Influence of Dynamic Stiffness in Contact Region on Disk Brake Squeal*

Yasunori OURA**, Yutaka KURITA** and Yuichi MATSUMURA***

**Department of Mechanical Systems Engineering, The University of Shiga Prefecture, 2500 Hassaka-cho, Hikone-shi, Shiga, 522-8533, Japan
E-mail: oura@mech.usp.ac.jp

***Interdisciplinary Graduate School of Medical and Engineering, University of Yamanashi, 4-3-11 Takeda, Kofu-shi, Yamanashi, 400-8511, Japan

Abstract

The squeal generation mechanism in disk brakes is clarified by measuring the stiffness between the disk and pad in the contact region (contact stiffness) and performing an analysis using the surface contact analysis model. The results of a squeal test using a squeal testing machine with a simple structure show that the squeal frequency becomes higher as the thrust pressure for braking becomes larger. Since we thought that the contact stiffness depends on the thrust pressure and influences the squeal frequency because the vibration characteristics of the disk do not change with thrust pressure, we measured the contact stiffness under squeal generating conditions. The pad was given constant pressure and excitation force generated by random noise with the squeal bandwidth. Measurement results show that the contact stiffness depends on the thrust pressure and is different for each pad. To clarify the influence of contact stiffness on squeal, the disk-pad-caliper system was analyzed by using the surface contact analysis model, which connects the disk with the pad via distributed springs (contact stiffness). The analytical results show that squeal is easily generated and its frequency easily changes when the dependence of contact stiffness on thrust pressure is large.

Key words: Self-Excited Vibration, Frictional Vibration, Modeling, Disk Brake, Squeal, Dynamic Stiffness

1. Introduction

Various methods of preventing disk brake squeal have been proposed because squeal is annoying and decreases the value of manufactured products. For instance, the shapes of all parts have been adjusted, new friction materials have been developed, and attenuation has been added to each part. For more efficient squeal prevention, the squeal generation mechanism has been researched for a long time (1) (2). The vibration characteristics of the disk and pad greatly influence squeal generation. It has recently been reported that the stiffness and shape of the contact region between a disk and pad also influence squeal generation (3) (4).

We examined the squeal generation mechanism by using a squeal testing machine with a simple structure instead of an actual brake system (5). The squeal test results clarified that a large thrust pressure for braking generates a high-frequency squeal. We thought that the stiffness of the contact region between a disk and pad (contact stiffness) is changed by thrust pressure and causes the dependence of squeal on thrust pressure because the vibration characteristics of the disk do not change with thrust pressure.

We constructed a surface contact analysis model that reproduces the contact region with distributed springs (contact stiffness) and examined the influence of contact stiffness on squeal \(^{(6)}\). The analytical results show that squeal is not generated when the contact stiffness is uniform throughout the contact region. Squeal is generated when the contact stiffness depends on the thrust pressure. The squeal frequencies in these analytical results depend on the thrust pressure like the squeal test results. We discovered that the dependence of contact stiffness on thrust pressure causes the squeal generation.

In the study reported in this paper, the contact stiffness between a disk and pad was measured, and the influence of the nonuniformity of contact stiffness on squeal was clarified. First, we compare squeal test results for two different kinds of pads. Then we show that the pad characteristics influence squeal generation and its frequency. Next, we present the measured contact stiffness of the pads used for the squeal test. An analysis by the surface contact analysis model was performed using the measured contact stiffness values, and the relationship between the contact stiffness and squeal generation is discussed.

2. Squeal test

2.1 Squeal testing machine

The influence of the pad material’s characteristics on squeal was examined using the squeal testing machine with a simple structure instead of an actual brake system. The shape of the disk is shown in Fig. 1(a). The disk was made of steel, and its boundary conditions were inner fixed and outer free. Thickness \(h\) of the outer part of the disk was 6 mm. The shape of the pad-caliper is shown in Fig. 1(b). From the pad for an actual car disk brake, we cut out a \(20 \times 20\)-mm piece for the contact region. Two kinds of pads (pad A and pad B) were prepared to investigate whether the squeal changed depending on the pad characteristics when the same caliper and disk were used. The friction coefficients \(\mu\) of both pads were found experimentally to be 0.3. This pad test piece was fixed to the center of a plate spring, and the ends of the spring were fixed to a rigid stand. To examine the influence of pad support stiffness on squeal, we used four thicknesses for the plate spring (\(t = 0.5, 1.0, 1.5,\) and \(2.0\) mm). The pad was pressed onto the disk rotating at a constant speed of 40 rpm by a thrust load \(F\) to generate a continuous squeal. The pressing position of the pad was at the same height and 110 mm away along the radius from the rotational center of the disk. The generated squeal was measured with a precision sound-level meter, placed 40 cm away from the center of the contact region.

