Vibration of a Wind Turbine Blade (Theoretical Analysis and Experiment Using a Single Rigid Blade Model)*

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Abstract
Recently, the use of the wind turbine generator has rapidly spread as the one of the clean energy resources, and its size is becoming larger. Because of the difficulty of the maintenance of such a large wind turbine generator, the vibration analysis and clarification of the dynamical characteristics of the wind turbine generator are important. However, the conventional researches aimed at the fluid engineering in order to gain higher efficiency, and only several investigations have been reported in the area of the vibration analysis and its suppression. The wind turbine is a special type of rotating machinery: It has a long heavy blade which rotates in the vertical plane under the action of the gravitational force, and the wind power acting on the wind turbine blade varies periodically because the wind power increases as the function of the height. Therefore, it requires the original point of view in the dynamical analysis. This paper investigates the fundamental vibration characteristic of the wind turbine blade by using a rigid blade model. The gravitational force causes the effect of parametric excitation on the blade system. The interaction effect of the gravitational force and the wind force on the vibration of the turbine blade is investigated, and the occurrence of super-harmonic resonance is clarified. Also, the occurrences of the unstable vibration ranges are indicated. Furthermore, these results are validated experimentally.

Key words: Vibration of Rotating Body, Wind Turbine Blade, Forced Vibration, Parametric Excitation, Super Harmonic Resonance, Stability

1. INTRODUCTION
The use of the wind power generation has rapidly spread as the one of the clean energy resources. Its size is becoming larger, and it is installed on the sea or on the mountain in order to increase the efficiency. Therefore, the high tolerance for trouble and damage is desired in order to reduce the maintenance cost. In this point of view, both the vibration analysis and its suppression are important. However, the conventional studies on this area have mainly focused on the fluid dynamics for turbine blade, and there are only several studies on the time history analysis of blade vibration. Recently, the elastic structure of turbine blade is examined for the large wind turbine. In this examination, the rated rotational speed is set at the higher side of the resonance point. Therefore, the investigation of the vibration characteristics in the wind turbine system will be more important subjects.

The wind turbine is the special type of rotor system which has a long and heavy blade rotating in the vertical plane under the action of the gravitational force. The wind force is known to increase depending on the height from the grand surface, and thus, the rotating blade is excited periodically by this wind force. In addition, the pulse-like force also acts on the blade when the blade passes in front of the main tower. Therefore, the vibration characteristics of
the turbine blade are expected to be different from those in the case of general turbine blade or the helicopter blade which rotates in the horizontal plane\(^9\)\(^{10}\). Chopra et al.\(^4\) considered the wind turbine with the rigid twin blades which have 3 degrees of freedom, and investigated its vibration taking the influence of nonlinearity into consideration. They clarified theoretically the softening of the resonance curve, the unstable vibration due to the parametric excitation effect generated by the gravitational force on the blade, and the flutter of the blade. However, the interaction of the parametric excitation and the forced vibration due to wind force is not investigated, and the obtained results have not been validated experimentally. Other conventional studies\(^5\)\(^6\) are mostly numerical investigation in the time domain, and the vibration phenomena due to the characteristics of the wind turbine have not been investigated.

This paper investigates the fundamental vibration characteristic of the wind turbine blade. The rigid blade is supported by spring plate and rotates in the vertical plane. The influence of the interaction between the gravitational force and the wind force during the blade rotation is considered, and the vibration characteristics are investigated. The influences of the parameters of blade shape are studied by both the numerical simulation and the theoretical analysis, and the occurrence of super harmonic resonance and the occurrence of the unstable vibration range are confirmed. Especially, both the case with only the constant wind force and the case with the wind force which varies depending on the height from the ground are noted. In the later case, the force acting on the blade becomes periodic. The interaction of the parametric excitation effect due to the gravitational force, which is pointed out by Chopra et al.\(^4\), and the wind force on the vibration characteristics of the blade is clarified. Moreover, the obtained theoretical results are validated experimentally.

2. Theoretical model and the equation of motion

2.1. Rigid blade model

Figure 1 shows the theoretical model of a wind turbine blade. The diameter of the rotating main shaft is assumed as 0, and the blade is assumed as the rigid body with uniform thin rectangular shape. One end of the rigid blade is supported perpendicular to the horizontal main shaft by the spring plate of spring constant \(k\) [Nm/ rad], and the other end is free end. The width, thickness, and length of the rigid blade are denoted by \(b\) [m], \(h_b\) [m], and \(l\) [m], respectively. The density of the blade is \(\rho\) [kg/m\(^3\)].

