Development of Automated Crack Propagation Analysis System

(Multiple Cracks and their Coalescence)*

Shuhei KANEKO**, Hiroshi OKADA** and Hiroshi KAWAI***

**Department of Mechanical Engineering, Faculty of Science and Technology,
Tokyo University of Science

2641 Yamazaki, Noda, Chiba 278-8510, Japan

***Department of Mechanical Engineering, Faculty of Systems Engineering,
Tokyo University of Science, Suwa

5000-1 Toyohira, Chino, Nagano 391-0292, Japan

E-mail: hokada@rs.noda.tus.ac.jp

Abstract

In this paper, the authors' recent developments on the crack propagation analysis software system are presented. The system has been developed based on three main parts i) finite element model generation, ii) finite element analysis and iii) stress intensity factor computations. We adopt the quadratic tetrahedral finite element and the processes in the finite element model generation are fully automated by using the Delaunay tessellation technique. The stress intensity factors are evaluated by using the virtual crack closure-integral method (VCCM) for the tetrahedral finite element. In previous papers, the authors have presented the crack analysis software system for single crack cases. It is extended to deal with the propagations of multiple cracks and their coalescences. The net-section yield criterion is adopted to judge the coalescence of adjacent cracks. In this paper, a brief review on the exiting crack propagation analysis system is presented first and present developments for the multiple cracks with their coalescence are discussed in detail. Finally, some numerical examples are presented.

Key words: Fracture Mechanics, Crack Propagation, Finite Element Method, Stress Intensity Factors

1. Introduction

In this paper, we present a new development in our on-going research work on the crack propagation analysis system. The system is extended so that it can deal with the crack propagation problems of arbitrary shaped multiple cracks. As many crack as the analyst desire can be modeled under the restriction of memory capacity of the computer. The system allows the cracks to coalesce each other. The processes of the multiple crack analysis with the crack coalescence are automated by extending the automatic finite element model generation schemes of Tokuda et al. [1], Kawai et al. [2] and Okada et al [3]. The crack propagation system and some numerical examples for the multiple cracks with their coalescence are presented in this paper.

It has been known that many cracks often initiate at a damage site. As seen in Wang et al. [4], fatigue cracks often initiate in welded structures. In nuclear industries, as presented by Shibata et al. [5], the initiations and the propagations of multiple cracks are of concern. They presented a detailed experimental study on interactions between adjacent cracks and their coalescence. Leek and Howard [6] presented a detailed study on the interactions and

*Received 18 May, 2012 (No. 12-0219)
[DOI: 10.1299/jcst.6.97]

the coalescences of neighboring surface flaws. They applied several criteria to judge the coalescence of the cracks. They assumed the surface flaws to be semi-elliptical in their shapes and the stress intensity factors at the deepest and the surface points are computed by the solutions of Newman, Jr. and Raju [7]. As the coalescence criteria, they discussed about those of ASME (ASME Boiler and Pressure Vessel Code, Section XI), BSI (British Standard Institution, PD 6493), Iida [8] and Leek and Howard [9]. The traditional codes assume one large semi-elliptical surface crack when two cracks meet the coalescence criteria. The experimental studies of Leek and Howard [6] and of Kamaya [10] showed that although the coalesced cracks eventually became elliptic in their shapes, they had different shapes immediately after the coalescences.

Finite element computations were also carried out by Kamaya [10,11]. The S-version finite element method (S-FEM) [12,13] was applied to the multiple crack problems by Kamaya, Miyokawa and Kikuchi [14]. With the conventional finite element method, the model generation is the bottle-neck process which generally takes much manual labor. Kamaya [10, 11] implemented a specialized procedure for the finite element model generation with the 20 node hexahedral finite element. Kamaya, Miyokawa and Kikuchi [14] applied the S-FEM to simplify the model generation process. In the S-FEM analysis, the finite element model to represent the crack and its vicinity only is superposed on that to model the structure as whole, as described by Okada et al [12]. It greatly reduces the model generation tasks.

In literature, we can find some software that can deal with crack problems such as FRANC3D/NG [15], FRANC3D [16] and ADPCrack3D [17,18]. They have been developed to perform crack propagation analyses. FRANC3D/NG and FRANC3D have been developed by a group of researchers at Cornell University. FRANC3D/NG and FRANC3D build the finite element models consisting of the tetrahedral elements except for at the immediate vicinity of the crack front. At the crack front region, the hexahedral finite elements are placed and are connected to the tetrahedral elements by using the pyramid elements. To evaluate the stress intensity factors, the interaction integral method [19,20,21] is adopted. ADAPCRACK3D was developed by a group of researchers of University of Paderborn. ADAPCRACK3D also uses the tetrahedral finite elements except for the crack region. At the vicinity of the crack, the hexahedral finite elements are placed. The crack region and the outside are connected by passing the displacements. The scheme is based on a sort of global-local analysis methodology. To evaluate the stress intensity factors, ADAPCRACK3D adopts the virtual crack closure-integral method (VCCM, also known as virtual crack closure-technique, VCCT) [22, 23, 24].

