible, which is presented later in 4.3.

Also, the reduction of model order leads to the reduction of necessary memory requirement. This enables the parallel processing on current standard multicore PCs, which contributes much to the reduction of computational time.

In the numerical example, the verification of the number of basis vectors is shown in 3.2, and the computational cost is discussed in 4.3.

2.2 Sampling scheme for Monte Carlo simulation

In the authors’ experience(17), the expected value of a quantity of interest (QoI) converges earlier than its standard deviation. This means that a very large number of analyses are necessary to determine the form of the probability density function of QoI. Usually, 10,000 cases are analyzed, but we sometimes encounter a problem where convergence is not obtained for standard deviation(17). In Ref. (17), 8 uncertainty factors are considered including 4 material parameters, 1 geometrical parameter and 3 parameters for initial stress distribution. Note that 4 material parameters were not expressed in normal distribution but in mixed Weibull distribution, which was determined by experimental measurement. The probability density function of QoI was not in normal distribution. It has been found that, in a sort of design, predicting an extreme case of failure, which may be seen in the tail probability, is more important than to obtain the global form of probability density function.

Based on this idea, a practical sampling scheme named stepwise limited sampling (SLS) is proposed to find efficiently an extreme case that may only occur with very low probability but may lead to fatal failure. The requirements for SLS are listed as follows.

(a) computationally cost-effective
(b) extensible to a large number of variables
(c) universally applicable to various types of problems
(d) easy implementation by automated algorithm

In pursuit of this, SLS takes the following 4 steps.

(1) STEP 1: Convergence check of expected value of QoI
Based on the idea that the expected value itself is distributed, its convergence is estimated, in the usual MC simulation, by the following equation of central limited theorem.

\[ E_i(E_{i-1}) \leq \frac{\sigma_{i-1}(i)}{\sqrt{n_{\text{max}}}} \]  

where \( E_i \) denotes the expected value after \( i \) sets of analyses in MC simulation, \( \sigma_{i-1}(i) \) denotes the standard deviation after first set of analyses. The right hand side is normalized as follows using the expected value after first set of analyses.

\[ E_i(E_{i-1}) \leq \frac{\sigma_{i-1}(i)}{E_{(1)} \sqrt{n_{\text{max}}}} \]  

In this study, one set of analyses includes 100 cases and \( n_{\text{max}} = 10,000 \). Finally, Eq. (5) is rewritten as follows.

\[ E_{(100)i} - E_{(100)(i-1)} \leq \frac{\sigma_{(100)i}}{100 \cdot E_{(100)}} \]  

Since the expected value oscillates as the increase of analyses in MC simulation, the expected value is supposed to be converged if Eq. (6) is satisfied three times continuously. The usual MC simulation is suspended when the convergence is obtained.

(2) **STEP 2: Determination of limited sampling criteria**

It is hard to describe the combination of many parameters that results in an extreme case such as a case with very high stress etc., but it becomes quite easy if two parameters among them are picked up. Therefore, we propose to determine the zones in multidimensional space of input parameters by a set of inequalities, Eq. (7).

\[ p_i x_i + p_j x_j + q \geq 0 \quad (i \neq j) \]  

where \( x_i \) and \( x_j \) are different input parameters, and \( p_i, p_j, q \) are scalar factors. The threshold is determined by \( \mu + 3\sigma \), where \( \mu \) is the average and \( \sigma \) is the standard deviation. This criterion can be automated and can be visualized as illustrated in the following numerical example.

(3) **STEP 3: Priori estimation of probability in the limited zone**

The probability in the limited zone can be estimated by multiplying the probabilities for multiple limited zones sequentially. The probability \( P_{i,j} \) for a set of two parameters, \( x_i \) and \( x_j \), is equivalent to the area ratio of the limited zone. In this paper, the zone is divided into a large number of pieces and the probability density function for each subsection is approximated to be constant for numerical integration in Eq. (8).

\[ P_{i,j} = \frac{1}{kh} \sum_s \sum_x f_{i,j}^s (x_i) f_{i,j}^x (x_j) \]  

where \( k \) and \( h \) are the number of subdivision for \( x_i \) and \( x_j \), respectively. \( f_{i,j}^s (x_i) \) stands for the value of probability density function for \( x_i \) at subdivided point \( k \).

In this study, \( k \) and \( h \) are determined after preliminary study to be 2,000 per \( \sigma \), i.e., 20,000 per \( \pm 5\sigma \), for normal distribution. However, this numerical algorithm is applicable to any kind of distribution, such as Weibull distribution.

