Verification of Representation of Analytical SEA Parameters for Structure-Borne Sound in Terms of Modal Density by Using FEM* 

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Abstract
This paper verifies the representation of power input and subsystem energy in terms of modal density of subsystems in Statistical Energy Analysis (SEA) by means of FEM calculations. First, vibration response in the modal form is used to derive the representations. It is shown that the utilization of values averaged over space and frequency for the power input and the subsystem energy of the subsystem can be represented in terms of the modal density. Furthermore, the coupling loss factor between plate subsystems is also analytically estimated by using the wave theory and is determined on the basis of the modal density. In order to verify these representations, FEM calculations are performed to compare subsystem energies, power inputs and coupling loss factors of three types of structures with different subsystem shapes and the same modal densities. The estimations obtained with analytical SEA are also compared with the FEM calculation results.

Key words: Statistical Energy Analysis, FEM, Vibration of Continuous System, Method of Vibration Analysis, Modal Analysis, Simulation

1. Introduction
Statistical energy analysis (SEA)\(^{(1)(2)}\) is an analysis method for vibration and noise in the high frequency region. SEA was developed in the 1960s for predicting high frequency vibrations in rockets and other structures, and for evaluating the reliability of onboard instrumentation. Since then, the systemization of SEA theory has been advanced with a focus on the fields of aircraft, marine vessels and buildings. SEA is used primarily for estimation of necessary parameters through theoretical evaluation equations and response prediction, and can be classified as analytical SEA. Since structures such as rockets are large and relatively simple, derivation of theoretical formulas for parameters is possible by applying the concept of statistical averages to plate theory and wave theory. Also, a parameter evaluation method\(^{(3)(4)}\) based on experimental measurements (experimental SEA) was developed in the 1980s, extending the application of SEA to various complex structures, such as automobiles. Accompanying the increase in computing speed and storage capacity in the 1990s, a calculation method for parameters utilizing the finite element method (FEM) was developed\(^{(5)\sim(8)}\). It is thought that applications for SEA will expand even further in future\(^{(9)\sim(11)}\).

Against this backdrop, the authors proposed a process for reducing structure-borne sound in machinery by using experimental SEA and achieved an reduction of noise in actual products\(^{(12)}\). This process aims at finding countermeasures in response to existing targets. On the other hand, structural design construction using analytical SEA can be expected...
since analytical SEA can be utilized in the initial stages of designing structures whose
detailed shape and dimensions have not yet been decided.

Accordingly, in this paper, with an emphasis on analytical SEA, we focus on the
representation of principle parameters, which form the core of analytical SEA, mainly in
terms of modal density. The principle parameters are power input, subsystem energy, and
coupling loss factor averaged over space and frequency. Information that verifies these
theoretical formulae is only found in papers related to coupling loss factors \(^{(13)}\), and
verification of these theoretical formulae is performed through FEM analysis in this paper.

First, the derivation of theoretical formulae is addressed, principle parameters are
expressed primarily in terms of modal density, and it is shown that they do not depend on
details such as subsystem shape or boundary conditions. Next, FEM analysis is performed
with respect to multiple structures with equal modal density, and it is verified that the
calculated principle parameters are equivalent regardless of the subsystem shape, as well as
that the results of the experiments coincide with those obtained with the theoretical
formulae.

2. Theoretical Formulae for Analytical SEA Parameters

In this section, we derive the theoretical formulae for the principle parameters of SEA,
namely the coupling loss factor, the subsystem energy and the power input averaged over
space and frequency, and show that they are mainly represented through the modal density.
The index for judging the applicability of analytical SEA using modal density is also
addressed.

2.1 Fundamental equations of SEA

In SEA, the system is regarded as an assembly of subsystems. The fundamental
equations are power balance equations consisting of power input into the system from outer,
power dissipated inside the subsystems and power transmitted between subsystems. The
amounts of dissipated and transmitted power are considered proportional to the vibration or
the acoustic energy (hereafter referred to as subsystem energy) carried by the subsystem.
The constants of proportionality for the fundamental equation of SEA for a system
consisting of \( r \) subsystems are expressed by using Internal Loss Factor (ILF) and Coupling
Loss Factor (CLF) as follows:

\[
P = \omega LE
\]

\[
L = \begin{bmatrix}
\eta_{1,1} + \sum_{i \neq 1} \eta_{i,1} & -\eta_{2,1} & \cdots & -\eta_{r,1} \\
-\eta_{1,2} & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
-\eta_{1,r} & \cdots & \cdots & \eta_{r,r} + \sum_{i \neq r} \eta_{i,r}
\end{bmatrix}
\]

Here, \( E \) is the vector of the subsystem energy and \( P \) is the vector of the power input from
outside as averaged over space and frequency, \( \omega \) is the band center angular frequency, and \( L \)
is the loss factor matrix, where the components \( \eta_{i,j} \) and \( \eta_{j,i} \) are the ILF for subsystem \( i \)
and the CLF from subsystems \( i \) to \( j \), respectively.

