Design Optimization of Shielding Effect for Aircraft Engine Noise*

Kazuhisa CHIBA**, Taro IMAMURA**, Kazuhisa AMEMIYA** and Kazuomi YAMAMOTO**
** Japan Aerospace Exploration Agency
7–44–1 Jindaiji-Higashi, Chofu, Tokyo 182–8522, Japan
E-mail: chiba@chofu.jaxa.jp

Abstract
The present study is a technical report of the multi-objective design optimization of the two-dimensional shielding effect for the reduction of aircraft engine fan noise. The design optimization has been performed for the simplified cross section of aircraft with fuselage and plane wing. Two objective functions are considered for minimizing the sound pressure level at the side and the bottom locations relative to the fuselage, where standard measuring locations are defined by International Civil Aviation Organization. Those values are evaluated by using the linearized Euler equations. Since a wing, which is defined as a plate without thickness, is assumed as a V-tail wing, it is described by using the length and cant angle relative to the fuselage. The kriging-based response surface model is selected as an optimizer for the reduction of optimization cost. As a result, it is revealed that engine fan noise described by a monopole sound source is reduced by the shielding effect. Moreover, there is no tradeoff between two objective functions, i.e., the sound pressure levels at the side and the bottom measuring locations can be simultaneously reduced. As it is better that the wing length is as long as possible, the cant angle is essential for the shielding effect to reduce engine fan noise.

Key words : Engine Fan Noise, Shielding Effect, Design Optimization, Kriging Model, Computational Aeroacoustics

1. Introduction

Engine noise is one of the most important items for a jet aircraft design. Although the devices for engine itself has been investigated as the manner for the noise reduction, their improvement reaches the ceiling. The other manners which great reduction is anticipated should be considered. One of the considerable manners is the shielding effect using wing and fuselage(1). An engine is mounted above an aft-circular fuselage portion and V-tail wing is set on. Engine noise has two specifications as jet and fan noises. As the source of jet noise sets on the rearward far-location from exhaust exit, it is practically difficult to shield. On the other hand, it is possible to put on the shield for fan noise because it propagates to side location. In this study, fan noise is focused.

The prediction of the aerodynamic sound induced by the unsteady flow field is important. There are various approaches to simulate the flow field with sound propagation. The direct numerical simulation (DNS) which treats density variation associated with a sound generation is a leading candidate for accurate simulation for the generation and propagation of aerodynamic sound. However, DNS is quite costly computation. On the other hand, the computation based on the linearized Euler equations (LEE) is one of the accurate methods capturing the propagation of sound. The essential key of this method is to reduce the computational time compared with DNS. But, it takes much computational cost for three-dimensional simulation yet. Therefore, the problem is simplified as two dimension on the plane to cut the aft-fuselage in a round slice.
The objective of this study is to investigate whether the shielding effect can be applied to engine fan noise. As a flow condition and practical frequency for fan noise are not considered because of the assumption of two-dimensional simplification and a mesh accuracy, practical design knowledge is not obtained. However, this is the first step to know whether the shielding effect can be applied, and to construct the design system for silent aircraft design.

2. Definition of Design Optimization

2.1. Objective Functions

Two objective functions are defined as the minimization of the sound pressure level (SPL) at the side and the bottom locations relative to the fuselage, because these measuring locations are defined by International Civil Aviation Organization. In this study, a plane at aft-fuselage shown in Fig. 1 is assumed as a computational configuration. The evaluation positions are set on (0, 2.5) and (-2.5, 0) shown in Fig. 2. An SPL value is evaluated as an integrated value for a wavelength around a measuring position. SPL is generally calculated as the following equation.

$$\text{SPL} = 20 \log \frac{P_1}{P_0}$$  \hspace{1cm} (1)

where,

$$P_1 = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} p_2^2 dt}$$ \hspace{1cm} (2)

The reference sound pressure $P_0$ is set on $P_0 = 2 \times 10^{-5}$ [N/m²].