In the numerical demonstration in chapter 3, the finally estimated probability \( P = \prod_{i,j} P_{i,j} \) is \( 3.864 \times 10^{-6} \). Its inverse means the computational efficiency.

(4) **STEP 4: Generation of random numbers in limited zone and analysis**

In the final step, random numbers are generated in a normal way, but only those contained in the limited zone by inequalities, Eq. (7), are used for the analysis.

3. **Numerical Example**

3.1 **Problem setting**

As a demonstrative example of dynamic problem, a frame structure in Fig. 2 is
analyzed by the model order reduction (BSOAR) and proposed stepwise limited sampling (SLS). The load \( \{ F(t) \} \) is expressed by Eq. (2) and Fig. 1. The material properties are shown in Table 1, where yield stress and tensile strength are utilized later in the Goodman diagram for fatigue life evaluation. The thickness is 60mm. The voxel type mesh is used, whose number of elements and so on are listed in Table 2.

The assumed time dependent factor \( \alpha(t) \) in Eq. (1) is shown in Fig. 3. Two parameters \( T \) and \( F \) are utilized. The uncertainty is supposed for three load parameters as shown in Table 3. The final time to be solved is 2.0s as shown in Fig. 3. The time step is 1ms and the number of time steps is 2,000.

![Frame structure](image)

![Cross section of top beam member](image)

![Cross section of column](image)

**Fig. 2  Numerical example**

**Table 1  Material properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>200</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Mass density (kg/m³)</td>
<td>7,870</td>
</tr>
<tr>
<td>Yield stress (MPa)</td>
<td>828</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>727</td>
</tr>
</tbody>
</table>

**Table 2  Voxel finite element mesh**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements</td>
<td>77,320</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>116,523</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>349,569</td>
</tr>
<tr>
<td>Element size (mm)</td>
<td>30</td>
</tr>
</tbody>
</table>

**Table 3  Definition of probabilities for load parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expected value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>30°</td>
<td>2°</td>
</tr>
<tr>
<td>( T )</td>
<td>0.3 s</td>
<td>0.04 s</td>
</tr>
<tr>
<td>( F )</td>
<td>( 5.0 \times 10^5 ) N</td>
<td>( 5.0 \times 10^4 ) N</td>
</tr>
</tbody>
</table>

(Normal distribution)
The purposes of this analysis are
(i) to obtain the expected value with moderate accuracy for the mean stress and stress amplitude at point A in Fig. 2, and
(ii) to predict an extreme case with high mean stress and high stress amplitude.

3.2 Verification of model order reduction

The displacements were compared at some representative points calculated using 5, 10, 15 and 20 basis vectors for each unit vector \( \{F_x\} \) and \( \{F_y\} \), respectively. First, 20 basis vectors were generated. Next, model order reduction and implicit time integration were carried out on four PCs in parallel using 20 basis vectors and 5, 10 and 15 among 20 vectors.

Figure 4 shows the displacement at point A in Fig. 2. Since the vibration mode was very simple in this problem setting, all results agreed quite well. In other words, 5 basis vectors were enough to provide accurate results. This verification should be done before MC simulation once. Since less number of basis vectors leads to small sized system, 5 basis vectors for each unit vector, i.e., totally 10 d.o.f. equation, were adopted in the subsequent MC simulation.

We have also verified the results by MOR through comparison with a result obtained by a commercial FEM code, which adopted an explicit time integration scheme. Again, both results agreed very well as shown in Fig. 4.

Note that the computational time by MOR was much faster than the commercial code, because the small sized voxel element required small time step to satisfy the Courant condition when explicit method was adopted.

3.3 Convergence check for expected values

As mentioned in the above sections, approximately 349,569 d.o.f. system was reduced to only 10 d.o.f. system to be solved in MC simulation. The random number generated by Mersenne twister was transformed to normal distribution by Box Muller method. The expected values of the mean Mises stress and Mises stress amplitude were monitored according to the algorithm in section 2.2 (1) STEP 1.

After the first set of analyses, i.e., 100 cases, the expected value \( E_{(100)} \) and standard deviation \( \sigma_{(100)} \) of the mean stress were 64.3MPa and 7.22MPa, and those of the stress amplitude were 3.5MPa and 1.54MPa, in Eq. (6). The right hand side of Eq. (6) is 0.11% for mean stress, and 0.44% for stress amplitude. To this end, the threshold for the convergence check was set to be 0.1% for both mean stress and stress amplitude.