As mentioned above, the hexahedral finite elements are usually adopted at the vicinity of the crack front. Present authors think that the reasons are that the hexahedral finite elements have been considered to have superior accuracy over the tetrahedral finite elements and that the methodologies and the algorithms to evaluate the crack parameters such as the stress intensity factors and the energy release rates seemed to be developed with assuming the use of hexahedral finite elements (see, for example, Nikishkov and Atluri [25], Shivakumar and Raju [26], Chang and Wu [27] and Okada et al. [28]). They are probably the reasons why many crack analyses have been performed using the hexahedral finite elements. Even though the automatic meshing technologies for the tetrahedral finite elements are widely available, the cracks are modeled with the hexahedral finite elements. There are only a few previous papers which concerns the evaluations of the J-integral and the stress intensity factors based on the tetrahedral finite elements. They are J-integral computations by Rajaram, Socrate and Parks [29] using the domain integral method and by Nagai, Ikeda and Miyazaki [30] using a moving least square approximation to smooth the stresses, etc.

Okada et al. [31,32,33] recently presented a virtual crack closure-integral method

(VCCM) for the quadratic tetrahedral finite element and succeeded to evaluate the mixed mode stress intensity factors accurately. This development enabled us to use the finite element models which are consisting only with the tetrahedral finite element. Therefore, the processes in the finite element model generation simplify significantly since the mixture of different element types is not required any more. The ordinary Delaunay tessellation technique can be adopted to build the finite element models for the fracture analyses.

So far, we successfully developed mesh generation schemes for single crack case. The schemes to create the finite element models for the stationary and propagating crack cases were presented by Tokuda et al. [1], Kawai et al. [2] and Okada et al [3, 34]. The authors have proposed a point base system for the mesh generations. To insert features such as the cracks in an analysis model, we insert points to those representing the structure as whole. Hence, the Delaunay tessellation technique [35] is applied to generate the finite element model with tetrahedral elements. By adopting this technique, the mesh generation processes can fully be automated.

Present work extends the crack propagation analysis system so that it can deal with the multiple crack problems. We can assume as many cracks as we desire within the capacity of computer memory. In this paper, a brief review of present analysis system is presented first. Next, the extension of present analysis system to the multiple crack case, criterion for the judgment of crack coalescence and mesh generation scheme when two cracks coalesce are described. Finally, some numerical examples demonstrating the capability of present system are presented.

2. Preliminary: A Brief Review of the Crack Analysis System

In this section, a brief review of the crack propagation analysis system that the authors have developed so far, for the single crack problems, is presented. In the system, the analysis procedures are consisting of i) generation of finite element model without the crack (we call this model as “base model”), ii) vertex nodes of the finite elements are extracted from the finite element model and they are called “base seed nodes”, iii) surface patches are generated from the surface group information of the finite element model, iv) seed nodes for the initial crack are generated (we call them as “seed nodes for the initial crack”), v) the seed nodes (the crack model) for the crack are inserted in the base seed nodes (the base model), vi) the finite element model with a crack is generated by using the Delaunay tessellation method, vii) the boundary conditions are assigned to the finite element model and the finite element analysis is performed, viii) the stress intensity factors are computed and the rates and the directions of crack extension are predicted and ix) new sets of seed nodes to represent the crack after its extension are generated. In the crack propagation analysis, the procedures (i)~(iv) are performed only once and (v)~(ix) are repeated for the number of crack propagation steps. It is noted that the size of finite elements are controlled by the distances between the seed nodes at the crack front. The finite element size is controlled to be smaller than 1/100 of the representative sizes of crack such as its depth or half-surface length at their initial state. According to Okada et al. [31] the finite element size is small enough to accurately evaluate the stress intensity factors by using VCCM.

In Fig. 1, the finite element mesh generation processes (i)~(vi) are schematically presented. They are the keys to successfully develop the crack propagation system. The base seed nodes and the surface patches are extracted from the finite element model which is generated by commercial finite element mesh generation software. In present analyses, we used TSV/Pre of Technostar Co. Ltd., Tokyo, Japan. The finite element model and the surface patch are presented in the top part of Fig. 1. The base seed nodes, which are extracted from the finite element model, are depicted in the middle part of Fig. 1. The seed nodes for the crack are shown in the middle left position in Fig. 1. The seed nodes for the crack are inserted in the base seed nodes and the seed nodes with a crack are obtained.

Fig. 1 Generation of a finite element model in present crack propagation analysis system

Then, by applying the Delaunay tessellation method along with the surface patches, the finite element model with a crack is generated. More details are described in Tokuda et al. [1].