This paper investigates only the out of plane vibration of the blade. The inertia coordinate system O-\(x'y'z\) with the origin O is set at the blade’s installation position as shown in Fig.1. The \(z\) axis is taken to be identical to the center axis of the horizontal main shaft, and the \(y\) axis is taken to the vertical direction. The rigid blade rotates with constant angular velocity \(\omega\) [rad/sec] about \(z\) axis. The rotational coordinate system O-\(x'y'z\) which rotates with the blade is considered, and the \(y'\) axis is taken to be identical to the blade’s center line. The \(x'\) and \(y'\) axes are identical to \(x\) and \(y\) axes at \(t = 0\) [s] as shown in Fig.2(a).

2.2. Energy derivation and equation of motion

2.2.1. Kinetic energy of the blade

The horizontal displacement of the free end of the blade is denoted as \(z_b\) [m]. The out of plane vibration is assumed to be small (\(\theta \ll 1\) [rad]), and approximations of \(\sin \theta \approx \theta = z_b/l\) and \(\cos \theta \approx 1 - (1/2)\theta^2 = 1 - (1/2)(z_b/l)^2\) are used. The arbitrary position of \(\xi\) in the blade at \(\theta = 0\) is noted, and its coordinate is represented as \((x_\xi, y_\xi, z_\xi)\) when the blade bends with angle \(\theta\). The kinetic energy of the small element at \(\xi\) is obtained, and then, it is integrated for all length of the blade. Thus, the kinetic energy of the blade \(T_b\) is derived as follows. Here, the cubic or higher order terms of \(\theta\) are neglected.

\[
T_b = \frac{1}{2} \rho bh_b \left( \int_0^l x_\xi^2 d\eta + \int_0^l y_\xi^2 d\eta + \int_0^l z_\xi^2 d\eta \right)
\]

\[
= \frac{1}{2} \rho bh_b l \left( \omega^2 + \omega^2 z_b^2 + z_b^2 \right)
\]

2.2.2. Potential energy of the blade

The blade’s potential energy consists of component due to the spring \(k\) and the component due to the gravity. The potential energy of the small
element at the position of $\xi$ is obtained, and then, it is integrated for all length of the blade. Thus, the potential energy of the blade $U_b [\text{N} \cdot \text{m}]$ is derived as follows.

$$U_b = \frac{k_2}{2} l^2 z^2 + \frac{1}{2} \rho bh l^2 g \left( 1 - \frac{z^2}{2 l^2} \right) \sin \omega t$$  (2)

2.2.3. Non-conservative force acting on the blade

The considered non-conservative forces acting on the blade are the damping force $D [\text{N}]$ and the wind force $Q_b [\text{N}]$.

The damping force is assumed to be viscous damping force, and it is represented as the function of the blade’s tip velocity $\dot{z}_b$. The total damping force of the blade is derived by integrating the damping force of the small element, $dD$, at the position of $\xi$ for all length of the blade as $D = \int dD$. Rayleigh’s dissipation function is derived as $(1/2)c_b \dot{z}_b^2$, and the damping coefficient $c_b [\text{N} \cdot \text{s} / \text{m}]$ is obtained from the damped free vibration of the experimental system.

The wind force $Q_b$ acting on the blade is also considered. Wind velocity varies depending on the height from the ground, and it causes the difference of the wind force between high and low positions\(^{(4)}\). Moreover, similar effect occurs when the wind direction suddenly changes and it is not perpendicular to the blade plane.

Imamura et al.\(^{(2)}\) clarified that the blade vibration has little influence on the wind force acting on the blade. Thus, in this paper, the wind force acting on the blade is assumed to depend only on the height from the ground as shown in Fig.1. Hence, the wind force acting on the rotating blade is considered to vary harmonically with the blade rotation due to the influence of the height.

