Fluid Force Acting on Square Cylinder of Finite Length under Lock-in Condition*

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Abstract
This paper describes the fluid force acting on a square cylinder of finite length in the lock-in region. The experiment was carried out in an N.P.L. blow-down type wind-tunnel with a working section of 500mm×500mm×2000mm at Reynolds number of 1.25×10⁴. The cylinder was forced to oscillate sinusoidally in the lift direction. The power spectrum of the fluctuating velocity in the wake behind a square cylinder was measured to show the lock-in region in the present experiment. The time-mean pressure distribution and fluctuating pressure distribution on the square cylinder were measured for the displacement in the vibration. Consequently, it was found that the mean drag and fluctuating lift increase and become maximum in the lock-in region, while the base pressure in the rear surface of the cylinder becomes low and attains minimum.

Key words: Vortex, Displacement, Pressure Distribution, Fluctuating Pressure, Fluid Force, Square Cylinder, Forced Vibration, Flow Induced Vibration

1. Introduction
In general, the oscillation excited by the Karman vortex is a well known oscillation caused by a fluid. Along with the shedding vortex, oscillation is excited in the direction perpendicular to the flow. When the natural frequency of an obstacle approaches the shedding frequency of the Karman vortex, a lock-in phenomenon occurs. Under this lock-in condition, the oscillation amplitude of the obstacle rapidly increases, and large-scale accidents might even occur in some cases. Therefore, to develop countermeasures against oscillation, it is important to study the fluid force acting on an obstacle under the lock-in condition, in particular, to evaluate the hydrodynamic excitation force(1). However, most of the reports on the fluid force acting on a square cylinder treat cases with a fixed support. The fluid force acting on a square cylinder in an oscillating state has been reported by Otsuki et al. (2), Sakamoto et al. (3), Mizota and Okajima(4)(5), and Bearman and Obasaju(6), but many points still require clarification.

In these reports, the following factors are related to the unsteady fluid force inside and outside the lock-in region: the amplitude dependence of the fluid force under the lock-in condition(2), the changes in the time-mean base pressure and fluctuating pressure depending on the aspect ratio of the square cylinder(3), the changes in the fluctuating pressure and phase with respect to the reduced flow velocity(6), and the amplitude dependence of the phase assuming the lift amplitude and oscillation displacement under the lock-in condition as references(4)(5). Thus far, the oscillation in the direction perpendicular to the flow has been focused on. Recently, however, the oscillation in the direction of the flow has also
been reported\(^7\). In addition, Enya et al.\(^8\) clarified the oscillation characteristics in the directions parallel and perpendicular to the flow for various natural frequencies of the square cylinder by numerical simulation. In our previous study\(^9\), a two-dimensional square cylinder was subjected to enforced oscillation in the direction perpendicular to the flow, and the flow characteristics around the square cylinder and the fluid force that induce vortex-excited oscillation inside and outside the lock-in region were examined in detail. However, in practice, for square cylinders of a finite length, which are frequently observed in structures with a rectangular cross section, such as high-rise buildings, the shedding vortices have a three-dimensional structure because of the effect of the free end. Therefore, the behavior of vortex formation is different from that for a two-dimensional square cylinder\(^10\)(\(^11\)). However, there has been no systematic study of the surface pressure around a finite-length square cylinder or the fluid force acting on a square cylinder inside and outside the lock-in region.

In this study, considering the above-mentioned points, a square cylinder of a finite length was subjected to enforced oscillation in the direction perpendicular to the flow to examine in detail the fluid force acting on the cylinder with respect to the forced oscillation frequency, which was varied between the inside and outside the lock-in region. Thus, an unstable fluid force that induces vortex-excited oscillation inside and outside the lock-in region was obtained to clarify the mechanism underlying the oscillation excitation.