2.2. Geometry Definition

It is assumed that an engine is mounted above an aft-fuselage portion, the shielding effect is investigated on the plane to cut the aft-fuselage in circular cross section slices. The cross
Table 1 Range of two design variables.

<table>
<thead>
<tr>
<th>serial number</th>
<th>correspondent design variable</th>
<th>range of design variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>wing cant angle $\gamma$</td>
<td>$30^\circ \leq \gamma \leq 120^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>wing spanwise length $\ell$</td>
<td>$1.2\text{m} \leq \ell \leq 5.7\text{m}$</td>
</tr>
</tbody>
</table>

section of the fuselage is defined by a circle. When its diameter is set on 3m, the monopole sound source is set over 0.45m from the top of fuselage shown in Fig. 2, and the measuring position is set on 7.5m from the origin. It is assumed that the sound source of engine noise is the monopole in this study. It is not appropriate to use statistical energy analysis (SEA) method for engine noise because the dominant frequency is relatively low and the SEA parameters for the computations are difficult to define. Also, far-field SPL estimation using Curle’s equation is difficult to apply because compact body assumption cannot be used to calculate the propagation of the acoustic wave. Furthermore, it is difficult to estimate the noise source of a jet engine numerically. In the present state of affair, both accurate noise source estimation of a jet engine as well as the accurate computation of the acoustic wave propagation are necessary for the purpose of the accurate prediction of engine noise. Based on these backgrounds, as for the noise source, fan noise which has discrete tones is considered. Engine fan noise is thus modeled as a monopole source. When the reference length is 3m, the frequency of monopole sound source is approximately 500Hz. Although the practical frequency of engine noise is higher, it takes impractical time to evaluate because the 10 mesh points in a wavelength should be generated to maintain the sufficient resolution for LEE evaluation. It takes roughly four hours for the mesh with sufficient resolution for 500Hz using five CPUs of a PC cluster with Pentium IV 2.2GHz processor.

The wing is defined by the two design variables as the cant angle $\gamma$ relative to the fuselage and the spanwise length $\ell$. The thickness is ignored because the wing is defined by giving of the wall boundary condition on mesh block boundary. The constraints in an optimizer are considered for the wing geometry. The range of the constraints is summarized in Table 1.

2.3. Evaluation Method

Fan noise source is modeled as a monopole source and wave propagation is estimated using the LEE. UPACS-LEE, developed as an extension of unified platform for aerospace computational simulation (UPACS) by multi-block structured computational fluid dynamics (CFD) solver, is employed for a computational aeroacoustics (CAA) solver on multi-block structured mesh shown in Fig. 3.

2.3.1. Governing Equations

The governing equations for the linearized Euler equations are derived as follows. First, the conservative variables $q$ are defined as the sum of mean flow components $\bar{q}$ and fluctuating components $q'$.

$$q = \bar{q} + q'$$ (3)

Equation 3 is substituted into Euler equations, and the fluctuating terms that are higher than second order are neglected under the assumption that those terms are sufficiently small. The governing equations are described as follows.

$$\frac{\partial \rho'}{\partial t} + \frac{\partial \left( \rho u_j \right)'}{\partial x_j} = S_\rho$$ (4)

$$\frac{\partial \left( \rho u_j \right)'}{\partial t} + \frac{\partial}{\partial x_j} \left[ -\bar{u}_j \bar{u}_i' + \bar{u}_i \left( \rho u_j \right)' + \left( \rho u_j \right)' \bar{u}_j + p' \delta_{ij} \right] = S_{\rho u_i}$$ (5)

$$\frac{\partial E'}{\partial t} + \frac{\partial}{\partial x_j} \left[ -\bar{u}_j B \rho' + \bar{u}_j \left( \rho H \right)' + \left( \rho u_j \right)' B + p' \delta_{ij} \right] = S_E$$ (6)

where,

$$p' = (\gamma - 1) \left( E' + \frac{1}{2} \bar{u}_k^2 - \bar{u}_k \left( \rho u_k \right)' \right)$$ (7)
\[ (\rho H)' = E' + p' \]  \hspace{1cm} (8)

The variables \( \rho, \rho u, E \) in the above equations are the density, momentum, and total energy, respectively. Also, \( p \) is the pressure, \( H \) is the total enthalpy, and \( \gamma \) is specific heat ratio. The right hand side (R.H.S.) of Eqs. 2-4 describes the noise source term. The governing equations are nondimensionalized by the mean flow density, sonic speed, and reference length of the flow.