2.2 Derivation of values averaged over space and frequency

In this section, we present the power input and the subsystem energy averaged over
space and frequency as derived from a modal expansion formula for response velocity.

2.2.1 Modal expansion formula for response velocity

Using a normalized mode, the steady state vibration response velocity \( V(x, x_e, \omega) \) at \( x \) in
a system with an excitation force \( F \) at \( x_e \) can be expressed as

\[
V(x_e, x_e, \omega) = \sum_{i=1}^{n} \frac{j\omega \phi_i(x) \phi_i(x_e) F}{\omega_i^2 (1 + j\eta_i) - \omega^2}
\]

Here, \( \omega \) is the excitation angular frequency, \( \omega_i, \phi_i \) and \( \eta_i \) represent the natural angular frequency, the mass normalized mode shape and the loss factor of subsystem \( i \), respectively.

Regarding a normalized mode shape, the following orthogonality relation holds true

\[
\int_A \phi_i(x) \phi_k(x) \, dx = \begin{cases} 0 & (i \neq k) \\ \frac{A}{m} & (i = k) \end{cases}
\]

where \( A \) represents the spatial domain of the system (length in one dimension, area in two dimensions, and volume in three dimensions), and \( m \) is the mass of the system.

### 2.2.2 Power input

The power input from outside into system \( P_n(x_e, \omega) \) can be obtained from the real part of the product of the excitation force \( F \) and the driving point velocity \( V(x_e, x_e, \omega) \). By substituting Eq.(3), we obtain

\[
P_n(x_e, \omega) = \frac{1}{2} \text{Re} \left( F V(x_e, x_e, \omega) \right) = \frac{1}{2} F^2 \sum_{i=1}^{n} \frac{\omega_i \eta_i \phi_i^2(x_e)}{\omega_i^2 (1 + j\eta_i) - \omega^2 + (\eta_i \omega_i)^2}
\]

where * denotes a complex conjugate.

Here, the excitation force is assumed to be a rain-on-the-roof excitation applied homogenously and without correlation inside the system and if Eq. (5) is averaged over the excitation points \( x_e \) in the spatial domain \( A \), the spatial average \( \overline{P_n}(\omega) \) for the power input is obtained from the following formula.

\[
\overline{P_n}(\omega) = \frac{1}{A} \int_A P_n(x_e, \omega) \, dx_e = \frac{F^2}{2m} \sum_{i=1}^{n} \frac{\omega_i^2 \eta_i}{\omega_i^2 (1 + j\eta_i) - \omega^2 + (\eta_i \omega_i)^2}
\]

Furthermore, from Eq.(6), the frequency average of the power input for bandwidth \( B \) between frequencies \( \omega_a \) and \( \omega_b \) is

\[
\langle \overline{P_n} \rangle = \frac{1}{B} \int_{\omega_a}^{\omega_b} \overline{P_n}(\omega) \, d\omega = \frac{F^2}{4mB} N (\alpha - \beta)
\]

In the above formula, integration by substitution is performed by letting

\[
z = \left( \frac{\omega_i^2 - \omega^2}{\eta_i \omega_i^2} \right)^2
\]

and the number of natural modes contained in bandwidth \( B \) is taken as \( N \). Also, in case the loss factor \( \eta_i \) is small, \( \alpha \) and \( \beta \) can be approximated as

\[
\alpha = \tan^{-1} \left( \frac{1}{\eta} \left( \frac{\omega_a}{\omega} \right)^2 \right) \approx \frac{\pi}{2} \left( \frac{\omega_a}{\omega} \right)
\]

\[
\beta = \tan^{-1} \left( \frac{1}{\eta} \left( \frac{\omega_b}{\omega} \right)^2 \right) \approx \frac{\pi}{2} \left( \frac{\omega_b}{\omega} \right)
\]

therefore,

\[
\alpha - \beta \approx \begin{cases} 0 & (\omega_a < \omega_b) \\ \pi & (\omega_a < \omega_b) \end{cases}
\]

and Eq.(7) can be rewritten as the following formula

\[
\langle \overline{P_n} \rangle = \frac{F^2}{4mB} N \pi = \frac{\pi F^2}{4m n}
\]