### Notations

- \(A\): oscillation amplitude
- \(C_D\): local drag coefficient
- \(C_{D'}\), \(C_L'\): local fluctuating drag coefficient and local fluctuating lift coefficient
- \(C_p\), \(C_{p'}\): pressure coefficient and fluctuating pressure coefficient
- \(D, H\): side length and height of square cylinder
- \(d\): oscillation displacement
- \(f, n\): vortex shedding frequency and forced oscillation frequency
- \(P_s\): pressure on surface of square cylinder
- \(S_T, S_v\): Strouhal number and nondimensional forced oscillation frequency
- \(U_\infty\): main flow velocity
- \(X, Y, Z\): direction of main flow (\(X\) axis), horizontal and vertical directions perpendicular to the main flow (\(Y\) and \(Z\) axes, respectively), on rectangular coordinates with the origin centered at the bottom of the square cylinder

### 2. Experimental Apparatus and Procedures

In this study, an N.P.L. blow-down wind tunnel with a 500 mm × 500 mm square blow-off exit and a working section of 2000 mm length was used. The model square cylinder used is made of aluminum and has a side length of \(D=40\) mm and a height of \(H=360\) mm. The square cylinder was perpendicularly attached on the bottom wall 500 mm downstream of the leading edge of the wind tunnel (Fig.1) and was oscillated sinusoidally in the direction perpendicular to the flow using a oscillator. In this experiment, the lock-in phenomenon is induced by forcibly oscillating the square cylinder and changing its natural frequency. According to our previous study\(^12\),\(^13\), the lock-in region expands with increasing oscillation amplitude, and the fluid force tends to change markedly. Here, the oscillation amplitude was fixed at a constant value during the experiment. To measure the pressure, \(\phi 1.2\)mm holes were prepared on the surface of the test square cylinder with 11 holes each on the front and rear surfaces and 10 holes on each side surface. The pressure was measured at the bottom of the square cylinder.

In this study, Scanivalve and a pressure converter were used in the measurement of the time-mean pressure on the square cylinder surface and the fluctuating pressure, respectively.
In order to confirm the occurrence or absence of vortex shedding, a constant-temperature hot-wire anemometer was installed in the near wake behind the square cylinder, and the power spectra were analyzed using an FFT analyzer.

In this experiment, the blockage ratio of the wind tunnel due to the square test cylinder was 0.06, which was considered to slightly affect the base pressure behind the square cylinder and the drag coefficient. Here, the effects are assumed to be sufficiently small and no correction for the blockage effect was carried out. Moreover, the finite-length square cylinder used in this study has a height to side length ratio of $H/D=9$. However, according to the report by Okamoto et al. (11), not the Karman vortex, which is an alternating vortex, but symmetric twin vortices were observed around the free end of the square cylinder, whereas the Karman vortex formed further from the free end. Therefore, in this study, $Z/D=1.5$, 5.0, and 6.5 were selected as representative measurement regions. The region in which $Z/D=1.5$, where a Karman vortex is observed, is approximated as a two-dimensional region that receives no effect from the free end. The region in which $Z/D=6.5$, where no Karman vortex is observed, receives a strong effect from the free end. In the region where $Z/D=5.0$, the Karman vortex forms but the effect of the free end is gradually weakened, which indicates a transition region (10).

Twenty-four different forced oscillation frequencies, $n$, which is a parameter in this experiment, were used within a range of $S_v=nD/U_∞=0$ - 0.200 across the in and out of the lock-in region. The experiment was carried out at an oscillation amplitude of 3.0 mm and a Reynolds number of $Re=1.25×10^4$.

![Fig.1 Experimental arrangement](image)

3. Experimental Results and Discussion

3.1 Lock-in region

Figure 2 shows the power spectra of the flow velocity fluctuation in the near wake for the square cylinder of finite length. The power spectra have a sharp peak at the oscillation shedding frequency of the vortex. In the figure, A and B represent the peaks at the shedding frequency of the vortex and at the forced oscillation frequency of the square cylinder, respectively. Therefore, in the case of a fixed support, only peak A appears. The positions at which strong peaks due to the vortex appeared were selected for the measurement. For the cases of $Z/D=1.5$, 5.0, and 6.5, a hot wire was fixed at the positions that satisfied $X/D=6.25$ and $Y/D=1.50$.