The wind force at the height of the main shaft is denoted as $Q_0$, and the wind force acting on the blade is represented as $Q_b = Q_0 + \Delta Q \cos \omega t$ which varies with the rotational speed $\omega$. From the above consideration, the equation of motion of the rigid blade of wind turbine is
derived from the Lagrange equation as:

$$m\ddot{z}_b + c_b\dot{z}_b + \left(\frac{mk\omega^2}{3} + \frac{k}{l}\right)z_b - \frac{mg}{2l}z_b \sin \omega t = Q_0 + \Delta Q \cos \omega t$$

(3)

Here, $m = \rho bh_b l$ is the total mass of the blade. The parameters of the blade used in the numerical simulation are the same with the parameters of experimental system. The width of the blade $b$ is set as the variable parameter.

The stiffness of the spring plate is represented as $k = (2EI)/l_s$, and $l_s$ is the length of spring plate. The area moment of inertia of the spring plate is $I = (b_s h^3)/12$ for width $b_s$ and thickness $h_s$. Two cases of blades are considered, and the parameters of these blades are denoted in Table 1. The spring constants of these two cases, $k_1$ and $k_2$, are obtained as:

$$k_j = \frac{2EI_j}{l_s}, \quad (j = 1, 2)$$

(4)

### 2.3. Dimensionless form

The dimensionless form of the equation of motion of Eq.(3) is derived. The thickness of the blade $h_b$ and the natural angular frequency $\omega_0 = \sqrt{k/m}$ in non-rotating condition are used as the representative values of length and angular frequency, respectively. Here, the dimensionless values are shown in the following with the symbol $\bar{\cdot}$:

\[
\begin{align*}
&\bar{z}_b = \frac{z_b}{h_b}, \quad \bar{c}_b = \frac{3c_b}{mp_0}, \quad \bar{\omega} = \frac{\omega}{\omega_0}, \quad \bar{t} = pt, \\
&\bar{l} = \frac{l}{h_b}, \quad \bar{g} = \frac{g}{h_b p_0^2}, \quad \bar{Q}_0 = \frac{3Q_0}{m h_b p_0^2}, \quad \bar{\Delta Q} = \frac{3\Delta Q}{m h_b p_0^2}
\end{align*}
\]

(5)

Moreover, the dimensionless value $\epsilon$ is introduced in order to represent the magnitude of parametric excitation term.

$$\bar{\epsilon} = \frac{3\bar{g}}{2\bar{l}} = \frac{mgl}{2k}$$

(6)

In the following sections of theoretical analysis, only the dimensionless values are used, and therefore, the symbol $\bar{\cdot}$ is omitted. The equation of motion in the dimensionless form is derived as:

$$\ddot{z}_b + c_b\dot{z}_b + (1 + \bar{\omega}^2)z_b = \frac{3g}{2\bar{l}}z_b \sin \bar{\omega} \bar{t} = Q_0 + \Delta Q \cos \bar{\omega} \bar{t}$$

(7)

or,

$$\ddot{z}_b + c_b\dot{z}_b + (1 + \bar{\omega}^2)z_b - \epsilon z_b \sin \bar{\omega} \bar{t} = Q_0 + \Delta Q \cos \bar{\omega} \bar{t}$$

(8)

### 2.4. The natural frequency of the rigid blade

By eliminating the values $c_b, \epsilon, Q_0$ and $\Delta Q$, the frequency equation is derived from the equation of motion of Eq.(8). The free vibration is assumed as $z_b = z_{0b} \sin pt$ and substituted to the frequency equation. By setting the coefficient of $\sin pt$ to 0, the undamped natural frequency is obtained as $p = \sqrt{1 + \bar{\omega}^2}$ in dimensionless form. Figure 3 shows the natural frequency $p$ for the rotational speed $\omega$. The natural frequency $p$ increases with the rotational speed $\omega$, and there is no main resonance $p = \omega$.

The lines $2\omega, 3\omega, \cdots, n\omega, (3/2)\omega, (5/2)\omega, \cdots, (2n-1)/2)\omega$ (n is the natural number) are also drawn in Fig.3 in order to indicate the possible ranges of the unstable vibrations due to the parametric excitation.
3. Numerical simulation

The numerical simulation of the equation of motion, Eq.(8), is performed. According to the definition of $\epsilon$ shown in Eq.(6), the influence of $\epsilon$ is investigated by changing both the spring constant $k$ and the mass of the blade $m$.

