Noise Reduction in a Rectangular Enclosure Using Active Wave Control*

Hiroyuki IWAMOTO**, Nobuo TANAKA** and Akira SANADA***

**Department of Aerospace Engineering, Tokyo Metropolitan University

6-6 Asahigaoka, Hino-city, Tokyo, 191-0065 Japan

E-mail: hiwamoto@sd.tmu.ac.jp

***Industry Technology Center of Okayama Prefecture

5301 Haga, Okayama-city, Okayama, 701-1296 Japan

Abstract

This paper is concerned with an active noise control for an enclosed sound field. It is the purpose of this paper to present an active wave control method of a three dimensional rectangular cavity and to clarify fundamental properties of the control system. Firstly, a transfer matrix method for the target system is newly introduced to describe the wave dynamics in the enclosure. This is followed by the derivation of feedforward control laws for absorbing reflected waves or eliminating transmitted waves. In the proposed method, the control laws are including the modal actuation scheme for uncontrolled direction. Then, from a viewpoint of numerical analyses, basic properties of the proposed method are verified. It is found that the transmitted wave eliminating control enables the generation of silent zone, and the reflected wave absorbing control enables the inactivation of acoustical modes in the controlled direction.

Key words: Active Wave Control, Wave Propagation, Rectangular Enclosure, Active Noise Control

1. Introduction

Since a noise problem is one of the seven typical pollutions and the number of its complaints is extremely numerous, development of noise control technologies is important⁽¹⁾. Especially in the case where noise sources are located in an enclosed sound field, the sound pressure may be significantly increased because of the resonance. Since low-frequency noise which is difficult to be controlled by a passive method is often the target in such a case, a number of active control methods have been proposed in recent years. Reviewing the active noise control in an enclosed sound field reported so far, the mainstream of the method is global noise attenuation using acoustical potential energy control^{(2), (3)} or energy density control^{(4), (5)}. However, these methods are based on the modal expansion theorem and aim at realizing a global noise control. Therefore, there are few methods proposed from a viewpoint of active wave control⁽⁶⁾⁻⁽⁸⁾.

Active wave control is the method which is based on the mechanism of modal excitation unlike the conventional modal-based approaches. It enables the inactivation of modes by absorbing the reflected waves and the generation of quiet zone by eliminating the transmitted waves. It was originally proposed for a one-dimensional structure, and recently the authors expanded the active wave control to a two-dimensional structure (rectangular panel). However, reviewing the active wave control proposed for an enclosed sound field⁽⁹⁾⁻⁽¹¹⁾, the method for a three-dimensional sound field has not been proposed. Therefore, this study presents active control of progressive waves in a rectangular cavity,

*Received 15 Oct., 2010 (No. T1-08-0651)
Japanese Original : Trans. Jpn. Soc. Mech.
Eng., Vol.75, No.750, C (2009),
pp.365-373 (Received 24 July, 2008)
[DOI: 10.1299/jee.6.107]

Copyright © 2011 by JSME

clarifying the validity of the proposed method from a viewpoint of numerical simulations.

Firstly, a one-dimensional transfer matrix method for the target system is newly introduced to describe the wave dynamics in the cavity. This is followed by the derivation of feedforward control laws for absorbing reflected waves or for eliminating transmitted waves. Then, from a viewpoint of numerical analyses, basic properties of the proposed method are verified. It is found that the transmitted wave eliminating control enables the generation of silent zone, and the reflected wave absorbing control enables the inactivation of acoustical modes in the controlled direction.

2. Transfer matrices of a rectangular cavity

In this study, a transfer matrix method based in a progressive wave solution is required in order to investigate active wave control. However, the conventional transfer matrix method is specified for a one-dimensional duct⁽¹²⁾. Therefore, this paper begins with proposition of a transfer matrix method for a rectangular cavity. Firstly, defining the velocity potential as ϕ , a wave equation of an acoustical field is described as

$$\nabla^2 \phi(x, y, z, t) - \frac{1}{c^2} \frac{\partial^2 \phi(x, y, z, t)}{\partial t^2} = 0, \quad (1)$$

where c is speed of sound, and Laplacian ∇^2 is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \quad (2)$$

Furthermore, using the velocity potential, sound pressure, p , and particle velocity for each direction, v_x , v_y , v_z , are described as

$$p(x, y, z, t) = -\rho_0 \frac{\partial \phi(x, y, z, t)}{\partial t}, \quad (3)$$

$$v_x(x, y, z, t) = \frac{\partial \phi(x, y, z, t)}{\partial x}, \quad (4)$$

$$v_y(x, y, z, t) = \frac{\partial \phi(x, y, z, t)}{\partial y}, \quad (5)$$

$$v_z(x, y, z, t) = \frac{\partial \phi(x, y, z, t)}{\partial z}, \quad (6)$$

where ρ_0 is the air density.