![Fig. 1 Specimen shape.](image)

2.2 Dependence of squeal on thrust pressure

The influences of the pad characteristics on squeal frequency and sound pressure level
(SPL) were examined. The results of the squeal test using pad A are shown in Fig. 2(a). The horizontal axis shows the average pressure given by the thrust load divided by the contact region. When plate spring thickness \( t \) was 0.5 or 1 mm, the squeal frequency changed from 1300 to 2032 Hz as the pressure increased. When \( t \) was 1.5 mm, the squeal frequency changed from 2032 Hz to 3024 Hz as the pressure increased. When \( t \) was 2 mm, the squeal frequency was only 3024 Hz, but the SPL gradually increased with the thrust pressure. The squeal test using pad A clarified that the squeal frequency becomes high when the pressure and pad’s support stiffness become large.

The results of the squeal test using pad B are shown in Fig. 2(b). When \( t \) was 0.5, 1, or 1.5 mm, the squeal frequency was only 2056 Hz. When \( t \) was 0.5 or 1.0 mm, squeal was generated for all the thrust pressures used in the squeal test. When \( t \) was 1.5 mm, squeal was generated only at thrust pressures between 0.1 and 0.3 MPa. When \( t \) was 2 mm, squeal was not generated. Figure 2 clarifies that squeal (frequency and SPL) differed depending on the pad material’s characteristics even when the vibration characteristics of the disk and caliper were the same.

![Graphs showing squeal frequencies and SPL for pads A and B with different plate spring thicknesses.](image)

Fig. 2  Thrust pressure versus sound pressure level.

3. Measurement of dynamic stiffness in frictional contact region

3.1 Measurement principle

We paid attention to the contact stiffness as the cause of squeal differing depending on
the kind of pad. When squeal is generated, the thrust load for braking generates thrust pressure and the squeal vibration generates pressure fluctuation in the frictional contact region. To measure the contact stiffness under squeal generating conditions, we gave the pad constant pressure $p$ instead of thrust pressure and excitation force $f$ generated by random noise with the squeal bandwidth, and we measured the transfer function (dynamic stiffness) from the deformation of the pad in response to the excitation force.

The excitation force $f$ is represented by relative displacement $x$ between the disk and pad in the contact region, contact stiffness $k$, and damping coefficient $c$.

$$ f = kx + cx $$  

(1)

The transfer function is obtained as

$$ \frac{F}{X} = k + cs = k \left( 1 + \frac{c}{k} \right) $$  

(2)

A Bode diagram of Eq. (2) is shown in Fig. 3. Contact stiffness $k$ was obtained from the gain of the transfer function at low frequency because the influence of damping was small. Damping coefficient $c$ was obtained from contact stiffness $k$ and the break frequency (cutoff frequency) of the transfer function because the break frequency is defined by $k/c$.

3.2 Measurement device

The experimental apparatus for measuring the dynamic stiffness is shown in Fig. 4. The pad was placed between the main body of the apparatus and the excitation board. The excitation board, the driving sources, and the driving source base were independent of the main body. A constant thrust pressure was applied to the contact region by the thrust screw. The displacement of the excitation board was measured as displacement $x$ of the contact region. The excitation force transmitted to the main body base was assumed to be excitation force $f$ added to the contact region. The vibration characteristics of the main body did not influence the measurement results because the main body was designed to have sufficiently large stiffness.

The driving source is required to generate a force larger than the thrust load and to have responses higher than the squeal frequency. We chose to use an accumulating piezoelectric actuator (NEC TOKIN, AE0505D16) as the driving source. Its maximum generated power is 850 N and the response is more than 20 kHz. Both stability and sufficient generated power were obtained by setting up one of these actuators in each of the four corners of the excitation board.

Displacement $x$, which had a high frequency and a minute amplitude, was measured as an acceleration by an acceleration pickup (Brüel&Kjær, Type 43) set up on the excitation board. This pickup could measure vibration at the squeal frequency because its
measurement bandwidth is from 0.1 Hz to 16.5 kHz.

The excitation force added to the contact region was measured with a piezoelectric power sensor (KISTLER, 9051A). The rigidity of this sensor is $9 \times 10^9$ N/m (ten times the pad dynamic stiffness), so it did not influence the measurement results. This sensor also was used to measure the thrust pressure because it can measure a semi-static load.