3.1. Case with no wind force ($Q_0 = \Delta Q = 0$)

By setting $Q_0 = \Delta Q = 0$ in Eq.(8), the system becomes the parametrically excited system with no excitation force. Chopra et al.(4) investigated the unstable vibration range due to parametric excitation at $p = 2\omega$ in such a system by theoretical analysis. Two values of $k$ shown in Table 1 are used, and the width of the blade $b$ is varied in order to change $m$.

Figure 4 shows the resonance curves for the various cases of $k$ and $m$. The unstable vibration range does not appear in the case of $k=8.58$ Nm/rad shown in Fig.4(a). In the case of $k=1.07$ Nm/rad shown in Fig.4(b), the unstable range appears at $p = (3/2)\omega$ when $m=0.35$ kg, and at both $p = (3/2)\omega$ and $2\omega$ when $m=0.45$ kg.

Furthermore, Fig.4(b) indicates that the unstable vibration range becomes wider when $m$ increases. These results are explained because the value of parametric excitation term $\epsilon$ increases when the spring constant $k$ decreases or the mass $m$ increases according to Eq.(6). The occurrence of the unstable vibration and its occurrence range are investigated in detail in the next section by analyzing the stability of the Mathieu equation with damping.
3.2. Case with constant wind force ($\Delta Q = 0$)

By setting $Q_0 \neq 0$, $\Delta Q = 0$ in Eq.(8), the system becomes the parametrically excited system with constant external force. The parameters of spring constant $k$ and mass $m$ are changed as the case of Fig.4. The resonance curves of the cases $k = 8.58$ Nm/rad and $k = 1.07$ Nm/rad are shown in Fig.5. The super harmonic resonance $2\omega$ occurs in the case of $k = 8.58$ Nm/rad shown in Fig.5(a). It is not the unstable vibration due to parametric excitation but the forced oscillation which can be explained as follows.

Equation (8) is considered to explain the occurrence of the super harmonic resonance. First, let the fourth term in the left side of Eq.(8), which shows the parametric excitation effect, move to the right side, and it is considered as the quasi-external force. The trivial solution is $z_0 = 0$ when the constant wind force is $Q_0 = 0$. As increasing the constant wind force $Q_0$ from 0, the constant component $Z_{b0}$ is caused in displacement $z_b$ due to the balance of $Q_0$ and the third term in the left side of Eq.(8). This caused constant component $Z_{b0}$ generates the $\sin \omega t$ component through the parametric excitation term which is moved to the right side. This generated periodic term can be considered as the quasi-external periodic force of frequency $\omega$, and this quasi-external force causes the periodic displacement component $Z_{b\omega}$ of frequency $\omega$ due to the balance of this periodic force and the third term in the left side of Eq.(8). However, there is no resonance point of the harmonic resonance $p = \omega$ as shown in Fig.3.

This caused periodic component $Z_{b\omega}$ of frequency $\omega$ generates the quasi-external periodic force of frequency $2\omega$ through the parametric excitation term. This quasi-external periodic force of frequency $2\omega$ causes the super harmonic resonance of the second order at the resonance point $p = 2\omega$ shown in Fig.3. This consideration also explains, as shown in Fig.5(a), that the amplitude of the super harmonic resonance increases as mass $m$ of the blade increases because the parametric excitation term becomes larger with mass $m$. Furthermore, as the constant wind force $Q_0$ grows, the series of quasi-external forces generated in the chain reaction through the parametric excitation term also become larger, and the super harmonic resonances of the third and the higher order may occur.

When the spring constant decreases to $k = 1.07$ Nm/rad, the parametric excitation term increases as shown in Eq.(6). As the result, the chain reaction through the parametric excitation term becomes larger, and the super harmonic resonances of the third and the higher order are also generated as shown in Fig.5(b). Moreover, the unstable vibration range appears...
at $p = \frac{3}{2}\omega$, which is already explained in Fig.4(b), in addition to these super harmonic resonances. Its occurrence range is not affected by the constant wind force $Q_0$.