For $Z/D=1.5$, peak B is weak for $S_v<0.08$. This is considered because the frequency of peak B is lower than that of peak A, and a highly regular vortex street for peak A appears downstream of the square cylinder for $X/D=6.25$, where the swell of the wake due to the forced oscillation is small. Peaks A and B are integrated at $S_v=0.100$. Therefore, the square cylinder enters the lock-in region at $S_v=0.100$ for $Z/D=1.5$. Afterwards, peak A separates from peak B at $S_v=0.144$, namely, the square cylinder escapes from the lock-in region. For $Z/D=5.0$, peaks A and B are integrated at $S_v=0.100$. Therefore, the square cylinder enters the
lock-in region at $S_v=0.100$ for $Z/D=5.0$. Afterwards, peak A separates from peak B at $S_v=0.136$, namely, the square cylinder escapes from the lock-in region.

For $Z/D=6.5$, similarly to the case with the fixed support\(^{(10)}\), peak A rarely appears and spectral peak B due to the forced oscillation only appears at $S_v=0.040$ because no regular vortex street forms owing to the effect of blow down from the free end of the square cylinder. When $S_v=0.048$, peak A appears because the symmetry vortices that form around the free end are affected by the swell of the wake due to the forced oscillation, become asymmetric, and join the wake. At $S_v=0.100$, peak A increases until it is integrated to peak B. Therefore, the square cylinder enters the lock-in region at $S_v=0.100$ for $Z/D=6.5$. Then, peak A gradually decreases and becomes separated from peak B at $S_v=0.128$. Because peak A thus decreases, the square cylinder escapes from the lock-in region at $S_v=0.128$ for $Z/D=6.5$. Both peaks A and B decrease as $Z/D$ increases from 1.5 to 5.0 to 6.5. This is because the higher the measurement position, the stronger the effect of downwash from the end of the square cylinder.

For $Z/D=5.0$ and 6.5, it is found that peak A increases with increasing $S_v$ after escaping from the lock-in region, and the intensity of peak A becomes greater than that before the
lock-in region. The reason for this is considered to be that an orderly vortex street forms in the wake of the square cylinder after escaping from the lock-in region. Sakamoto et al.\(^{(3)}\) reported that the swell due to forced oscillation flows into naturally discharged vortices after the lock-in region, which supports our experimental result of a small peak B.

Figure 3 shows \(S_t=S_v\) diagrams for \(Z/D=1.5, 5.0,\) and 6.5, which were obtained from the measurement of the power spectra. The result for a two-dimensional square cylinder is also shown for reference. Here, \(S_t=\left(f_o D/U_o\right)\) is the Strouhal number and is proportional to \(S_v\) in the lock-in region. For \(Z/D=1.5, S_v\) remains almost constant for small \(S_v\) and approaches \(S_t\) at \(S_v=0.100\) and higher. As the forced oscillation frequency increases, \(S_t\) increases, maintaining the proportional relationship of \(S_t=S_v\), then \(S_t\) deviates from \(S_v\) at \(S_v=0.144\). Therefore, it is confirmed that the lock-in region ranges over \(0.100\leq S_v\leq 0.140\) for \(Z/D=1.5\). Next, for \(Z/D=5.0, S_t\) approaches \(S_v\) at \(S_v=0.100\) and higher. Then, as the forced oscillation frequency increases, \(S_t\) increases, maintaining the proportional relationship of \(S_t=S_v\), then \(S_t\) deviates from \(S_v\) at \(S_v=0.136\). Therefore, the lock-in region ranges over \(0.100\leq S_v\leq 0.132\) for \(Z/D=5.0\). For \(Z/D=6.5,\) only the data for \(S_v\geq 0.048\) are shown in this figure because no vortex street forms in the range of \(S_v<0.048\). \(S_t\) approaches \(S_v\) at \(S_v=0.100\) and higher. Then, \(S_t\) increases with increasing forced oscillation frequency, maintaining the proportional relationship of \(S_t=S_v\); and \(S_t\) deviates from \(S_v\) at \(S_v=0.128\). Therefore, the lock-in region ranges over \(0.100\leq S_v\leq 0.124\) for \(Z/D=6.5\).

From the above results, the lock-in region is found to narrow as the measurement position approaches the free end of the square cylinder. The lock-in region for the square cylinder of finite length is wider than that for the two-dimensional square cylinder, which has been reported to be in the range of \(0.124\leq S_v\leq 0.148\) by Okamoto et al.\(^{(9)}\).