In the LEE, the mean flow properties such as \([\rho, \rho u, E]\) and the noise source on the R.H.S. \([S_{\rho}, S_{\rho u}, S_E]\) are given as the computational conditions, and the fluctuating components, such as \([\rho', (\rho u)' , E']\), are solved. The mean flow can be given by assuming uniform flow or by using the numerical result which is computed a priori. The noise source can be given by assuming the monopole noise source, which is given as

\[
S = \begin{bmatrix}
S_{\rho} \\
S_{\rho u} \\
S_E
\end{bmatrix} = \begin{bmatrix}
S \\
\bar{u} S \\
\left( \frac{c^2}{\gamma^2} + \frac{1}{2}\eta^2 \right) S
\end{bmatrix}
\]  \hspace{1cm} (9)

where, \( c \) is the sonic speed. \( S \) describes the Gaussian distribution as,

\[
S = A \exp \left[ -\ln(2) \left( \frac{(x-x_s)^2 + (y-y_s)^2}{b^2} \right) \right] \sin \omega t
\]  \hspace{1cm} (10)

where, \((x, y)\) and \( t \) are the Cartesian coordinate and time, respectively. Also, \( A, b, (x_s, y_s)\), and \( \omega \) are the amplitude, half-value width of Gaussian distribution, the coordinate of the noise source, and the angular frequency of the monopole. \( \omega \) is described as \( \omega = 2\pi f_n \). As \( f_n \) is the nondimensionalized frequency of monopole sound source, \( f_n \) is described as \( f_n = 500 \text{Hz} \cdot \ell_{\text{ref}} / c \). \( \ell_{\text{ref}} \) denotes the reference length.

### 2.3.2. Computational Methods

CAA computations focus on the propagation of the acoustic waves, thus to prevent the dissipation and dispersion of the acoustic waves, careful treatments are required in the simulations. In general, number of mesh points within the wave length of the particular frequency can be reduced by using the high-order schemes. In UPACS-LEE code, finite volume type of sixth order compact scheme developed by Kobayashi(5) is implemented. To avoid the numerical oscillation, 10th order filtering method proposed by
Gaitonde-Visbal\(^6\) is applied, and the filter control parameter is set as \(\alpha = 0.45\). For time integration, fourth order Jameson-Baker Runge-Kutta method\(^7\) is used. In the present study, the size of the computational domain is \(-2.6 < x < 2.6, 0 < y < 2.6\), and around the region of interest, sponge region\(^8\) is generated to damp the acoustic wave. Note that the mesh for CAA is generated isotropically in the region of interest, and 10 points per wave is sufficient to capture the acoustic wave propagation accurately. The mesh is stretched to generate sponge region around the region of interest. This treatment is necessary to prevent the reflection of the waves at the outer boundary. The boundary condition for slip wall suggested by Chakravarthy et al.\(^9\) is used for the wall boundary condition.

### 2.4. Optimizer

Current engineering analyses heavily depend on expensive and complex computation. Statistical techniques are widely used in engineering design to construct approximations of these analyses. Therefore, these approximations are used in place of the exact analyses to obtain the following benefits; 1) Approximations yield insight into the relationship between output responses \(y\) and input design variables \(x\) for small evaluation time. 2) Approximations provide fast analysis tools for optimization and design space exploration since inexpensive approximations are employed instead of expensive exact computations. A common method for constructing approximations of computer analyses is to apply design of experiments (DOE)\(^{10,11}\), response surface models (RSMs)\(^{12,13}\), and regression analysis to construct polynomial approximations of computationally expensive analyses. Since computer experiments typically lack random error, a more appropriate and more statistical method for approximating deterministic computer experiments is investigated through the use of kriging model. The validity of kriging model does not depend on the existence of random error and may be better suited for applications involving computer experiments.