### 3.3. Case with periodic wind force ($Q_0 \neq 0, \Delta Q \neq 0$)

Figure 6 is the resonance curve when the wind force is $Q_b = Q_0 + \Delta Q \cos \omega t$. Figure 6(a) and (b) are the cases of $k = 8.58 \text{ Nm/rad}$ and $k = 1.07 \text{ Nm/rad}$, respectively. In the case of $k = 8.58 \text{ Nm/rad}$, the super harmonic resonances of the second and the third order occur through the parametric excitation. Furthermore, the whole vibration amplitude level is larger than the case of Fig.5. It can be explained in the same manner of previous section as follows: First, let the fourth term of the parametric excitation effect in the left side of Eq.(8) moves to right side. The constant component $Q_0$ in the wind force causes the constant displacement component $Z_{b0}$ due to the balance of $Q_0$ and the third term in the left side of Eq.(8). This caused constant displacement component $Z_{b0}$ generates the quasi-external periodic force of frequency $\omega$ through the parametric excitation term. When there is the time variation $\Delta Q$ in the wind force, the resultant force of both the periodic component $\Delta Q \cos \omega t$ and the quasi-external periodic force of frequency $\omega$ causes the periodic vibration component $Z_{b\omega}$ of frequency $\omega$. Therefore, its amplitude is larger than the case of Fig.5 in which the wind force is constant.

The caused periodic component $Z_{b\omega}$ with frequency $\omega$ in displacement $z_b$ generates periodic force with frequency $2\omega$ through the parametric excitation term. The chain reaction through the parametric excitation term appears similar to the case of Fig.5, and its reaction is stronger than the case of Fig.5 because of the direct action of periodic wind force component $\Delta Q \cos \omega t$. As the result, the super harmonic resonance of the third order is also observed.

Figure 6(b) for $k = 1.07\text{[Nm/rad]}$ shows the case with larger parametric excitation term. Similar to the case with constant wind force shown in Fig.5(b), the super harmonic resonances of $2\omega, 3\omega, \ldots$ occur at the resonance points shown in Fig.3, and the unstable vibration occurs at the frequency of $p = \frac{3}{2}\omega$ corresponding to the unstable range shown in the case with no wind force. The vibration amplitude level of $z_b$ increases and the unstable range becomes wider as blade mass $m$ increases because the parametric excitation term becomes larger with mass $m$. However, as the characteristic of the chain reaction through the parametric excitation term.
term is same as the case with constant wind force, the qualitative characteristics of generated vibrations are the same.

Such super harmonic resonances caused by the interaction of both the parametric excitation and the external periodic force has been investigated$^{[11][12]}$. For example, Blankenship et al.$^{[11]}$ investigated the forced oscillation in the mechanical system which has both the parametric excitation and nonlinearity due to clearance theoretically and experimentally, and showed the occurrence of super harmonic resonance up to the third order. Mazzei et al.$^{[12]}$ considered the two rotating shafts connected by the universal joint, and investigated numerically the interaction of both the parametric excitation due to the universal joint and the external periodic force generated when the initial joint angle is differ from 0. He clarified the occurrence of the unstable vibration range of the super harmonic resonance due to the external force at the rotational speed of a half of the natural frequency. Most of these studies considered the case with periodic external force and investigated its interaction with the parametric excitation. On the contrary, this paper considers also the case with only the constant external force, and clarifies the occurrence of the super harmonic resonance by the interaction of the constant external force and the parametric excitation.

4. Theoretical Analysis

The unstable vibration is observed in the numerical simulation at both $p = 2\omega$ and $p = (3/2)\omega$ as shown in Fig.4(b). The occurrence range of the unstable vibration is investigated by the stability analysis of the Mathieu’s equation$^{[13][14]}$. By setting $\beta = c_b/2$ and by using the natural frequency $p = \sqrt{1 + \omega^2}$, the equation of motion of Eq.(8) is represented in the damped Mathieu equation$^{[14]}$ as follows:

$$\ddot{z}_b + 2\beta \dot{z}_b + (p^2 - \epsilon \sin \omega t)z_b = 0$$  \hspace{1cm} (9)

Cases of frequency $n\omega$ and frequency $([2n − 1]\omega)/2$ are considered, and their stabilities are analyzed. First, the case of frequency $n\omega$ is considered. The solution is assumed as follows:

$$z_b(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$  \hspace{1cm} (10)

Equation (10) is substituted into Eq.(9), and the coefficients of $\cos n\omega t$ and $\sin n\omega t$ of both side are equated, respectively. The obtained equations are represented in the matrix form for the vector $[a_0, \cdots, a_n, b_1, \cdots, b_n]^T$. The condition for the determinant of the coefficient matrix, $\phi_1(\epsilon, \omega) = 0$, is derived as the vector holds $[a_0, \cdots, a_n, b_1, \cdots, b_n]^T \neq [0]$. The case of frequency $([2n − 1]\omega)/2$ is considered in the similar manner. The solution is assumed as follows:

$$z_b(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n-1}{2}\omega t + b_n \sin \frac{2n-1}{2}\omega t\right)$$  \hspace{1cm} (11)