### 3.2 Surface pressure distribution for square cylinder

The distribution of the surface pressure on the oscillating square cylinder is examined with respect to the oscillation displacement. The oscillation phase angle \(\omega t\) at intervals of \(60^\circ\) over one cycle of the displacement is used as a parameter. Figure 4 shows the surface pressure distribution in the three regions where \(Z/D=1.5, 5.0,\) and 6.5: (a) \(S_v=0.080,\) which is lower than the frequency in the lock-in region, (b) \(S_v=0.128, 0.120,\) and 0.112 in the lock-in region, and (c) \(S_v=0.160\) after escaping from the lock-in region.

First, for \(Z/D=1.5,\) few changes are observed in the surface pressure distribution for the square cylinder with respect to \(\omega t\) at \(S_v=0.080,\) which is before the lock-in region. However, the surface pressure distribution markedly changes with \(\omega t\) at \(S_v=0.128\) in the lock-in region, showing significant asymmetry between the left and right sides of the square cylinder. Thus, it is clear that a lift force acts on both sides of the square cylinder at this value of \(S_v.\) Moreover, at \(S_v=0.160,\) which is after the lock-in region, the surface pressure distribution changes with \(\omega t,\) showing asymmetry between the left and right sides, similarly to that exhibited in the lock-in region. In some cases, the pressure coefficient is lower than that before entering the lock-in region from both sides.

Also, for \(Z/D=5.0,\) few changes are observed in the surface pressure distribution for the square cylinder with respect to \(\omega t\) at \(S_v=0.080,\) which is before the lock-in region. At \(S_v=0.120\) in the lock-in region, changes in the surface pressure distribution increase with \(\omega t,\) showing significant asymmetry between the left and right sides of the square cylinder. This clearly reveals the generation of a lift force, similarly to the case of \(Z/D=1.5\). Moreover, some changes are also observed in the surface pressure distribution on the square cylinder with respect to \(\omega t\) at \(S_v=0.160,\) which is after the lock-in region, showing asymmetry between the left and right sides, similarly to that exhibited in the lock-in region. In the pressure distribution for \(Z/D=6.5,\) no change is observed with respect to \(\omega t\) at \(S_v=0.080,\) which is before the lock-in region. However, some changes are observed with respect to \(\omega t\) at \(S_v=0.112\) and 0.160, which are in and after the lock-in region, respectively, similarly to
Fig. 4 Surface pressure

the cases of $Z/D=1.5$ and 5.0. In particular, significant asymmetry appears between the left and right sides.

Here, at $S_v=0.160$ after the lock-in region, the change in the pressure with respect to the oscillation displacement is generally larger than that before the lock-in region. As reported by Sakamoto et al., the reason for this is as follows. Because the shedding frequency of the vortex is higher than the forced oscillation frequency before entering the lock-in region, the irregularity of the vortex street increases owing to the swell of the wake flow excited by the oscillation. However, after the lock-in region, the swell of the wake due to the forced oscillation is drawn into the vortex because the outflow oscillation frequency of the vortex is lower than the forced oscillation frequency. Therefore, the order of the vortex street is greater than that before the lock-in region.

### 3.3 Base pressure coefficient and local drag coefficient

From the surface pressure distribution for the square cylinder with respect to the oscillation displacement in Fig. 4, the mean surface pressure distribution is obtained for a whole cycle. Moreover, because the friction drag force applied to the square cylinder is considerably smaller than the pressure drag force in the range of Reynolds number used in this experiment, it can be assumed that only the pressure drag force acts on the square
cylinder. Therefore, the local drag coefficient $C_D$ was obtained from the measurement result of the surface pressure.

Figure 5 shows the base pressure coefficient $C_{pb}$ and the local drag coefficient $C_D$ for $Z/D=1.5$, 5.0, and 6.5. The result for the two-dimensional square cylinder is also shown in the figure. The area between the dotted lines represents the lock-in region. Here, $C_{pb}$ is obtained from the mean surface pressure distribution over a whole cycle. $C_{pb}$ and $C_D$ change similarly for $Z/D=1.5$, 5.0, and 6.5. This is because the change in the base pressure directly affects the local drag force at the surface pressure.