In a crack propagation analysis, the seed nodes are updated as the size and the shape of the crack evolve, as shown in Fig. 2. Seed nodes for the extended part of the crack are generated and are merged with those that are generated in previous crack propagation step. The seed nodes for the extended part and those from the previous step are presented in Figs. 2 (a) and (b). Hence, the seed nodes for preceding crack propagation step are created as shown in Fig. 2 (c). The points whose colors are dark blue and red are those on the crack front and on the crack face. Those with light blue color are all the others.

Once the stress intensity factors are computed along the crack front, the rate and the direction of crack propagation are predicted by using appropriate formulae. There are many formulae as listed in the survey paper of Qian and Fatemi [36]. In present research, we adopted the criterion of Richard et al. [37]. That is because the criterion of Richard et al. [37] seems to be most capable in accounting for the influence of the mode III stress

intensity factor. The rate of crack propagation is predicted by the Paris law.

The criterion of Richard et al. [37] expresses the kink and tilt angles (φ and ψ , as depicted in Fig. 3) of the crack, as:

$$\varphi = - \left[140^\circ \left(\frac{|\Delta K_{II}|}{\Delta K_I + |\Delta K_{II}| + |\Delta K_{III}|} \right) - 70^\circ \left(\frac{|\Delta K_{II}|}{\Delta K_I + |\Delta K_{II}| + |\Delta K_{III}|} \right)^2 \right] \text{sgn}(\Delta K_{II} / \Delta K_I) \quad (1)$$

$$\psi = - \left[78^\circ \left(\frac{|\Delta K_{III}|}{\Delta K_I + |\Delta K_{II}| + |\Delta K_{III}|} \right) - 33^\circ \left(\frac{|\Delta K_{III}|}{\Delta K_I + |\Delta K_{II}| + |\Delta K_{III}|} \right)^2 \right] \text{sgn}(\Delta K_{III} / \Delta K_I) \quad (2)$$

It is noted that although the tilt angle ψ is defined as shown in Eq. (2) the tilt of crack front results in the formation of the so-called factory roof crack face and it is almost impossible to generate such an analysis model. Therefore, we only adopt the kink angle in our crack propagation analyses. It is also noted that in absence of the mode III stress intensity factor, the kink angle of Eq. (1) gives almost the identical result as the maximum tangential stress (MTS) criterion. The readers are referred to Recharad et al. [37] for further details.

The amplitude of equivalent stress intensity factor is expressed by:

$$\Delta K_{eq} = \frac{\Delta K_I}{2} + \frac{1}{2} \sqrt{\Delta K_I^2 + 4(\alpha_1 \Delta K_{II})^2 + 4(\alpha_2 \Delta K_{III})^2} \quad (\alpha_1 = 1.155, \alpha_2 = 1) \quad (3)$$

Thus, ΔK_{eq} is employed to determine the rate da/dN , of crack propagation by using the Paris law [38], as:

$$\frac{da}{dN} = C(\Delta K_{eq})^n \quad (4)$$

3. Multiple Crack Analysis, Crack Coalescence-Criterion and Generation of Coalesced Crack Model

3.1 Multiple crack analysis

The crack propagation analysis system is extended so that multiple crack analyses can be performed. When the seed nodes for the multiple cracks are generated, each crack can be treated independently. The seed nodes for each crack are generated by exactly the same procedures as the single crack case. The insertion processes of the seed nodes for the cracks into the base seed nodes can be performed one by one, by using the existing program. Delaunay tessellation method is applied to the multiple crack case without any modifications from the single crack case. The evaluations of the stress intensity factors can also be carried out independently for each crack. The generations of seed nodes for the

Fig. 2 Generation of seed nodes for the crack propagation

Fig. 3 Kink and tilt angles of crack propagation

crack extension can also be performed independently for each crack.

Therefore, the processes of the generations of seed nodes for the initial cracks, their insertion, the evaluations of the stress intensity factors and the generations of seed nodes for the extended cracks are repeated for the number of cracks, as summarized in Fig. 4. The extension from the single to multiple crack propagation analysis system does not require modifications in many individual parts of the existing crack propagation analysis system. Their executions are repeated for the number of cracks.

When adjacent cracks approach to each other as the results of their propagations, their coalescence is judged and they are merged to be a single crack. Thus, the finite element model with the coalesced crack is generated and the crack propagation analysis resumes as a multiple or single crack problem. The procedures are presented in the flow chart of Fig. 4.

In subsequent subsections, the crack coalescence criterion and the generation of seed nodes for the coalesced crack are described in detail.

3.2 Net-section yield criterion for the judgment of crack coalescence

We adopt the net-section yield criterion as the crack coalescence criterion. The net-section yield criterion which is sometimes called as the plastic collapse criterion was applied to the problems of multi-site-damage cracks in panels and plates (see, for example, Tong et al. [39], Singh, Park and Atluri [40]). In this criterion, it is assumed that the load

Fig. 4 Analysis procedures of multiple crack propagation analysis considering their coalescence

carrying capacity of the material degrades once it experiences plastic deformation and the rate of crack propagation accelerates. Thus, it has been considered to be a good engineering crack coalescence criterion for multiple-site damage problems in aging aircraft structures. By adopting this criterion in analysis system, we expect that reasonably good engineering predictions on the crack coalescences can be achieved. It is also noted that the crack coalescence may be judged to occur when the distance between two adjacent cracks becomes zero, as suggested by Iida [8]. However, the finite element model generation and the evaluations of the stress intensity factors by VCCM become increasingly difficult as the distance between the adjacent cracks gets smaller. Therefore, we did not adopt such a coalescence criterion.