Here, \( n = N / B \) is the modal density. The power input averaged over space and frequency is expressed in terms of modal density from Eq. (9), and the only other information regarding the subsystem is its mass.
2.2.3 Subsystem energy

Next, we show the derivation of the subsystem energy averaged over space and frequency. If the subsystem energy is considered to be twice the kinetic energy (14), then in case the excitation force $F$ is applied at $x_e$, the total response energy of a system $E(x_e, \omega)$ is as follows:

$$E(x_e, \omega) = 2 \times \frac{1}{2} \int \left[ V(x, x_e, \omega) V^*(x, x_e, \omega) dx \right] = \frac{1}{2} F^2 \sum_{i=1}^{n} \frac{\omega_i^2 \phi_i^2(x_e)}{(\omega_i^2 - \omega^2)^2 + (\eta_i \omega_i)^2}$$  \hspace{1cm} (10)

Furthermore, if spatial averaging with respect to the excitation points is performed in the same way as in Eq. (6) and subsequently the frequency is averaged in the same way as in Eq. (7). If the loss factor for all modes $\eta_i$ is given by the constant $\eta$, then the value averaged over space and frequency of the subsystem energy is as follows:

$$\langle \bar{E} \rangle = \frac{1}{4} \int A \left[ E(x_e, \omega) dx_e \right] = \frac{\pi F^2}{4m} \frac{1}{\omega \eta}$$  \hspace{1cm} (11)

As shown in this equation, the subsystem energy averaged over space and frequency is also expressed in terms of modal density, and depends only on the loss factor $\eta$ (i.e., ILF) in addition to the subsystem mass $m$, while it does not depend on the subsystem shape or the boundary conditions. Furthermore, in Eqs. (9) and (11), the relation satisfies

$$\bar{P}_{12} = \omega \eta \langle \bar{E} \rangle$$  \hspace{1cm} (12)

2.3 Derivation of CLF by wave theory

Here, we focus on CLF, which is related to bending waves between plates (1).

Assume that subsystems 1 and 2 are separated by a straight boundary line of coupling length $L$, as shown in Fig. 1. Focusing on a single wave in subsystem 1 moving towards the coupling boundary at angle $\theta$, we derive the power $P_{1,2}$ transmitted to subsystem 2 by that wave.

If the energy per unit area (energy density) carried by that wave is denoted as $e_1^+$, the intensity associated with this energy can be expressed as $e_1^+ c_{g1}$ by using the bending wave group velocity $c_{g1}$. The wave propagates in the range between $\theta = -\pi/2$ and $\pi/2$ at the coupling boundary, and if it is transmitted homogenously regardless of $\theta$, the incident power $P_1^+ (\theta)$ at angle $\theta$ for an infinitesimal length of the coupling boundary $dL$ can be expressed as:

$$P_1^+ (\theta) = \frac{e_1^+ c_{g1}}{\pi}$$  \hspace{1cm} (13)

Accordingly, using the energy transmissibility $\tau(\theta)$, the power transmitted to subsystem 2, $P_{1,2}^+ (\theta)$, is:

$$P_{1,2}^+ (\theta) = \tau(\theta) P_1^+ (\theta) \cos \theta$$  \hspace{1cm} (14)

Fig. 1 Energy transmitted through the coupling between subsystems
Integrating this in relation to the angle of incidence and the coupling length, the transfer power through the boundary $P_{1,2}$ is given by the following formula.

$$P_{1,2} = \int_{0}^{\pi} \int_{-\pi/2}^{\pi/2} P_{1,2}^{(\theta)} d\theta dL.$$  \hfill (15)