For $Z/D=1.5$, $C_{pb}$ sharply decreases before the lock-in region as $S_v$ increases from 0.096, and the rate of decrease accelerates after entering the lock-in region. $C_{pb}$ becomes minimum at $S_v=0.128$ within the lock-in region. Afterwards, $C_{pb}$ gradually returns to its initial value and this tendency continues even after escaping from the lock-in region. In the lock-in region, shear layers separated from the square cylinder are rolled into the vortex, and an orderly vortex street forms. When the rolling position of the separated shear layers approaches the immediate vicinity of the rear of the cylinder, the base pressure decreases.

Although the result for the power spectra reveals agreement between the naturally discharged vortex and the swell of the wake due to the forced oscillation in the range of $0.128 \leq S_v \leq 0.140$, their peaks tend to become weaker and separate with increasing $S_v$. Therefore, the vortex gradually weakens and the base pressure increases.

For $Z/D=5.0$, $C_{pb}$ starts decreasing at $S_v=0.096$, which is before the lock-in region, reaches its minimum value at $S_v=0.112$ in the lock-in region, and then tends to return to its initial value. For $Z/D=6.5$, $C_{pb}$ starts decreasing at $S_v=0.080$, which is before the lock-in region, reaches its minimum value at $S_v=0.112$ in the lock-in region, and then tends to return to its initial value. Moreover, the difference between $C_{pb}$ immediately before the lock-in region and the minimum $C_{pb}$ in the lock-in region is large for $Z/D=1.5$ and 5.0, showing almost the same rate of change. However, the difference is small for $Z/D=6.5$. This is because less rolling of the vortex occurs towards the rear face owing to the effect of
downwash from the free end. The results that the base pressure decreases in the lock-in region for \(Z/D=1.5, 5.0,\) and 6.5 have already been reported for the lock-in phenomenon of a cylinder with a rectangular cross section (two-dimensional rectangular cylinder) and for a two-dimensional square cylinder by Sakamoto et al.\(^{(3)}\) and Okamoto et al.\(^{(9)}\), respectively. The same result is obtained for a finite-length square cylinder. Moreover, outside the lock-in region, the natural shedding vortex interferes with the swell of the wake due to the forced oscillation, and no clear vortex street forms in contrast to that forming in the lock-in region, resulting in the increased base pressure.

The result for \(Z/D=1.5\) is considered to indicate the presence of a two-dimensional flow. However, when the result for \(Z/D=1.5\) is compared with that for the two-dimensional square cylinder, some differences are observed. For example, a regular vortex street forms at the rear of the finite-length square cylinder, which is different from the result for the two-dimensional square cylinder, because of the effect of downwash from the free end. In addition, the peaks of \(C_{pb}\) and \(C_D\) for the two-dimensional square cylinder are observed in the latter half of the lock-in region, similarly to the result reported by Sakamoto et al.\(^{(3)}\), whereas these peaks for the finite-length square cylinder are observed at the center of the region.

![Fig. 6 Fluctuating pressure](image-url)

(a) \(Z/D=1.5\)  
(b) \(Z/D=5.0\)  
(c) \(Z/D=6.5\)
3.4 Fluctuating pressure distribution for square cylinder surface

The fluctuating pressure distribution on the surface of the square cylinder is obtained with respect to the oscillation displacement obtained by the phase-averaging method. For \( Z/D = 1.5, 5.0, \) and 6.5, the fluctuating pressure coefficient \( C_p' \) is calculated with respect to the oscillation phase angle at intervals of 60° over one cycle of the oscillation displacement, the results of which are shown in Fig.6. In the figure, the results for \( S_v = 0.080 \) and 0.160 are shown for the cases before and after the lock-in region, and those for \( S_v = 0.128, 0.120, \) and 0.112 for the above \( Z/D, \) respectively, are shown for the cases of the lock-in region.

For \( Z/D = 1.5 \) in Fig. 6(a), few changes are observed at \( S_v = 0.080 \) before the lock-in region, whereas the fluctuating pressures on the right, left, and rear change greatly with the oscillation phase angle \( \omega t \) at \( S_v = 0.128 \) in the lock-in region, where the direction of the fluctuating pressure acting on the surface of the square cylinder reverses at approximately \( \omega t = 0-60° \). Moreover, the same phenomenon is also observed at approximately \( \omega t = 180-240° \). In addition, a similar change is observed at \( S_v = 0.160 \) after the lock-in region, where the change in the fluctuating pressure is slightly smaller than that at \( S_v = 0.128 \).