Equation (11) is substituted into Eq.(9), and the coefficients of $\cos ([2n − 1]/2)\omega t$ and $\sin([2n−1]/2) \omega t$ of both side are equated, respectively. The obtained equations are represented in the matrix form for the vector $[a_0, \cdots, a_n, b_1, \cdots, b_n]^T$. The condition for the determinant of the coefficient matrix, $\phi_2(\epsilon, \omega) = 0$, is derived as the vector holds $[a_0, \cdots, a_n, b_1, \cdots, b_n]^T \neq [0]$.

This study considers the unstable vibration up to the third order, $n = 3$. Figures 7(a) and 7(b) are the $\epsilon - \omega$ planes derived from both $\phi_1 = 0$ and $\phi_2 = 0$ for the parameters $k = 8.58[N\cdot m/\text{rad}]$ and $k = 1.07[N\cdot m/\text{rad}]$, respectively. These curves are the boundaries between the stable and the unstable regions. Because of the influence of damping $\beta(= c_b/2)$, the unstable range does not appear when the parameter $\epsilon$ of the parametric excitation term is 0 or very small. As the parameter $\epsilon$ increases, the unstable range appears and becomes wider.

In the case of Fig.7(a) with large value of spring coefficient $k$, the unstable vibration does not occur since the values of $\epsilon$, 0.05 and 0.06, do not reach to the boundary curve of the unstable region. It explains that there is no range of the unstable vibration in Fig. 4(a). In the case of Fig.7(b) with smaller value of spring coefficient $k$, the values of parametric excitation
coefficient $\epsilon$ increases, and the unstable vibration occurs at the frequency range where the value of $\epsilon$ is larger than the boundary curve of the unstable region. In the case with blade mass of $m = 0.35$, the value of $\epsilon$ is larger than the boundary curve at only the frequency range around $(3/2)\omega$. When the blade mass is increased to $m = 0.45$, the value of $\epsilon$ increases and it is larger than the boundary curve at both frequency ranges of $2\omega$ and $(3/2)\omega$. The positions, number, and width of these ranges correspond to those of the unstable vibration ranges shown in the numerical results of Fig. 4(b).

Both these numerical and theoretical results clarify following points. The parametric excitation effect is generated by the gravitational force acting on the blade, and it increases when the blade mass $m$ increases or the spring coefficient $k$ of the blade decreases. The interaction of both the parametric excitation effect and wind force occurs when the wind force exists, and the super harmonic resonances may appear in the blade vibration. The magnitude of the harmonic component grows and super harmonic resonances also occur when the constant wind force $Q_0$, variation of the wind force $\Delta Q$, and the parametric excitation effect increases. When the value of parametric excitation coefficient $\epsilon$ is larger than the boundary curve of the unstable region at some frequency ranges, the unstable vibrations occur at these ranges even if there is no wind force.

5. Experiment

5.1. Experimental Setup

Figure 8 shows the experimental setup. The main shaft of the blade is supported by the ball bearings, and it rotates by a motor via pulleys. The stainless spring steel plate is attached at the tip of the main shaft via a flange, and the rigid blade is attached to the other end of the spring steel plate. The spring steel plate is length of $l_s = 100\text{mm}$, width of $b_s = 25\text{mm}$ and thickness of $h_s = 1\text{mm}$, respectively. The blade is mass of $m = 350\text{g}$, length of $l = 150\text{mm}$, width of $b = 60\text{mm}$ and thickness of $h_b = 6\text{mm}$, respectively. The mass of the blade is changeable by attaching the additional masses of $0.250\text{kg}$, and the spring coefficient of the spring steel plate is also changeable by re-arranging the length of the spring steel. The load torque on the rotation of the main shaft, which is generated by the gravitational force acting on the blade, is counterbalanced by attaching the counter masses to the flange in the opposite side of the blade, and the unevenness of the shaft’s rotating speed is reduced. The vibration
of the blade is measured by using the strain gage (Kyowa Electronic Instruments Co., Ltd.) attached on the spring steel plate at the position of 30mm apart from the attached point.