Furthermore, assuming the respective energy densities of all propagating waves are equal to $e_{1,2}^{(\theta)}$, it can be expressed as $e_{1,2}^{(\theta)} = E_{1}/S_{1}$ ($S_{1}$ is the surface area of subsystem 1) by using the total energy $E_{1}$ of subsystem 1. Accordingly, $P_{1,2}^{(\theta)}$ of Eq.(13) becomes constant, and Eq.(15) becomes

$$P_{1,2} = \frac{E_{1}c_{1,2}}{\pi S_{1}} L \tau_{1,2}.$$ \hfill (16)

Here,

$$\tau_{1,2} = \int_{-\pi/2}^{\pi/2} \tau(\theta) \cos \theta \, d\theta.$$  \hfill (17)

Furthermore, in SEA it is considered that the power proportional to the energy $E_{1}$ of the subsystem is transmitted to adjacent subsystems, where the constant of proportionality is CLF. For this reason, CLF is derived from Eq.(16) as

$$\eta_{1,2} = \frac{c_{1,2}}{\omega S_{1}} \tau_{1,2} = \frac{L}{\pi \omega c_{1,2} \eta_{1}} \tau_{1,2}.$$ \hfill (18)

Here, $n_{1}$ is the modal density of subsystem 1 expressed by Eq.(19).

$$n_{1} = \frac{S_{1} \omega}{\pi c_{1,2}^{2}}.$$ \hfill (19)

The following relationship holds true for the modal density and CLF.

$$n_{i} \eta_{i,j} = n_{j} \eta_{i,j}.$$ \hfill (20)

Similarly to the power input and the subsystem energy as averaged over space and frequency, CLF is also expressed in terms of modal density due to the assumption of uniformity of the transmission waves, and does not depend on the subsystem shape or the boundary conditions.
2.4 Application index of analytical SEA

When the structural field does not satisfy uniformity of the assumed wave motion in the derivation of Eq.(18), accurate analysis can not be expected from SEA. Structural fields that satisfy uniformity of wave motion can be regarded as equivalent to the case where adjacent peak resonances influence each other and individual peak resonances cannot be distinguished. In such structural fields, the resonance peak half bandwidth (damping) becomes larger than the natural frequency interval.

In this perspective, as a factor representing the state of the structural field, the Modal Overlap Factor (MOF) is defined as the ratio of the natural frequency interval $\Delta \omega$ and the half bandwidth $\omega \eta$, and can be expressed with the following equation (21).

$$MOF = \frac{1}{N} \sum_{i=1}^{N} \frac{\omega_i}{\Delta \omega_i} = \frac{\omega \eta}{\Delta \omega} = \omega \eta$$

In the above equation, the frequency interval and the half bandwidth are expanded as a constant $\Delta \omega = 1/n$ ($n$ is the modal density, $N$ is the number of modes) and $\omega \eta$. Here, $\eta$ also represents ILF, in the same way as in Eq.(12). MOF is evaluated for each subsystem, and if the MOF of all subsystems of the system is greater than 1, the application of analytical SEA to that system is regarded as satisfactory (1),(13).

3. Verification by FEM Analysis

In the previous section, the principle parameters of analytical SEA were described primarily in terms of modal density. It was shown that they do not depend on the subsystem shape or the boundary conditions. In this section, the non-dependence of power input, subsystem energy and coupling loss factor on the subsystem shape is verified by FEM analysis. In cases where two types of loss factors (all modes are uniformly 0.05 and 0.025) are verified in order to discuss the applicability of analytical SEA (determining the applicable lower limit of the frequency band) by means of MOF.

3.1 Verification method

There are three types of structural fields (triangular, rectangular, and pentagonal subsystem shapes) used in the verification, as shown in Fig. 2. The symbol “#” in the figure denotes the subsystem number, and the dotted lines in (b) and (c) indicated the rectangular shape of (a). Only the shape of equivalently numbered subsystems in (a) to (c) is transformed to make their respective surface areas and plate thicknesses equal in order for the modal density given by Eq.(19) to be equal. Furthermore, since Eq. (19) does not hold true in the low frequency region, the number of modes for each structure in Fig. 2 is calculated through eigenvalue analysis, and it is confirmed in advance that the argument presented here that the mode densities are equal provides the effective lower limit for the frequency band.

First, the values of the power input of Eq. (9) and the energy of the subsystem of Eq. (11) as averaged over space and frequency will be verified. For this, FEM analysis is carried out only with respect to subsystem 1 (free boundary) of each of the structures of Fig. 2, and a comparison is made with the results of Eqs. (9) and (11).