For \( Z/D = 5.0, \) no change is observed in the fluctuating pressure at \( S_v = 0.080 \) before the lock-in region, similarly to the case for \( Z/D = 1.5. \) However, the fluctuating pressure begins changing at \( S_v = 0.120 \) in the lock-in region, forms an asymmetric pressure distribution around the two sides of the square cylinder, and then the asymmetry reaches a peak. Similarly to the case of \( Z/D = 1.5, \) the direction of the fluctuating pressure acting on the surface of the square cylinder reverses at approximately \( \omega t = 0-60° \) and 180-240°. The change becomes slightly smaller at \( S_v = 0.160 \) after the lock-in region, but is similar to that in the case of \( S_v = 0.120 \).

For \( Z/D = 6.5, \) no change is observed in the fluctuating pressure at \( S_v = 0.080. \) However, at \( S_v = 0.112 \) in the lock-in region, the fluctuating pressure forms a highly asymmetric distribution on the right, left, and in the vicinity of the rear of the square cylinder compared with the cases of \( Z/D = 1.5 \) and 5.0. At \( S_v = 0.160, \) the change in the fluctuating pressure becomes slightly smaller and shows a tendency similar to that at \( S_v = 0.112, \) which is also observed in the cases of \( Z/D = 1.5 \) and 5.0.

3.5 Local fluctuating lift coefficient and local fluctuating drag coefficient

Using the fluctuating pressure coefficient \( C_p' \) calculated by the phase-averaging method, the local fluctuating drag coefficient \( C_D' \) and local fluctuating lift coefficient \( C_L' \) that act on the surface of the square cylinder were calculated. Figure 7 shows the maximum absolute values of \( C_D \) and \( C_L \) at each \( S_v \) for \( Z/D = 1.5, 5.0, \) and 6.5. The result for the two-dimensional square cylinder is also shown in the figure for reference. The ordinate represents the maximum absolute values of \( C_L \) and \( C_D \), \( |C_L|_{\text{max}} \) and \( |C_D|_{\text{max}} \), respectively, and the abscissa represents \( S_v \). In the figure, the area between the dotted lines represents the lock-in region.

\( C_L \) for \( Z/D = 1.5 \) is approximately 0.1 before the lock-in region and begins to increase at approximately \( S_v = 0.100, \) where the lock-in region starts, and reaches its maximum at \( S_v = 0.128. \) \( C_L \) then sharply decreases until approximately \( S_v = 0.140, \) which is the end of the lock-in region, and remains almost constant after the lock-in region. Moreover, \( C_D' \) is smaller than \( C_L \) and almost 0 before the lock-in region. \( C_D \) begins to increase at approximately \( S_v = 0.100, \) where the lock-in region starts, and remains almost constant after the lock-in region, namely, after approximately \( S_v = 0.140. \)

For \( Z/D = 5.0, \) \( C_L \) increases before entering the lock-in region; this increase sharply accelerates at approximately \( S_v = 0.090 \) immediately before the lock-in region, and \( C_L \) reaches its maximum at \( S_v = 0.120. \) After the peak, \( C_L \) sharply decreases until approximately \( S_v = 0.135, \) at the end of the lock-in region, and subsequently remains almost constant. Moreover, \( C_D' \) is smaller than \( C_L \) and reaches a maximum at approximately \( S_v = 0.120 \) in the
lock-in region, after which \( C_D' \) decreases and remains almost constant after the lock-in region. In addition, \( C_L' \) for \( Z/D = 6.5 \) is similar to that for \( Z/D = 5.0 \). Namely, \( C_L' \) begins to increase before entering the lock-in region, rapidly increases at approximately \( S_v = 0.090 \) immediately before the lock-in region, and reaches a maximum at \( S_v = 0.112 \). \( C_L' \) then, slowly decreases until approximately \( S_v = 0.140 \), which is after the lock-in region. However, the maximum value of \( C_L' \) for \( Z/D = 6.5 \) is greater than those for \( Z/D = 1.5 \) and 5.0. Moreover, \( C_D' \) is smaller than \( C_L' \) and reaches its maximum within the lock-in region, after which \( C_D' \) decreases and remains almost constant, similar to the case of \( Z/D = 5.0 \).