The size of plastic zone is estimated by a formula:

$$r_p = \frac{k}{2\pi} \left(\frac{K_I}{\sigma_Y} \right)^2 \quad (5)$$

where σ_Y is the yield stress of the material. In equation (5), the plastic zone size is expressed by the mode I stress intensity factor. k is a parameter whose typical value is 1. The mode I stress intensity factor is replaced by its amplitude in fatigue crack propagation problems. The equivalent stress intensity factor may replace the mode I stress intensity factor. It is noted that the mode I component dominates over the other components, after some amount of crack propagation. Therefore, the mode I and the equivalent stress intensity factor may give the similar results.

The radius r_p varies along the crack front. We assume a virtual circular pipe whose center is at the crack front and radius is r_p for each crack, as depicted in Fig. 5 (a). When the virtual circular pipes of adjacent cracks touch each other, we assume that the cracks coalesce. In the case of planar cracks, the coalescence of two adjacent cracks is as illustrated in Fig. 5 (b). The point A in Fig. 5 (b) indicates that the outer surfaces of the virtual pipes overlap each other. We define the point A to be a control point to define the crack front curve after the cracks coalesce.

3.3 Generation of coalesced crack model

Once the crack coalescence is found to occur, the crack front curves are connected before they advance according to the crack propagation law. The new crack front curve is defined so that it passes through the point "A", as also shown in Fig. 5. A spline approximation is adopted to create a smooth crack front curve connecting two original ones. Thus, sharp corners in the crack front curve which result in singular behavior in the distributions of the stress intensity factors along the crack front are eliminated. The detail of the spline approximation is depicted in Fig. 6. In Fig. 6, the point "A" in Fig. 5 is shown as

Fig. 5 Coalescence of two co-planar cracks

point "c". B-spline approximation for the crack front curve "a-c" is given by:

$$x_i^{a-c}(t) = (1-t)^2 x_i^a + 2t(1-t)x_i^b + t^2 x_i^c \quad (6)$$

where the range of variable t is $0 \leq t \leq 1$ and it takes 0, 0.5 and 1 at the x_i^a , x_i^b and x_i^c , respectively. x_i^a , x_i^b and x_i^c are the coordinate values of the points "a", "b" and "c", respectively. The spline curve "c-e" can also be defined in the same manner. The points "b" and "d" are at the intersecting points of the line parallel to the free surface passes through the points "c" and the crack fronts.

When the two cracks are not on the same plane, a plane parallel to the free surface is assumed. Intersecting points between that plane and the crack front curves are defined to be "b" and "d" in Fig. 6. Then, the plane that includes the line "b-d" and is perpendicular to the free surface is assumed to be the plane of coalesced part of the crack.

New seed nodes are generated on the coalesced part of the crack front first, as shown in Fig. 7 (a). Then, seed nodes on the crack face and at the ligament side are generated. They are placed on the straight lines passing through the points on the crack front curve. The lines are generated so that they make right angles with the free surface of the body. They are schematically illustrated in Fig. 7 (b). Then, seed nodes are placed on the lines so that finite elements having the same element widths across the crack front are generated and the new crack face at the vicinity of the coalesced part is filled by the points, as also shown in Fig. 7 (b). Then, they are combined with those for the existing part of the crack front. An example, after the generation processes are completed is depicted in Fig. 8 (a). The figure is drawn as the points are viewed from the perpendicular direction from the original planes of the cracks. The points whose colors are dark blue and red are those on the crack face and on the crack front. Those with light blue color are all the others.

The points are stacked in the perpendicular directions, as shown in Fig. 8 (b). Finally, they are combined with those of cracks before the coalescence as shown in Fig. 8 (c) and the seed nodes for the coalesced crack are generated as presented in Fig. 8 (d). These seed nodes are inserted in the global model by using the same way as those for the initial flaw.

3.4 Crack propagation after the coalescence

After the cracks coalesce, the crack propagation analysis resumes. Exactly the same algorithms as those for the crack propagation analysis without the coalescence are adopted.

4. Crack Propagation Analyses with the Crack Coalescences

Fig. 6 B-spline curves to represent the coalesced part of the crack front

Fig. 7 Seed nodal points that are placed at the part of crack coalescence

Fig. 8 The seed nodal points for the coalesced part of the crack

In order to demonstrate the present software system, four example problems are presented in this section. They are i) two cracks in the same plane, ii) two parallel cracks (iii) that do not coalesce, iii) two parallel cracks up to their coalescence and iv) three cracks in the same plane.