**3.3 Measurement results**

The dependence of contact stiffness on thrust pressure and frequency were examined on the basis of the measurements results for dynamic stiffness. The measurement results for pad A are shown in Fig. 5(a) and those for pad B are shown in Fig. 5(b). The upper figures show the dynamic stiffness for each unit area and the lower figures show the phases. The contact region was excited by random noise with a bandwidth from 0 to 4 kHz and maximum amplitude of ±1 µm. The thrust pressure ranged from 0.25 MPa (pressure in the squeal test) to 2.5 MPa (braking pressure in an actual car).

![Experimental apparatus](image)

**Fig. 4** Experimental apparatus.

![Dependence of dynamic stiffness on frequency](image)

**Fig. 5** Dependence of dynamic stiffness on frequency.
The relationship between the dynamic stiffness and contact stiffness was examined. The dynamic stiffness did not depend on the frequency, and the phases are almost 0 in Fig. 5. The break frequency was at least ten times the measured frequency of 4 kHz. It can be considered that the influence of damping coefficient $c$ on squeal was small compared with the influence of contact stiffness $k$. Therefore, dynamic stiffness $F/X$ can be considered to be contact stiffness $k$. Consequently, the measurement results show that contact stiffness $k$ depended on the thrust pressure. Contact stiffness $k$ differed for the two kinds of pads (value and extent of dependence on thrust pressure). The mechanism of how this difference in contact stiffness influences squeal needs to be clarified.

### 3.4 Dependence of contact stiffness on thrust pressure

The contact stiffness of each pad was evaluated. The relationship between contact stiffness and thrust pressure is shown in Fig. 6. The value of contact stiffness in the figure is the dynamic stiffness at 2 kHz in Fig. 5. Marks ▲ are the contact stiffness of pad A and marks ● are the contact stiffness of pad B. When the pressure was lower than 0.5 MPa, the contact stiffness of pad A was smaller than that of pad B. However, when the pressure became large, the contact stiffness of pad A became larger than that of pad B. The contact stiffness of pad A changed more easily with the pressure than that of pad B did. The relationship between contact stiffness $k$ and pressure $p$ is expressed as

$$k = K_N p^n.$$  \hspace{1cm} (3)

Contact stiffness $k$ was considered to be 0 when the pressure was 0 because it was small enough compared with contact stiffness $k$ when the pressure was non-zero. Nonlinearity index $N$ shows the extent to which the contact stiffness depended on pressure. $K_N$ is the coefficient of rigidity.

![Fig. 6 Dependence of contact stiffness on thrust pressure.](image)

<table>
<thead>
<tr>
<th>$N$</th>
<th>$K_N$</th>
<th>$n$</th>
<th>$k_n$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$5.75 \times 10^{10}$</td>
<td>1.00</td>
<td>$5.75 \times 10^{10}$</td>
<td></td>
</tr>
<tr>
<td>0.09</td>
<td>$1.86 \times 10^{10}$</td>
<td>1.10</td>
<td>$1.78 \times 10^{11}$</td>
<td></td>
</tr>
<tr>
<td>0.35</td>
<td>$6.98 \times 10^{8}$</td>
<td>1.55</td>
<td>$2.59 \times 10^{13}$</td>
<td>Pad B</td>
</tr>
<tr>
<td>0.50</td>
<td>$1.15 \times 10^{8}$</td>
<td>2.00</td>
<td>$3.30 \times 10^{15}$</td>
<td></td>
</tr>
<tr>
<td>0.53</td>
<td>$7.68 \times 10^{7}$</td>
<td>2.14</td>
<td>$1.45 \times 10^{16}$</td>
<td>Pad A</td>
</tr>
<tr>
<td>0.60</td>
<td>$3.32 \times 10^{7}$</td>
<td>2.50</td>
<td>$6.41 \times 10^{17}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Nonlinearity of pad stiffness.
The solid lines in Fig. 6 are approximations of contact stiffness $k$. When the pressure was 0.25 MPa, the contact stiffness values of pads A and B were close and the mean contact stiffness of both pads was $5.75 \times 10^{10}$ N/m$^2$. The value of $K_N$ was set to the value at which the contact stiffness became $5.75 \times 10^{10}$ N/m$^2$ at 0.25 MPa. The values of $N$ and $K_N$ used for the approximation lines are given in Table 1. The approximation lines can reproduce the contact stiffness of pad A for $N = 0.53$ and that of pad B for $N = 0.35$. The extent to which the contact stiffness depended on thrust pressure was larger for pad A than for pad B.