The maximum value of \( C_L' \) in the lock-in region gradually increases with increasing \( Z/D \). The reason for this is considered to be as follows. With increasing \( Z/D \), the vortex undergoes less rolling into the rear owing to downwash from the free end. Nevertheless, the vortices separated at the right and left corners of the front face do not reach the rear of the square cylinder and remain on their respective sides\(^{(11)}\). Owing to these vortices, the fluctuating lift force increases.

Next, \( C_L' \) and \( C_D' \) are compared between the square cylinder of finite length at \( Z/D = 1.5 \) and the two-dimensional square cylinder. \( C_L' \) and \( C_D' \) change negligibly before entering the lock-in region. A difference is observed in the lock-in region, that is, the peak of \( C_L' \) sharply increases for the two-dimensional square cylinder, and is greater than that for the square cylinder of finite length at \( Z/D = 1.5 \), which can be considered as a two-dimensional region despite its finite length. This is because a complex three-dimensional flow field forms in the wake of the square cylinder of finite length owing to downwash from the free end, which suppresses the formation of a regular vortex street compared with the case of the two-dimensional square cylinder. Moreover, the peak of \( C_L' \) is observed in the latter half of the lock-in region for the two-dimensional square cylinder, similarly to the base pressure coefficient and local drag coefficient described in section 3.3. On the other hand, for the square cylinder of finite length, the peak of \( C_L' \) for \( Z/D = 1.5 \) is observed in the latter half of the lock-in region, similarly to that for the two-dimensional square cylinder, whereas the peaks of \( C_L' \) for \( Z/D = 5.0 \) and 6.5 are observed near the center of the lock-in region.
3.6 Excitation mechanism of finite-length square cylinder in lock-in region

Tanaka\(^{(13)}\) has reported that when the phase of the fluctuating lift force leads the oscillation displacement, the field becomes aerodynamically unstable and flow-induced vibration occurs. However, when the phase of the fluctuating lift force lags the oscillation displacement, the field is aerodynamically stable and no vibration occurs. Figure 8 shows the changes in the phase difference between the fluctuating lift force and the oscillation displacement. Here, the leading phase is assumed to be positive.

In Fig. 8, the phase difference $\phi$ before the lock-in region is approximately 120° for $Z/D=1.5$ and 5.0, and 110° for $Z/D=6.5$. Upon entering the lock-in region, the phase difference changes sharply, and the phase becomes closest to 0 at approximately $S_v=0.122$, 0.110, and 0.104 for $Z/D=1.5$, 5.0, and 6.5, respectively. The phase difference subsequently changes, and beyond the lock-in region, it remains at approximately $\phi=-35^\circ$, -75°, and -90° for $Z/D=1.5$, 5.0, and 6.5, respectively. As a result, for $Z/D=1.5$, the local fluctuating lift force, the phase of which leads the oscillation displacement at $S_v<0.122$, is aerodynamically unstable and acts on the surface of the square cylinder as an excitation force in a direction that increases the oscillation. Moreover, this tendency is also observed for $Z/D=5.0$ and 6.5, that is, the phase leads the oscillation displacement for $S_v<0.110$ and $S_v<0.104$ for $Z/D=5.0$ and 6.5, respectively, and the local fluctuating lift force acts on the surface of the square cylinder as an excitation force that increases the oscillation.

Here, the excitation force and damping force are examined for each position. For $Z/D=1.5$, the local fluctuating lift force caused by the vortex acts as an excitation force for values of $S_v$ up to approximately the center of the lock-in region, and then subsequently changes to a damping force. For $Z/D=5.0$, the behavior of the excitation force ends at $S_v$ below the center of the lock-in region and thereafter changes to that of a damping force. In addition, for $Z/D=6.5$, the excitation is observed only immediately after entering the lock-in region, and the local fluctuating lift force generally acts as a damping force. Here, the maxima of the local fluctuating lift acting as an excitation force, i.e., the maximum excitation force, are compared between different positions of the square cylinder on the basis of the value of $S_v$ at which $\phi=0^\circ$, using the results in Fig. 7. The maximum excitation forces are 0.68, 0.72, and 0.76 for $Z/D=1.5$, 5.0, and 6.5, respectively, revealing that the maximum slightly increases as the position approaches the free end. Moreover, the oscillation frequency at which the excitation force becomes maximum depends on the position. Namely, at lower positions, the oscillation frequency that gives the maximum excitation force is higher. In other words, for the square cylinder of finite length, the oscillation is excited from the upper part and propagates downward with increasing oscillation frequency.