4.1 Two cracks on the same plane and their coalescence

Comparisons are made with those available in Leek and Howard [6]. Present solutions are compared with the experimental data and those using the solutions of Newman and Raju [7]. They are available in Leek and Howard [6]. Computations were also performed with assuming the coalescence criteria as summarized in Table 1. We call them as ASME and ASME 2010. They are usually adopted to judge the coalescence of and to recharacterize the shape and the size of neighboring flaws when they are detected by an inspection. However, it is noted that such criteria have sometimes been used to judge the crack coalescence in crack propagation analyses as presented in Leek and Howard [6, 9]. Thus, we perform analyses by using present analysis system with these criteria to examine the influences of the crack shape recharacterization to the crack propagation behaviors.

The geometry of the specimen and the configurations of the cracks are depicted in Fig. 9. The material was assumed to be A508 class 3 steel. The parameters for the Paris law that is presented as equation (4) are set to be $C=3.318 \times 10^{-12}$ and $n=3.056$. Young's modulus and Poisson's ratio were set to be 205.8 GPa and 0.3. Yield stress σ_Y is 441 MPa. Uniformly distributed cyclic load at the end plane of the tensile plate with the cracks is assumed. The amplitude of cyclic tension $\Delta\sigma$ is 80 MPa. The finite element discretization is given in Fig. 10 for the case of two cracks in the same plane as an example. Very fine finite elements are placed at the vicinity of the crack. Typical numbers of nodes and elements at the initial state are about 1,100,000 and 800,000, respectively, and they increase to about 1,350,000 and 1,000,000.

In Figs. 11, the initial cracks are shown by their surface meshes. The variation of the stress intensity factor amplitudes is also presented. The stress intensity factors are normalized by $\Delta\sigma\sqrt{\pi\bar{a}}$. $\bar{a}$ is the depths of initial cracks. In Fig. 12, the crack shapes when the crack coalescence occurs are depicted. It is seen in Fig. 12 (a) that the distance between the cracks becomes small so that the crack coalescence criterion is satisfied. The magnitude of the stress intensity factor increases at where the cracks approach each other. That is due to the mechanical interactions between the cracks. The crack coalescence is judged to occur according to present criterion. The shape of the crack after the coalescence and the variation of the stress intensity factor are shown in Fig. 12 (b). It is seen that the stress intensity factor increases at the portion of the crack coalescence. In Fig. 13, the crack after the growth is depicted. It is seen that the shape looks like semi-elliptic. That is because the stress intensity factor at the coalesced portion of the crack is much larger than that at the other locations. Thus, the rate of crack propagation is much faster at the coalesced portion than the others.

As the crack grows, after the coalescence, the magnitude of the stress intensity factor rises at both the sides of the surface crack. Therefore, the crack tends to enlarge along the surface directions and becomes a very shallow semi-elliptic in its shape, as depicted in Fig. 13.

In Figs 14 (a) and (b) the normalized depth and width of the cracks are presented. The definitions of the normalized depth and width are also depicted in the figures. It is seen that present analysis with the crack coalescence gives much closer predictions to the experiment than those using the Newman-Raju solutions and the ASME criterion for the crack coalescence. Comparing with that adopted the ASME 2010 criterion, present solution is closer to the experiment. The ASME 2010 and the net-section yield criteria judged crack coalescence at the same load cycle (205×10^3 cycles). Differences between them are only in the ways in the crack shape recharacterization when the cracks coalesce. The ASME criterion predicts the crack coalescence earlier than the others, resulting in much faster crack propagation.

4.2 Two parallel cracks without coalescence

Two initial cracks are placed parallel to each other but not in the same plane. The initial crack configurations are as depicted in Fig. 9 (c). The loading conditions and the specimen geometries are presented in Fig. 9 (a). The loading conditions and the material constants are the same as those in previous subsection. The results are compared with Leek and Haward [6]. The finite element mesh at the initial state is presented in Fig. 15 (a). As they grow, they

Fig. 9 The configurations of initial cracks which are analyzed as example problems

Table 1 The Coalescence criteria (ASME and ASME 2010)

Distance between the cracks	ASME	ASME 2010
Horizontal direction (s)	$s \leq 2 \times \max(a_1, a_2)$	$s \leq 0.5 \times \max(a_1, a_2)$
Vertical direction (d)	$d \leq 12.7 \text{ mm}$	$d \leq 12.7 \text{ mm}$
After the coalescence	$a = \max(a_1, a_2), 2c_{New} = 2c_1 + 2c_2 + s$

(a) Whole view

(b) Surface magnified view at the vicinity of the cracks

(c) Crack faces

Fig. 10 A typical finite element mesh discretization for the plural crack problems

approach to each other, but they do not collide with each other. The normalized depth and the width of the crack are compared with those in the paper. Present predictions result in much faster crack propagation than those using the solutions of Newman-Raju [7] (ASME). ASME gives slower crack propagation than the experiment whereas present prediction is faster than the experiment.