Figure 5 shows the values of the left hand side of Eq. (6). To satisfy Eq. (6) three times after each set of analyses, 1,500 analyses were needed for mean stress, whilst 3,400 analyses were needed for stress amplitude.
Therefore, the usual MC simulation is suspended after 3,400 analyses, and the resting computational effort is devoted to find an extreme case with high mean stress and stress amplitude.

![Convergence check for expected value](image)

**Fig. 5** Convergence check for expected value

3.4 Sampling

This problem includes three uncertainty parameters, $T$, $F$ and $\theta$. To determine the limited sampling criteria expressed by Eq. (7) according to 2.2 (2) STEP 2, choice of two parameters among three results in three sets of two parameters per QoI. Since two QoI are considered, i.e., mean stress and stress amplitude, six sets of two parameters were examined.

Figure 6 illustrates two important cases, i.e., one case for mean stress w.r.t. $F$ and $\theta$ and one for stress amplitude w.r.t. $\theta$ and $T$. Two inequalities Eq. (9) were obtained as shown in Fig. 6(a) and (b). On the other hand, no correlation was found in the resting four cases.

![Limited sampling highlighting on tail probability](image)

**Fig. 6** Limited sampling highlighting on tail probability

\[
F + 0.0999 \theta - 9.64 \geq 0
\]

\[
-T + 0.189 \geq 0
\]
\[
\begin{align*}
F + 0.0999\theta - 9.64 & \geq 0 \\
-T + 0.189 & \geq 0
\end{align*}
\]

Next, the probability in the above limited zone was estimated according to 2.2 (3) STEP 3. The probability in zone shown in Fig. 6(a) was 0.140\%, and that shown in Fig. 6(b) was 0.276\%. By multiplying them and taking the inverse, the computational efficiency was 258,799.

Since only 3,400 analyses were needed to obtain the convergence of the expected values of QoIs, 100 sampling points in the limited zone were analyzed. It means that totally 3,500 analyses are worth analyzing approximately 26 million analyses in the usual MC simulation.

3.5 Results

The mean stress and stress amplitude obtained by 3,500 analyses were plotted in the Goodman diagram, which is often used to evaluate the fatigue life, as shown in Fig. 7. Here, safety factor was supposed to be 5. In Fig. 7, red points are the results by the first 3,400 analyses to obtain the expected values, and blue points are the results by the subsequent 100 analyses to predict an extreme case.

Figure 7(a) implies that the frame structure is safe because all results are below the limit lines. Moreover, the blue points are obviously close to the limit lines. It means that the tail probability was evaluated by the proposed SLS scheme. Further discussion is described in the following chapter.

4. Discussion

4.1 Correlation between stress and load parameters

The results with high stress in the tail probability were due to unexpected combination of input parameters. Six cases with very high stress were examined in Table 4. It seems to
Table 4  Parameters that resulted in high stress amplitude

<table>
<thead>
<tr>
<th>No.</th>
<th>Stress amplitude (MPa)</th>
<th>Mean stress (MPa)</th>
<th>θ (°)</th>
<th>T (s)</th>
<th>F (×10^5N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>12.1</td>
<td>92.0</td>
<td>35.5</td>
<td>0.182</td>
<td>6.54</td>
</tr>
<tr>
<td>②</td>
<td>11.8</td>
<td>91.9</td>
<td>32.9</td>
<td>0.182</td>
<td>6.80</td>
</tr>
<tr>
<td>③</td>
<td>11.8</td>
<td>90.4</td>
<td>36.2</td>
<td>0.185</td>
<td>6.36</td>
</tr>
<tr>
<td>④</td>
<td>11.7</td>
<td>90.2</td>
<td>35.4</td>
<td>0.166</td>
<td>6.43</td>
</tr>
<tr>
<td>⑤</td>
<td>11.7</td>
<td>88.9</td>
<td>36.2</td>
<td>0.173</td>
<td>6.57</td>
</tr>
<tr>
<td>⑥</td>
<td>11.6</td>
<td>90.2</td>
<td>33.6</td>
<td>0.179</td>
<td>6.60</td>
</tr>
</tbody>
</table>

Fig. 8  Distribution of parameters $F$ and $\theta$ that resulted in high stress amplitude

(a) Distribution of parameter $T$

(b) Stress amplitude vs. parameter $T$

Fig. 9  Distribution of parameter $T$ that resulted in high stress amplitude
be reasonable that higher $F$ value leads to higher stress. However, $\theta$ and $T$ are not so straightforward. Figure 8 shows the distribution of $\theta$ in the limited zone, but a tendency is hardly found. Figure 9 shows the distribution of $T$ in the limited zone. In Fig. 9(a), we can not say that the extreme $T$ leads to the extremely high stress. Figure 9(b) compares the stress amplitude predicted by the first 3,400 analyses dotted in blue and those by the next 100 analyses dotted in red. This figure clearly tells the correlation between high stress and $T$. Interestingly, 100 results by the combination of input parameters in the limited zone have similar tendency but show higher stress. Therefore, we can understand that the usual MC simulation is suited to obtain the whole probability density function but may not be able to predict the tail probability with only 10,000 analyses.