4. Analysis using surface contact analysis model

4.1 Surface contact analysis model

The influence of the contact stiffness on squeal was considered using the surface contact analysis model shown in Fig. 7. In this model, the disk is expressed by a vibration system with one degree of freedom (translational direction) and the pad is expressed by a vibration system with two degrees of freedom (translational and rotational directions). The frictional contact region is reproduced by distributed springs. The equations of motion are

$$
\int -f dA = -Kx - \int f dA \tag{4}
$$

$$
m \ddot{x}_p = -k x_p + \int f dA \tag{5}
$$

$$
J \ddot{\phi} = -k_\phi + \int \mu f \int dA + \int f dA \tag{6}
$$

Equivalent mass $M$ was calculated by taking the kinetic energy of the entire disk (treated as a continuous body) as being equal to the kinetic energy of a 1-DOF solid body. The value of $M$ is different in each vibration mode of the disk. Equivalent spring constant $K$ was calculated from $M$ and the disk’s natural frequency $f_d$. The values of $m$ are the total mass of the pad-caliper; those of $J$ are inertial moments assuming that the center of the plate spring (caliper) is the rotation center. Spring constant in the translational direction $k_t$ was calculated from $m$ and natural frequency in the pad translational direction $f_t$. Spring constant in the rotational direction $k_\phi$ was calculated from $J$ and natural frequency in the pad rotational direction $f_\phi$.

Pressure fluctuation $f$ in the contact region is expressed by contact stiffness $k$ and relative displacement $x$ between the disk and pad.

$$
f(t) = kx = k(x_d - x_p - \ell \phi) \tag{7}
$$

Here, $\ell$ expresses the position in the perpendicular direction in the contact region. Equation (7) is substituted into the equations of motion, and the characteristic equation is obtained. The characteristic equation is the third-order equation of $s^2$ (the sixth-order equation of $s$). If the characteristic equation has a conjugate complex root $s = X \pm iY$, and the real part $X$ has a positive value, a self-excited vibration (squeal) is generated. The real part $X$ is called the “squeal index” because it decides the speed to which vibration grows up. The imaginary part represents the squeal frequency.

![Fig. 7 Surface contact analysis model.](image-url)
4.2 Contact stiffness

The surface contact analysis model was improved by utilizing the measurement results of contact stiffness. In the frictional contact region, the thrust load for braking generates thrust pressure \( p \) and the squeal vibration generates pressure fluctuation \( f \). In a previous report \(^6\), the relationship between the thrust pressure and displacement in the frictional contact region was expressed by the following equation.

\[
p + f = k_1 (x_0 + x) + k_2 (x_0 + x)^2 \quad (8)
\]

Here, \( x_0 \) is the constant value of the relative displacement between the disk and pad caused by the thrust load and \( x \) is the fluctuation value of that caused by squeal vibration. The relationship between thrust pressure and contact stiffness was assumed to be expressed by

\[
k = \sqrt{k_1^2 + 4k_2 p} \quad (9)
\]

The contact stiffness defined by Eq. (9) becomes \( k_1 \) when the thrust pressure \( p \) is 0 and increases at the rate of the 0.5th power of thrust pressure \( p \) in proportion to the value of \( k_2 \). This dependence of contact stiffness on thrust pressure corresponds to nonlinearity index \( N = 0.5 \) in Fig. 6. Equation (9) cannot reproduce the dependence of contact stiffness on thrust pressure because both pads did not have \( N = 0.5 \).

Equation (9) was improved to reproduce the nonlinearity of each pad. In the research described in the present paper, we expressed the relationship between the thrust pressure and displacement in the frictional contact region by using the following equation.

\[
p + f = k_n (x_0 + x)^n \quad (10)
\]

Here, \( n \) shows the nonlinearity (dependence ratio of contact stiffness on thrust pressure) of a pad. The coefficient \( k_n \) determines the contact stiffness. When the pressure is 0, the contact stiffness is 0 because it is small enough compared with the contact stiffness when the thrust pressure is large. Equation (10) is expanded and pressure \( p \) is obtained as

\[
p = k_n x_0^n \quad (11)
\]

The pressure fluctuation \( f \) caused by squeal vibration \( x \) is expressed by the following equation, disregarding higher-orders of \( x \) because they are minute.

\[
f = n k_n x_0^{n-1} x \quad (12)
\]