4.3 Two parallel cracks with coalescence.

A plate with two parallel cracks is subject to a cyclic tension. The specimen geometries and loading conditions are depicted in Fig. 17 (a). As depicted in Fig. 17 (b), two initial cracks are placed parallel to each other but not in the same plane. The material is assumed to be SUS316 stainless steel. Young's modulus and Poisson's ratio are set to be 210 GPa and 0.3, respectively. The constants in Paris law C and n are 2.085×10^{-12} and 0.3, respectively. The amplitude of applied tensile stress $\Delta\sigma$ is 137 MPa. The distance between the cracks in their perpendicular direction is smaller than the case of previous subsection. The cracks eventually collide with each other. This is a hypothetical problem and we do not have experimental data to compare with. Therefore, only a qualitative discussion is presented. The crack geometries at the step of crack coalescence are presented

Fig. 11 Distribution of normalized mode I stress intensity factor for the initial crack configuration and its discretization of crack surface

Fig. 12 Variations of the mode I stress intensity factor with respect to the shape of the cracks when the crack coalescence is judged to occur (205×10^3 cycles)

Fig. 13 Variation of the mode I stress intensity factor with respect to the shape of the cracks after the crack coalescence (314×10^3 cycles)

(a) Depth of the crack(s) [$a_{\max}$: maximum depth, t : plate thickness]

(b) Total width of the crack(s) [l : total width of cracks ($2c_1+2c_2+s$), $2w$: total plate width]

Fig. 14 The depth and the total surface width of the crack(s)

(a) Initial model (crack faces and plate surface)

(b) After 392×10^3 cycles (crack faces and plate surface)

Fig. 15 Mesh discretizations of initial cracks and after 392×10^3 cycles (crack faces and the

(a) Depth of the cracks [$a_{\max}$: maximum depth, t : plate thickness]

(b) Total width of the cracks [l : total width of cracks ($2c_1+2c_2+s$), $2w$: total plate width]

Fig. 16 The variations of the depth of and the total width of the cracks

Figs. 18 (a) and (b). When the crack geometries are as presented in Fig. 18 (a), the crack coalescence is judged to take place. Then, the coalesced crack is generated as depicted in Fig. 18 (b). In Fig. 19, a three-dimensional view of the crack surface is depicted. It is seen from Figs. 18 (b) and 19 that a smooth crack surface is produced after the coalescence.

4.4 Three cracks in the same plane and their coalescence

This is also a hypothetical problem to demonstrate the capabilities of present software

system. The specimen geometries and the loading conditions are the same as those in previous subsection. They are depicted in Fig. 17 (a). Three initial cracks are placed in the same plane as depicted in Fig. 17 (c). Their initial shapes are different. The finite element mesh view of the cracks at the initial state is presented in Fig. 20 (a). After they grow, they eventually coalesce. Figs. 20 (b) and (c) show the cracks just before and after the coalescences are judged to occur. The crack coalescences take place simultaneously. Similar to the case of two initial cracks, the stress intensity factor at the part of crack coalescence is much larger than that at the other locations. Then, the crack grows and the crack shape becomes a large shallow crack as shown in Fig. 20 (d).

5. Conclusions

In this paper, the outlines of crack propagation analysis system to deal with multiple crack problems with their coalescence and some numerical examples are presented. The outlines of the software are described for a general case that the surface of the body has some curvatures. Although present example problems have only planar external surfaces, it can deal with the case of more complex cases as described in Okada et al. [34]. The results of present analysis system can give us some detail information about the crack propagation behavior before and after their coalescence. Thus, present method may give somewhat different predictions from those using the conventional coalescence criteria which we call as ASME and ASME 2010 in this paper. From a limited number of examples which were compared with the existing predictions and experimental data, present method seems to give closer solutions to the experiments. That is because present method does not recharacterize the crack to be half-elliptical one after the crack coalescence. Even when the finite element method is adopted, it is sometimes difficult to create the finite element models for arbitrary shaped cracks. In such cases, the crack shapes must be recharacterized as suggested by ASME and ASME 2010 criteria.

When there are more than two cracks connecting one after another, present way may give more realistic predictions than the conventional criteria. The result of the three-crack problem that is presented in this paper indicates that small surface cracks coalesce one after another, resulting in a large shallow crack. When there are more cracks, such behavior is expected to be more pronounced. It is known that many surface cracks initiate due to stress corrosion cracking (SCC) and present method may become a useful tool for the problems of SCCs.

Finally, it is pointed out that present method can be applied to general three-dimensional structures. The applications are not limited in simple laboratory specimens. Therefore, the authors believe that present method will be a very useful tool for the structural integrity problems of real structures.