Next, in verifying the CLF of Eq. (18), FEM analysis is carried out on the three types of composite structures (free boundary) shown in Fig. 2, where subsystems 1 and 2 are the coupled orthogonally at the side with a length of 0.35m, and the CLF as obtained with the Power Injection Method (PIM) is compared with the result of Eq. (18).

Furthermore, the subsystem energy is estimated by using the analytical SEA model, and the analysis results of FEM performed on the three types of composite structures are compared.
3.2 Analysis procedure

3.2.1 FEM Analysis

The FEM model for the target structure is created using ANSYS Rev.8.0, after which an eigenvalue analysis is performed and the natural frequencies and natural modes are extracted. Next, the forced vibration response is calculated with MATLAB by means of modal superposition. The target frequency band is taken between the 100Hz band and the 800Hz band in the 1/3 octave and the applicability of analytical SEA, and the responses are calculated in 1Hz increments in the interval between 80Hz and 900Hz. Shell elements are used in FEM models, and the length of the side of the mesh is taken to be less than 3.5mm (1/6 of a bending wavelength at 2kHz).

The excitation conditions are such that all node points inside the subsystem (in case of a composite structure, node points located on the coupling edge are excluded) are regarded as excitation points for the purpose of simulating rain-on-the-roof excitation\(^{(11),(13),(15)}\). The excitation force \(F=1\text{N}\) is applied on each excitation point, and both the power input \(P_{in}(f)\) and the average in space and frequency of subsystem energy \(E_i(f)\) are calculated from the following formula.

\[
P_{in}(f) = \frac{1}{2} \text{Re}(F V^*_e)
\]

\[
E_i(f) = \frac{1}{2R} m_i \sum_{k} V_k V^*_e
\]

Here, \(m_i\) and \(N_i\) are the total mass and the number of nodes of subsystem \(i\), \(V_k\) is the response velocity spectrum of node \(k\), \(e\) is the excitation node, and \(R\) is the number of excitation points.

3.2.2 Estimation of subsystem energy with analytical SEA model

Each composite structure is taken to be a system of two subsystems (#1 and #2) which take into account only the bending vibration (out-of-plane vibration), whose contribution to sound is considerable. The analytical SEA model of the system of two subsystems is governed by the following equation.

\[
\begin{bmatrix}
E_1 \\
E_2
\end{bmatrix} = \frac{1}{\omega} \begin{bmatrix}
\eta_{11} + \eta_{12} & -\eta_{12} \\
-\eta_{12} & \eta_{22} + \eta_{21}
\end{bmatrix} \begin{bmatrix}
P_1 \\
P_2
\end{bmatrix}
\]

(24)

Here, ILF \((\eta_{11,1} \text{ and } \eta_{22,2})\) is the loss factor used in FEM analysis (0.05 or 0.025) and the CLF \((\eta_{12,2} \text{ and } \eta_{21,1})\) is calculated from Eq.(18). Furthermore, the transmissibility \(\tau_{i,j}\) within Eq. (18) is calculated by

\[
\tau_{i,j} = 2 \left(\psi_{0,i}^{0.5} + \psi_{0,j}^{0.5}\right)^{-2} \frac{2.754 h_{i,j}}{1 + 3.24 h_{i,j}}
\]

(25)

\[
\psi_{i,j} = \rho_i \left(\frac{c_{ij}}{c_{ij}}\right)^{1.5} h_{i,j}^{2.5}
\]

(26)

Here, \(h_{i,j}\) represents the plate thickness ratio \((=t_i/t_j)\), and \(\rho_i\), \(c_{ij}\) and \(t_i\) represent the density, the longitudinal wave velocity and the plate thickness of system \(i\) respectively.

3.3 Results and considerations

3.3.1 Target structures

The number of natural modes and the modal densities of the subsystems in Fig. 2 as calculated with FEM under the condition of free boundary are shown in Table 1 and Fig. 3. Discrepancies such as subsystems with no natural modes due to the subsystem shape are
evident below the 200Hz band. However, the number of mode increases together with the frequency, and discrepancies due to subsystem shape become fewer. Also, mode densities which do not depend on subsystem shape become almost identical at the 315Hz band and above, which is in agreement with the result calculated from Eq. (19). As a result, it can be said that the argument of this paper (namely that the mode densities can be regarded as equivalent) holds true for the 315Hz band and above.