In conclusion, if one needs to predict such an extreme case so as not to miss a single possibility that leads to fatal failure, the proposed SLS works better. It is not predictable by simply taking the combination of tail probability for all parameters, but the determination of the limited sampling criteria is important.

4.2 Comparison with usual Monte Carlo simulation

The advantage of SLS over usual MC simulation was pointed out in the demonstrative example. For more discussion, usual MC simulation with 10,000 sampling points was carried out.

Figure 10 shows the 10,000 random numbers typically shown for parameters $F$ and $\theta$. The sampling points in the tail probability zone are irregularly distributed. It implies that even using the Mersenne twister algorithm, if one wants to put highlight on the tail probability, a special care is needed like SLS.

The Goodman diagram is shown in Fig. 11. Comparing with Fig. 7 obtained by SLS, the usual MC simulation appended the results near the expected value. This is natural and, as mentioned before, usual MC simulation is suited to obtain the whole probability density function. Figure 12 shows the probability density function for the stress amplitude. Many analyses were devoted to raise the accuracy of the expected value. However, the maximum stress predicted was 10.9 MPa. On the contrary, the maximum stress predicted by SLS was approximately 12 MPa as shown in Table 4. We can say that the proposed SLS provides the expected value with allowable accuracy and quite reliable tail probability.

Finally, the correlation between the mean stress and the load parameter $F$ is shown in Fig. 13. We wrote in 4.1 that higher $F$ may lead to higher stress, but a positive correlation is not so clearly seen. This implies that we should not miss a case with unexpected combination of input parameters.

4.3 Computational cost

In this demonstrative analysis, four standard Windows X64 PCs were used. Every PC
Fig. 11  Goodman diagram with 10,000 cases without SLS

Fig. 12  Probability of stress amplitude for 10,000 cases

Fig. 13  Mean stress vs. load parameter F for 10,000 cases

had 8 GB core memory and 7.1 GB were available for calculation. Figure 14 shows the elapsed time to complete the analyses. The procedure by MOR and SLS are summarized in 5 steps as shown in Fig. 14.

In the first step [1] in Fig. 14, the calculation of the inverse matrix of the whole system requires long computational time and large amount of memory. To verify the necessary number of basis vectors in steps [2] and [3] in Fig. 14, four cases were analyzed in parallel and then examined on one PC in step [4] as described in 3.2. In the final step [5], totally 3,500 analyses were carried out on four PCs. Note that, by virtue of MOR, the required memory for the time integration (step [3] in Fig. 14) was dramatically reduced to 1.1 GB. This contributed to the parallel processing even on one PC.

Also, Fig. 14 implies that the overhead time for the verification of the number of basis vectors (steps [2] to [4] in Fig. 14) becomes negligible after MC simulation. Needless to say, the usual MC simulation with 10,000 analyses requires longer time in a scalable manner. Although we should tune the FORTRAN code to generate basis vectors, MOR and
SLS can put the uncertainty simulation into practice with highlighting on the tail probability.

<table>
<thead>
<tr>
<th>PC</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>A</td>
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<td>B</td>
<td></td>
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<tr>
<td>C</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
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</tbody>
</table>

![Schematic of computational time by parallel processing](image)

**5. Conclusion**

This paper aimed at predicting an expected value of stress under the dynamic load condition with uncertainty factors together with an extremely high stress that may happen with very low probability to be used in the fatigue life estimation based on Monte Carlo (MC) simulation. A model order reduction (MOR) technique, block second order Arnoldi (BSOAR) method, was adopted to reduce the computational time and memory requirement. The reduction of memory led to the parallel processing in the MC simulation. The demerit of MOR was the verification of necessary number of basis vectors, but it became negligible in the MC simulation. To put highlight on the tail probability not to miss a single case with high stress that may lead to fatal failure, a new sampling scheme named stepwise limited sampling (SLS) was proposed. Through demonstrative analysis, the advantage and efficiency of SLS was presented. Although the evaluation of the tail probability may not frequently be required, this practical sampling scheme will be effectively used in the reliability evaluation and design of structures.

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