In Eq. (12), a coefficient of \( x \) is contact stiffness \( k \). The relationship between the contact stiffness and pressure is obtained by eliminating \( x_0 \) from \( k \) by using Eq. (11) to get

\[
k = n k_n x_0^{n-1} p \quad (13)
\]

By comparing Eqs. (13) and (3), we obtained

\[
N = n - \frac{1}{n}, K_n = n k_n \quad (14)
\]

Here, \( n \) and \( k_n \) correspond to \( N \) and \( K_n \) in Table 1. The stability can be analyzed by substituting Eq. (12), \( k_n \), and \( n \) into the equations of motion. The stability analysis method is similar to that in the previous report \(^6\).

4.3 Analytical results

An analysis by the surface contact analysis model was performed using the measured contact stiffness values (see Section 3). The analytical results for the contact stiffness values of pads A and B are shown in Fig. 8. The vertical axis shows the disk’s natural frequency \( f_d \) and the horizontal axis shows its natural frequency in the pad’s rotational direction \( f_\phi \). Horizontal broken lines are \( f_d \) (\( h = 6 \) mm) and vertical broken lines are \( f_\phi \) (\( t = 0.5, 1.0, 1.5 \) and \( 2.0 \) mm). The marked intersections of the broken lines indicate the conditions under which squeal was generated in the squeal test. Those marked with a circle are when the thrust load was 50 N (average thrust pressure in contact region was 0.125 MPa). Those marked with a cross are when it was 100 N (0.25 MPa), and those marked with a triangle
Fig. 8  Instability areas.
are when it was 150 N (0.375 MPa). The area between the solid lines is the squeal generation area, where the squeal index had positive values. Thin and regular solid lines show the areas for thrust loads of 50 and 150 N, respectively.

The squeal generation areas of pad A are shown in Fig. 8(a). Squeal is generated at a high disk frequency \( (f_d) \) when the pad support stiffness \( (t) \) is large. The squeal generation area moves upward as the thrust pressure changes from 0.125 to 0.375 MPa. These analytical results correspond to the squeal test results that the squeal frequency became high when the pad support stiffness and thrust pressure became large.

The squeal generation areas of pad B are shown in Fig. 8(b). Compared with Fig. 8(a), the squeal generation area hardly moves as the pressure changes from 0.125 to 0.375 MPa. These analytical results correspond to the experimental finding that only 2-kHz squeal was generated when pad B was used. However, 3-kHz squeal is generated in the analysis though squeal was not generated in the squeal test for \( t = 2.0 \text{ mm} \).

The relationship between contact stiffness nonlinearity and squeal generation was examined. In the squeal test, pad A generated squeal for all pressures and pad support stiffness. On the other hand, for pad B, there was a condition in which no squeal was generated. The nonlinearity index of pad B is smaller than that of pad A. We think that squeal is not generated when the nonlinearity index of the pad is small. Figure 8(c) shows the squeal generation area when the nonlinearity index is small \( (n=1.1) \). The squeal generation area is narrower than that for pad B. The squeal generation area is made narrower by reducing nonlinearity \( n \). Moreover, it disappears at \( n = 1 \) (uniform contact stiffness throughout the contact region).

The tendency of squeal differs according to the dependence of contact stiffness on thrust pressure (nonlinearity index \( N \)) even when the value of contact stiffness \( k \) is the same.

**Conclusion**

We investigated the contact stiffness between a disk and pad to clarify the mechanism through which the squeal frequency and SPL depend on the thrust pressure for braking. By measuring the contact stiffness and performing an analysis using the surface contact analysis model, we clarified the squeal generation mechanism.

1. The dynamic stiffness of the contact region does not depend on vibration frequency. Therefore, the dynamic stiffness can be regarded as the contact stiffness.
2. Each pad has different contact stiffness (value and extent of dependence on thrust pressure).
3. Squeal frequency and SPL differ according to the contact stiffness.
4. Squeal is easily generated and its frequency easily changes when the dependence of contact stiffness on thrust pressure is large.

**Acknowledgment**

We thank Yukio Nishizawa and others at ADVICS Co., Ltd. for providing the brake pad sample and good advice based on actual car disk brake squeal.

**References**

3. Sueoka, A., Ryu, T. and Shirozu, K., Squeal of a Disk Brake of Floating Type for Cars: 1st Report; Relationship between Occurrence of Squeal and Contact Region between Rotor and Pads in Experiment, *Transactions of the Japan Society of Mechanical Engineers*,