Also, the lower limit frequency band for analytical SEA for which the MOF of each subsystem is 1 or higher is 315Hz when the loss factor is 0.05 and 630Hz when the loss factor is 0.025.

3.3.2 Verification of space and frequency average value

The verification results for the values of the power input in Eq. (9) and the of subsystem energy in Eq. (11) as averaged over space and frequency are shown below. The respective spatial averages of the power input and the subsystem energy obtained with FEM analysis performed only on subsystem 1 in Fig. 2 are shown in Fig. 4 together with the results of Eqs. (9) and (11). It is clear that the power input and the subsystem energy change in accordance to the subsystem shape.

Figure 5 shows the results presented in Fig. 4 as averaged over frequency. Figures 5 (a)
and (c) present the results for the case where the loss factor is 0.05. All results coincide and do not depend on the subsystem shape for the 315Hz and above, for which MOF > 1. Also, there is good agreement at high frequencies, which shows that analytical SEA is effective at high frequencies. In addition, as it can be seen from Figs. 5 (b) and (d), when the loss factor is 0.025, all of the results above the 630Hz band for which MOF > 1 are also in agreement, and the agreement is stronger at higher frequency bands.

From the above results, it is found that the subsystem energies averaged over space and frequency are equal as long as the modal densities are equal, regardless of the shape of the
subsystems, and the validity of Eqs. (9) and (11) can be confirmed.

3.3.3 Verification of CLF

Regarding the CLF of a composite structure where subsystems 1 and 2 are orthogonal, as shown in Fig.2, the calculation result of Eq. (18) and CLF, \( \eta_{1,2} \), as calculated using FEM and PIM, are shown in Fig. 6.

In the 315Hz band and above, where the modal density does not depend on the shape and all results generally coincide, the representation of CLF primarily in terms of modal density can be verified. Furthermore, since the theoretical formula for CLF shown in Eq. (18) does not depend on the loss factor, the analytical results in Figs. 6 (a) and (b) are the same. Meanwhile, the results calculated with FEM change in accordance with the loss

(a) Loss factor 0.05
(b) Loss factor 0.025

Fig. 6 Comparison of coupling loss factors estimated with analytical SEA and those calculated with FEM

(a) Exciting subsystem, #1
(b) Received subsystem, #2

Fig. 7 Comparison of energies estimated with analytical SEA and those calculated with FEM in the case of a loss factor of 0.05

(a) Exciting subsystem, #1
(b) Received subsystem, #2

Fig. 8 Comparison of energies estimated with analytical SEA and those calculated with FEM in the case of a loss factor of 0.025
factor. This is frequently observed in experimental SEA\(^{(15)}\).

### 3.3.4 Verification of energy estimated with analytical SEA

Figures 7 and 8 show the results for the energy of each subsystem as estimated with analytical SEA for the case where 1W of power is inputted into subsystem 1 in accordance with section 3.2.2, as well as the results for each subsystem calculated in accordance with section 3.2.1, which are the basis of the rain-on-the-roof excitation with FEM.

There is a satisfactory agreement between the results obtained with FEM and those for all composite structures estimated with analytical SEA ("Estimated with Analytical SEA" in the figure legend) in the case where the loss factor in Fig. 7 is 0.05 and for the 315Hz band and above, for which \(\text{MOF} > 1\). Also, all results for the 630Hz band and above, for which \(\text{MOF} > 1\), also agree well when the case loss factor is 0.025 in Fig. 8.

From the above, analytical SEA is valid for frequency bands for which \(\text{MOF} > 1\), and it can be confirmed that the response averaged over space and frequency does not depend on the subsystem shape. In addition, the response depends on the subsystem shape in frequency bands where \(\text{MOF} < 1\) even if averaged over frequency and space, and discrepancies become apparent.

### 4. Conclusion

Analytical SEA is an effective tool which can be used in analyzing vibration and noise in the high frequency region from the initial stages of design. Along with showing that the power input, the subsystem energy and the coupling loss factor as averaged over space and frequency are expressed primarily in terms of modal density, this paper clarifies the derivation process for the above for the efficient utilization of analytical SEA. Also, in order to verify the validity of these results, three types of systems with equal modal density and different subsystem shapes were prepared and FEM analysis was performed. Also, it was confirmed that analytical SEA can be effectively applied in problems targeting frequency regions for which the modal overlap factor (\(\text{MOF}\)) exceeds 1.

### References

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