Fig. 17 The configurations of initial cracks which are analyzed as example problems (the propagation of two parallel cracks and three cracks and their coalescence)

Fig. 18 The shapes of the crack(s) before and after the coalescence (plane view and surface of the specimen)

Fig. 19 A three-dimensional view of the crack after the coalescence

(c) Coalesced crack and the variation of the mode I stress intensity factor (300×10^3 cycles) (d) Coalesced crack and the variation of the mode I stress intensity factor (399×10^3 cycles)

Fig. 20 The evolutions of shapes of cracks and the variations of the mode I stress intensity factors after the crack coalescence

Acknowledgement

A part of present research was conducted under the support of Grant in Aid for Scientific Research (c), No. 22560149. H.O. expresses his sincere gratitude to the support.

References

1. Tokuda, T., Kawai, H., Okada, H. and Fukui, Y., Development of Automated Crack Propagation Analysis System (1st Report, Outlines of the System and Finite Element Model Generation), Trans JSME Series A, Vol. 76, No. 770, pp. 1255-1262, 2010.
2. Kawai, H., Okada, H. and Araki, K., A mesh generation technique for three dimensional crack propagation analysis using virtual crack closure-integral method (VCCM) and tetrahedral finite element, Trans JSME Series A, 74-742, pp. 819-826, 2008.

3. Okada, H., Kawai, H., Tokuda, T., and Fukui, Y., Development of Automated Crack Propagation Analysis System (2nd Report, the Crack Propagation Analysis System and Finite Element Model Generation for the Crack Propagation), Trans JSME Series A, Vol. 76, No. 772, pp. 1681-1688, 2010.
4. Wang, Y., Tomita, Y., Hashimoto, K., Osawa, N. and Terai, K., Study on setting the conditions of initial cracks for fatigue strength evaluation of welded structures, Journal of the Society of Naval Architects of Japan (192), pp. 545-553, 2002.
5. Shibata, K. Yokoyama, N., Ohba, T., Kawamura, T. and Miyazono, S., Growth evaluation of fatigue cracks from multiple surface flaws, Journal of the Atomic Energy Society of Japan, Vol. 27, No. 3, pp. 250-262, 1985.
6. Leek, T. H. and Howard, I. C., An examination of method of assessing interacting surface cracks by comparison with experimental data, Int. J. Pres. Ves. & Piping, Vol. 68, pp. 181-201, 1996.
7. Newman, Jr. J.C. and Raju, I. S., An empirical stress intensity factor equation for the surface cracks, Engineering Fracture Mechanics, Vol. 15, pp. 185-192, 1981.
8. Iida, K., Shapes can coalescence of surface fatigue cracks, Proceedings of I.C.F. International Symposium on Fracture Mechanics, (1983), pp. 679-693.
9. Leek, T. H. and Howard, I. C., Rules for the assessment of interacting surface cracks under mode I load, Int. J. Pres. Ves. & Piping, Vol. 60, pp. 323-339, 1994.
10. Kamaya, M., Growth evaluation of multiple interacting surface cracks. Part I: Experiments and simulation of coalesced crack, Engineering Fracture Mechanics, vol. 75 (2008) 1336-1349.
11. Kamaya, M., Growth evaluation of multiple interacting surface cracks. Part II: growth evaluation of parallel cracks, Engineering Fracture Mechanics, Vol. 75 (2008) 1350-1366.
12. Okada, H., Endoh, S. and Kikuchi, M., On fracture analysis using an element overlay technique, Engineering Fracture Mechanics, Vol. 72, (2005) 773-789.
13. Fish, J., The -s-version of the finite element method, Computers & Structures, Vol. 43, No. 3, pp. 539-547, 1992.
14. Kamaya, M., Miyokawa, E. and Kikuchi, M., Growth prediction of two interacting surface cracks of dissimilar sizes, Engineering Fracture Mechanics, Vol. 77, (2010) 3120-3131.
15. Wawrzynek, P. A., Carter, B. J. and Ingraffea, A. R., Advances in simulation of arbitrary 3D crack growth using FRANC3D/NG, 12th International Conference on Fracture (ICF12) : held July 12-17, 2009 in Ottawa, Canada.
16. Ural, A., Heber, G., Wawrzynek, P. A., Ingraffea, A. R., Lewicki, D. G. and Neto, J. B. C., Three-dimensional, parallel, finite element simulation of fatigue crack growth in a spiral pinion gear, Engineering Fracture Mechanics, Vol. 72, (2005), pp. 1148-1170.
17. Schöllman, M., Fulland, M. and Richard, H. A., Development of a new software for adaptive crack growth simulations in 3D structures, Engineering Fracture Mechanics, Vol. 70, (2003), pp. 249-268.
18. Richard, H. A., Sander, M., Fulland, M. and Kullmer, G., Development of fatigue growth in real structures, Engineering Fracture Mechanics, Vol. 75, (2008), pp. 331-340.
19. Banks-Sills, L., Update: Application of the finite element method to linear elastic fracture mechanics, Applied Mechanics Reviews, Vol. 63, (2010), 020803.
20. Banks-Sills, L., Hershkovitz, I., Wawrzynek, P. A., Eliasi, R. and Ingraffea, R., Methods for calculating stress intensity factors in anisotropic materials: Part I- $z=0$ is a symmetric plane, Engineering Fracture Mechanics, Vol. 72, (2005), pp. 2328-2358.
21. Gosz, M. and Moran, B., An interaction energy integral method for computation of mixed-mode stress intensity factors along non-planar crack fronts in three dimensions, Engineering Fracture Mechanics, Vol. 69, (2002), pp. 299-319.
22. Rybicki, E. and F. Kanninen, M. F. A finite element calculation of stress intensity factors by a modified crack closure integral, Engineering Fracture Mechanics, Vol. 9 (1977), pp.