Constructing approximations of computer analyses typically involve the following processes; a) selecting an experimental design to sample computer analysis, b) selecting a model to represent data, and c) fitting the model to the observed data. There are a variety of options for each of these selections, and several advantages and disadvantages with emphasis on response surface methodologies, neural networks, inductive learning and kriging model.

#### 2.4.1. Response Surface Model

RSMs are surrogate metamodels produced by curve-fitting techniques to samples of computationally expensive data. They are widely used in the design community when performing optimization studies on expensive simulation codes. There are a number of variations and refinements that may be applied to the basic RSM approach. One of the possible alternatives is DOE to generate the initial set of points and a kriging model applied to construct an RSM. Most DOE methods seek to sample efficiently the entire design space by constructing an array of possible designs with relatively even but not constant spacing between the points.

Techniques of response surface modeling were originally developed to analyze the results of physical experiments and create empirically-based models of the observed response values. Response surface modeling postulates a model of the following form;

\[
y(x) = f(x) + \epsilon
\]

where \(y(x)\) denotes the unknown function of interest, \(f(x)\) is a known polynomial function of \(x\), and \(\epsilon\) is random error which is assumed to be normally distributed with mean zero and variance \(\sigma^2\). The individual errors \(\epsilon_i\) at each observation are also assumed to be independent and identically distributed. The polynomial function \(f(x)\) used to approximate \(y(x)\) is typically a low order polynomial described as the following linear or quadratic equations.

\[
\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i
\]

\[
\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{j=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j
\]
The parameters $\beta_0, \beta_i, \beta_{ii},$ and $\beta_{ij}$ of the polynomials in Eqs. 12 and 13 are determined through least-squares regression which minimizes the sum of the squares of the deviations of predicted values $\hat{y}(x)$ from the actual values $y(x)$. The coefficients of Eqs. 12 and 13 used to fit the model can be found using least-squares regression given by the following equation.

$$\beta = \frac{X'Y}{X'X}$$

where $X$ denotes the design matrix of sample data points, $X'$ is its transpose, and $y$ is a column vector containing the values of the response at each sample point.

The general approach for constructing polynomial RSMs is shown in Fig. 4. A three-step process, screening, model constructing, and model exercising, is typically employed. As shown in Fig. 4, the first step involves screening which may be employed when there is a large number of factors to reduce the design space to an appropriate region of interest. In the second step, the approximation models are constructed from sample data which is obtained from an appropriately selected experimental design. When there are noise factors in the design, robustness models of the mean and variance of each response would be also created. When the models are sufficiently accurate, the model is exercised in the last stage of the process to search the design space and find improved or robust solutions.

2.4.2. Kriging Model

The most obvious forms of regression are those using least-squares polynomials. However, they are not good at modeling complex surfaces that have many local basins and bulges in them. In the present study, a kriging approach is used instead because this allows the use to control the amount of regression as well as providing a theoretically sound basis for judging the degree of curvature needed to model adequately the user’s data. In addition, kriging provides measures of probable errors in model being built that can be used when assessing where to place any further design points.

In kriging, the inputs $x$ are assumed to be related to the outputs, that is, responses $y$ by an expensive function $f(x)$. Here, this function is UPACS-LEE code to analyze sound pressure. The response of the code is then evaluated for combinations of inputs generated by the DOE
Construct inputs using DOE

Evaluate \( f_c \) at these points

Construct Krig for \( f_c \)

Find \( x^* \) such that
\[
\begin{align*}
\arg & \min_{x} f_c(x) \\
\text{subject to} & \text{any constraints}
\end{align*}
\]

Convergence? %

\%6

\%6

\%6

Evaluate \( f_c(x^*) \) and add to model

Stop

Fig. 5 Kriging procedure for function \( f_c \).

with D-optimality and used to construct an approximation.