The strain signal measured by the strain gage is sent via the slip ring, and it is amplified by the amplifier (NEC, AS1603) and recorded. The rotational speed of the main shaft, $\omega$, is measured by the pulse counter.

The wind force is applied by using the electric fan (Abitelax, AF162E). The horizontal distance between the blade’s rotating plane and the electric fan is set to 0.65m. In the case with constant wind force, the center of electric fan is adjusted to the main shaft of the blade. While, in the case with the periodic wind force, the center of the electric fan is set apart to the upper side from the main shaft of blade and the wind force is applied mainly on the upper half of the blade’s rotating plane.

The parameters of the experimental system are shown as:

$$
\begin{align*}
c_b &= 0.024 \text{ N} \cdot \text{s/m}, \\
k &= 8.58 \text{ N} \cdot \text{m/rad}, \\
m &= 347 \text{ g}, \\
l &= 0.25 \text{ m}, \\
b &= 0.06 \text{ m}, \\
h_b &= 6.0 \text{ mm}
\end{align*}
$$

(12)

5.2. Experimental Results

5.2.1. Case with constant wind force ($\Delta Q = 0.0N$) The centers of both main shafts of blade and the electric fan are adjusted, and wind force is applied from the electric fan. The main shaft is driven by the motor, and its rotational speed is changed statically. The observed resonance curve is shown in Fig.9(a). The super harmonic resonances of the second and the third order are observed at $\omega = 2.81\text{Hz}$ and $\omega = 1.65\text{Hz}$.

Figures 9(b)(c) are the spectrum diagrams at the resonance points of super harmonic resonances of 2$\omega$ and 3$\omega$, and indicates the occurrences of 2$\omega$ and 3$\omega$ vibration components at these resonance points, respectively.

5.2.2. Case with varying wind force ($\Delta Q \neq 0.0N$) The center of the electric fan is set apart with $\delta = 0.21m$ from the center of the main shaft of blade, and the wind force is applied mainly on the upper half of the blade’s rotating plane. The main shaft of the blade is driven by the motor, and its rotational speed is changed statically. The observed resonance curve is shown in Fig.10(a). Same as the case with constant wind force, the super harmonic resonances of the second and the third order are observed at $\omega = 2.80\text{Hz}$ and $\omega = 1.61\text{Hz}$. Figures 10(b)(c) are the spectrum diagrams at the super harmonic resonances of 2$\omega$ and 3$\omega$, and indicates the occurrences of 2$\omega$ and 3$\omega$ vibration components at these resonance points.

In the numerical simulation analysis, the vibration amplitude increases when the variation of the wind force $\Delta Q$ is added. However, in the case of this experiment, the variation of the wind force $\Delta Q$ increases but the constant wind force component $Q_0$ decreases as the discrepancy between the center lines of the electric fan and the main shaft of the blade increases. As the result, the overall vibration amplitude level of the resonance curve of Fig.10(a) only slightly increased from the case with constant wind force shown in Fig.9(a). However, these
experimental results indicates that the variation of the wind force does not affect on the qualitative vibration characteristics of the blade, and this result agrees with the numerical results of Figs.5 and 6.

**Conclusion**

The vibration characteristics of the single rigid blade which is rotating vertically are investigated, and the following results are obtained:

(1) The parametric excitation effect is generated by the gravitational force acting on the blade. This effect may cause the unstable vibrations even if there is no wind force if the
spring coefficient of the blade is small or the blade mass is large. The ranges of these unstable vibrations are clarified considering the blade rotational speed.

(2) When the constant wind force acts on the blade, the interaction of both the parametric excitation effect and wind force occurs and it causes the super harmonic resonances. As the wind force or the blade mass increases, or the spring coefficient of the blade decreases, this interaction becomes stronger and the magnitude of the harmonic component grows. As the result, super harmonic resonances of the second and the third order occur because of a chain reaction through it.

(3) When the variation of the wind force on the blade exists because of the difference of the wind force depending on the height, the interaction of both the parametric excitation effect and wind force occurs in the same manner. The super harmonic resonances are caused due to this interaction, and the vibration magnitude level is larger than the case with only constant wind force. However, the qualitative vibration characteristics are the same as the case with only constant wind force.

(4) The occurrences of these vibrations are confirmed experimentally.

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