- 931-938.
23. Shivakumar, K. N., Tan, P. W. and Newman Jr., J. C., A virtual crack-closure technique for calculating stress intensity factors for cracked three dimensional bodies, *International Journal of Fracture*, Vol. 36, (1988), pp. R43-R50.
 24. Fawaz, S. A., Application of the virtual crack closure technique to calculate stress intensity factors for through cracks with an elliptical crack front, *Engineering Fracture Mechanics*, Vol. 59, (1998), pp. 327-342.
 25. Nikishkov, G. P. and Atluri, S. N., Calculation of fracture Mechanics parameters for an arbitrary three-dimensional crack, by the equivalent domain integral method, *International Journal for Numerical Methods in Engineering*, Vol. 24, Issue 9, pp. 1801–1821, 1987.
 26. Shivakumar, K. N. and Raju, I. S., An equivalent domain integral method for three-dimensional mixed-mode fracture problems, *Engineering Fracture Mechanics*, Vol. 42, No. 6, pp. 935-959.
 27. Chang, J. H. and Wu, D. J., Stress intensity factor computation along a non-planar curved crack in three-dimensions, *International Journal of Solids and Structures*, Vol. 44, (2007), pp. 371-386.
 28. Okada, H., Higashi, M., Kikuchi, M., Fukui, Y. and Kumazawa, N., Three dimensional Virtual Crack Closure-Integral Method (VCCM) with skewed and non-symmetric mesh arrangement at the crack front, submitted to *Engineering Fracture Mechanics*, vol. 72, pp. 1717-1737, 2005.
 29. Rajaram, H., Socrate, S. and Parks, D. M., Application of domain integral methods using tetrahedral elements to the determination of stress intensity factors, *Engineering Fracture Mechanics*, Vol. 66, pp. 455-482, 2000.
 30. Nagai, M., Ikeda, T. and Miyazaki, N., Stress intensity factor analysis of a three-dimensional interface crack between dissimilar anisotropic materials, *Engineering Fracture Mechanics*, Vol. 74, (2007), pp. 2481-2497.
 31. Okada, H., Kawai, H. and Araki, K., A virtual crack closure-integral method (VCCM) to compute the energy release rates and stress intensity factors based on quadratic tetrahedral finite elements, *Engineering Fracture Mechanics*, Vol. 75, (2008), pp. 4466–4485.
 32. Okada, H., Araki, K. and Kawai, H., Stress intensity factor Evaluation for large scale finite element analyses (Virtual Crack Closure-Integral Method (VCCM) for Tetrahedral Finite Element), *Trans JSME Series A*, 73-728, (2007), pp. 498-505.
 33. Okada, H., Araki, K. and Kawai, H., Stress intensity factor Evaluation for large scale finite element analyses (virtual crack closure-integral method (VCCM) for mixed mode/complex shaped crack using tetrahedral finite element) , *Trans JSME Series A*, 73-733, (2007), pp. 997-1004.
 34. Okada, H., Kawai, H., Tokuda, T. and Fukui, Y., Fully automated mixed mode crack propagation analysis based on tetrahedral finite element and VCCM (Virtual Crack Closure-Integral Method), *International Journal of Fatigue*, In Press, Available online, 2012.
 35. Taniguchi, T. and Moriwaki, K., Automatic Mesh Generation for 3-D FEM, (2006), Morikita.
 36. Qian, J. and Fatemi, A., Mixed mode fracture crack growth: A literature survey, *Engineering Fracture Mechanics*, Vol. 55, (1996), pp. 969-990.
 37. Richard, H. A., Fulland, M. and Sander, M. Theoretical crack path prediction, *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 28, (2005), pp. 3-12.
 38. Paris, P. and Erdogan, F., A critical analysis of crack propagation laws, *J. Basic Engng.*, Vol. 85, (1963), pp. 528-534.
 39. Tong, P., Greif, R. and Chen, L., Residual strength of aircraft panels with multiple site damage, *Computational Mechanics*, Vol. 13, (1994), pp. 285-294.
 40. Singh, R., Park, J. H. and Atluri, S. N., Growth of multiple cracks and their linkup in a fuselage lap joint, *AIAA Journal*, Vol. 32, (1994), pp. 2260-2268.