The present kriging model describes the unknown function \( y(x) \) as follows;

\[
y(x) = \beta + Z(x)
\]

where \( x \) is an \( m \)-dimensional vector (\( m \) design variables), \( \beta \) denotes a constant global model, and \( Z(x) \) represents a local deviation from the global model. In kriging, \( Z \) is taken to depend on the distance between corresponding points. The distance measure is employed instead of the Euclidean distance as follows;

\[
d(x^{(i)}, x^{(j)}) = k \sum_{h=1}^{k} \theta_h \| x^{(i)}_h - x^{(j)}_h \|^2
\]

where \( \theta_h \) denotes the \( k \)-th element of the correlation vector parameter \( \theta \). The correlation \( R \) between points \( x^{(i)} \) and \( x^{(j)} \) is given as follows;

\[
R(x^{(i)}, x^{(j)}) = \exp \left( -d(x^{(i)}, x^{(j)}) \right)
\]

When the response at a new point \( x \) is required, a vector of correlations between the point and those used in the DOE is formed, \( r(x) = R(x, x^i) \). The prediction is then given by

\[
\hat{y}(x) = \hat{\beta} + r^T R^{-1} (y - I \hat{\beta})
\]

where \( \hat{\beta} \) denotes the estimated value of \( \beta \) and \( y = [y(x)^1, \cdots, y(x)^n] \).

The unknown parameter to be estimated for constructing the kriging model is \( \theta \). This parameter can be estimated by maximizing the following likelihood function \( f_\ell \)

\[
f_\ell(\hat{\beta}, \sigma^2, \theta) = \frac{1}{(2\pi)^{n/2}(\sigma^2)^{n/2}|R|^{1/2}} \exp \left[ -\frac{(y - I \hat{\beta})^T R^{-1} (y - I \hat{\beta})}{2\sigma^2} \right]
\]

Maximizing the likelihood function is an \( m \)-dimensional unconstrained nonlinear optimization problem. In this study, the alternative method is employed to solve this problem. For a given \( \theta, \hat{\beta} \) and \( \sigma^2 \) is defined as

\[
\hat{\beta} = \frac{I^T R^{-1} y}{I^T R^{-1} I}
\]
Fig. 6  Exact and approximate solutions obtained by kriging-based RSM plotted on the two-dimensional plane of the objective functions. An orange arrow denotes the optimum direction because of the two-objective minimization problem.

\[ \hat{\sigma}^2 = (y - \hat{y})^T R^{-1} (y - \hat{y}) / n \]  

(21)

\( n \) is the number of points used in the DOE. The mean-squared error of the prediction is

\[ \hat{s}^2(x) = \hat{\sigma}^2 \left[ 1 - r^T R^{-1} r + \frac{(1 - I^T R^{-1} r)^2}{I^T R^{-1} I} \right] \]  

(22)

The root mean squared error (RMSE) is described as \( s = \sqrt{\hat{s}^2(x)} \).

This basic approach can be used to model any response quantity, including constraints. Here, because the constraints may be rapidly computed, there is no need to apply the RSM process to them at all. Thus, a kriging is built just for the predicted noise. The general strategy is shown in Fig. 5.

It is notable that kriging-based RSM is not always sufficient for all problems because of the difficulty to set up such models for many design variables. Also, the approach is numerically expensive when there are more than a few hundred data points because the set up process requires the repetitive lower-upper (LU) decomposition of the correlation matrix \( R \), which has the same dimensions as the number of points used. Moreover, the number of such LU steps is strongly dependent on the number of design variables, and the likelihood is commonly highly multimodal.