Figure 9 shows the relationship between the local fluctuating lift coefficient and the oscillation displacement at $S_v=0.080$, 0.112 and 0.128, and 0.160, which are before, during, and after the lock-in region, respectively, for $Z/D=1.5$. First, at $S_v=0.080$ before the lock-in region, (Fig. 9(a)), the local fluctuating lift force is small, showing a flat elliptic locus.
However, at $S_v=0.112$ in the lock-in region, (Fig. 9(b)), the local fluctuating lift force is large, showing a clockwise elliptic locus, is unstable, and acts in the direction that increases the oscillation. Moreover, the elliptic locus of the local fluctuating lift force at $S_v=0.112$ shows a larger area than that at $S_v=0.080$, which indicates a greater amount of excitation energy supplied to the square cylinder. Next, at $S_v=0.128$ (Fig. 9(c)), the local fluctuating lift force shows a counterclockwise elliptic locus. The damping force acts on the square cylinder, the elliptic locus can be shown as a larger area, and the suppression energy reaches its maximum in the lock-in region. In addition, at $S_v=0.160$ after the lock-in region (Fig. 9(d)), a counterclockwise locus is shown, similarly to the case at $S_v=0.128$. The local fluctuating lift force stably acts on the surface of the square cylinder as a damping force in the direction that attenuates the oscillation. The area of the ellipse is less than that at $S_v=0.128$ in Fig. 9(c), indicating that the suppression energy at $S_v=0.160$ is lower than that at $S_v=0.128$.

The above results reveal that the maximum coefficient of the fluctuating lift force becomes large around the free end of the square cylinder of finite length, and the lift force acts as a damping force that attenuates the oscillation of the square cylinder owing to the relationship between the phase and the oscillation displacement. In short, for the square cylinder of finite length, the fluid force due to the vortex is different between the vicinity of the free end and the lower part, even within the lock-in region. Namely, the fluid force mainly acts as a damping force that attenuates the oscillation at the free end, but it also partially acts as an excitation force that induces the oscillation in the lower part. The damping force has an effect even in the lower part with increasing $S_v$.

![Fig. 9 Relation between oscillating displacement and fluctuating lift coefficient ($Z/D=1.5$)](image)

### 4. Conclusions

The results obtained in this study are summarized below.

1. The lock-in region for the square cylinder of finite length is larger than that for the two-dimensional square cylinder. When $Z/D$ increases, the effect of downwash from the end of the square cylinder becomes strong and the lock-in region narrows: $0.100 \leq S_v \leq 0.140$ for $Z/D=1.5$, $0.100 \leq S_v \leq 0.132$ for $Z/D=5.0$, and $0.100 \leq S_v \leq 0.124$ for $Z/D=6.5$.

2. The base pressure coefficient $C_{p0}$ in the lock-in region is lower than those before and
after the lock-in region, and reaches its minimum in the lock-in region. With decreasing $C_{pb}$, the local drag coefficient $C_D$ increases to its maximum value.

(3) In the lock-in region, the fluctuating pressure greatly varies with the oscillation phase angle, and the local fluctuating lift coefficient $|C_{L_{max}}|$ rapidly increases to its maximum value. In addition, the maximum $|C_{L_{max}}|$ slowly increases with increasing $Z/D$. On the other hand, the local fluctuating drag coefficient $|C_{D_{max}}|$ is smaller than $|C_{L_{max}}|$.

(4) The fluctuating lift force unstably acts on the square cylinder surface around the center of the lock-in region and acts in the direction that increases the oscillation. The excitation energy reaches its maximum value in the lock-in region. In addition, the fluctuating lift force stably acts on the square cylinder surface as a damping force that attenuates the oscillation after escaping from the lock-in region. The damping energy at this time is smaller than the excitation energy.

References