3. Results of Design Optimization

The design variable as the cant angle is selected at every 10deg and the design variable as the length is selected at every 0.9m. The 25 couples are extracted from a total number of 60 by using the DOE for the initial sample points of kriging model. Note that the number of initial sample points generally needs 10 times a number of design variables for the accuracy of generated RSM and the efficient optimization convergence. Three additional sample points are added per one update, which include two weak and one strong non-dominated solutions. And then, update is stopped when the optimum solutions are converged. It is notable that exact solution denotes a result evaluated practically by using LEE computation, and approximate solution is a result evaluated approximately on response surface.

The update number of 15 was performed and the total number of 70 exact solution was evaluated for the optimization convergence. Exact and the approximate solutions obtained by kriging-based response surfaces are shown in Fig. 6. It is shown that the efficient search is performed by using kriging-based RSM because there are many exact solutions near the optimum region where Pareto solutions may exist. This figure shows that both objective functions...
can be minimized simultaneously because Pareto-optimum surface does not appear despite the optimization convergence. That is, there is no tradeoff between the objective functions in the present design space. Pareto solutions do not focus on a point but has width because the evaluated values of the present objective functions include perturbation due to unsteady phenomenon.

Kriging-based response surfaces for each objective function are shown in Fig. 7. The comparison of those two figures reveals that the location to reduce SPL is similar, and there is no tradeoff between the objective functions. Moreover, the cant angle to reduce SPL is approximately 65 deg, which value is expedient for the practical design of V-tail wing. Of course this optimum value of the cant angle cannot be directly used due to the several assumptions. However, it is revealed that the reduction of the present objective functions depends on the cant angle. That is, the shielding effect is useful for fan noise reduction to adjust the wing cant angle. When the appropriate cant angle is designed, the SPL is reduced as high as with the optimized wing length. Figure 7(a) shows the response surface at the side measuring location. This figure reveals that the SPL at the side location is always high, when the cant angle surpasses 90 deg, because there is no shield for the sound pressure from the noise source. It does not depend on the wing length for that situation. Figure 7(b) shows that the SPL at the bottom location is always large when the cant angle becomes low. This fact reveals that low cant angle weakens the shielding effect due to the strong reflection and diffraction. There is an appropriate angle for the shielding effect.

Figure 8 shows the comparison of the root mean square (RMS) distributions of sound pressure using logarithm \( \log(\sqrt{p^2}) \) between no wing and V-tail one. This value means the noise intensity. When there is no wing, two strong pressure distributions occur from the noise source. One diagonally radiates above the fuselage, the other occurs to sideward of the fuselage. The wing should properly shield the sideward radiation to reduce the SPL at the side location. The right side of Fig. 8 shows \( \log(\sqrt{p^2}) \) in the case with V-tail wing of the cant angle of 67 deg and the length of 5.1 m, design variables of which give one of the strong optimum solutions. This wing effectively generates the shielding effect for the strong sideward radiation, and the values at the side and the bottom locations are restrained, simultaneously. That is, it is an essential key of the effective shielding to reduce the radiation to sideward of the fuselage.

4. Conclusion

The multi-objective design optimization has been performed to investigate the usefulness of the shielding effect using a V-tail wing and a fuselage to reduce engine fan noise. In the present study, the problem has several assumptions such as two-dimensional, no flow condition, and low-frequency monopole sound source. The noise propagation is evaluated by
using the linearized Euler equations at the side and the bottom measuring locations relative to the fuselage. As a result, it is revealed that the shielding effect can be applied to engine fan noise. In the present computational condition, there is no tradeoff between the objective functions, i.e., the noise at two positions of the side and the bottom relative to the fuselage can be reduced simultaneously. It is essential that the wing shields the sideward radiation of the sound pressure relative to the fuselage to reduce the sound pressure level at both the side and the bottom measuring locations. When a spanwise length of V-tail wing becomes larger, the shielding effect properly becomes strong. The appropriate adjustment of wing cant angle is essential for the shielding effect to reduce engine fan noise.

References