In general, it is possible to replace the velocity potential by the sound pressure or each particle velocity in the wave equation described in Eq. (1). Henceforth, this paper treats the wave equation of the sound pressure. The solution of the wave equation is assumed to be written in the form, $p(x, y, z, t) = \tilde{p}(x, y, z) e^{j\omega t}$ where j and ω are the imaginary unit and the angular frequency, respectively. Based on this assumption, the Helmholtz equation is derived as

$$\nabla^2 \tilde{p}(x, y, z) + k^2 \tilde{p}(x, y, z) = 0, \quad (7)$$

where k is the wavenumber defined as

$$k = \omega/c. \quad (8)$$

Then, the general solution to Eq. (7) is described as

$$\tilde{p}(x, y, z) = \sum_{l,m,n=0}^{\infty} \tilde{p}_{x,l}(x) \tilde{p}_{y,m}(y) \tilde{p}_{z,n}(z), \quad (9)$$

where $\tilde{p}_{x,l}(x)$, $\tilde{p}_{y,m}(y)$ and $\tilde{p}_{z,n}(z)$ are the acoustical mode elements for each coordinate. In general, an acoustical mode is termed (l, m, n) mode based on its indices of each coordinate. Assuming that $\tilde{p}_{x,l}(x)$ and $\tilde{p}_{y,m}(y)$ are harmonic functions, these satisfy the following equations:

$$\frac{\partial^2 \tilde{p}_{x,l}(x)}{\partial x^2} = -\alpha_l^2 \tilde{p}_{x,l}(x), \quad (10)$$

$$\frac{\partial^2 \tilde{p}_{y,m}(y)}{\partial y^2} = -\beta_m^2 \tilde{p}_{y,m}(y), \quad (11)$$

where α_l and β_m are the wavenumber for x and y coordinates, respectively. Then, substituting Eqs. (9)-(11) into Eq. (7), we have

$$\sum_{l,m,n=0}^{\infty} \left\{ -\alpha_l^2 \tilde{p}_{z,n}(z) - \beta_m^2 \tilde{p}_{z,n}(z) + \frac{\partial^2 \tilde{p}_{z,n}(z)}{\partial z^2} + k^2 \tilde{p}_{z,n}(z) \right\} = 0. \quad (12)$$

Then, applying the operator method to the above equation, a general solution of Eq. (12) is derived as

$$\tilde{p}_{z,n}(z) = c_{1,lm} e^{-k_{lm}z} + c_{2,lm} e^{k_{lm}z}, \quad (13)$$

where

$$k_{lm} = \sqrt{\alpha_l^2 + \beta_m^2 - k^2}. \quad (14)$$

As is apparent from the above equation, k_{lm} becomes an imaginary number if the exciting frequency becomes higher than a certain threshold. The frequency of this threshold is termed cut-on frequency and defined as

$$\omega_{c,lm} = c\sqrt{\alpha_l^2 + \beta_m^2}. \quad (15)$$

If the exciting frequency is higher than cut-on frequency, each term of Eq. (13) becomes a complex exponential function, and hence the progressive wave appears in the z direction. In contrast, if the exciting frequency is lower than cut-on frequency, those terms become real exponential function, and hence the near-fields appear in the z direction. Furthermore, at the cut-on frequency, the wave components become spatially constant in the z direction.

Then, substituting Eq. (13) into Eq. (9), a wave solution of a rectangular cavity is derived as

$$\tilde{p}(x, y, z) = \sum_{l,m=0}^{\infty} \tilde{p}_{x,l}(x) \tilde{p}_{y,m}(y) (c_{1,lm} e^{-k_{lm}z} + c_{2,lm} e^{k_{lm}z}). \quad (16)$$

Moreover, particle velocity in the z direction is obtained from Eqs. (3) and (6), and a state vector in a rectangular cavity is defined as

$$\mathbf{z}_z(x, y, z) = (\tilde{p}(x, y, z) \quad \tilde{v}_z(x, y, z))^T = \sum_{l,m=0}^{\infty} \tilde{p}_{x,l}(x) \tilde{p}_{y,m}(y) \mathbf{z}_{z,lm}(z), \quad (17)$$

where $\mathbf{z}_{z,lm}(z)$ is a state vector of $(l, m, *)$ mode group which is written as

$$\mathbf{z}_{z,lm}(z) = \begin{pmatrix} \tilde{p}_{lm}(z) \\ \tilde{v}_{z,lm}(z) \end{pmatrix} = \mathbf{K}_{lm} \mathbf{D}_{lm}(z) \mathbf{c}_{lm}, \quad (18)$$

where

$$\mathbf{K}_{lm} = \begin{pmatrix} 1 & 1 \\ -jk_{lm} & jk_{lm} \\ \rho_0 \omega & \rho_0 \omega \end{pmatrix}, \quad (19)$$

$$\mathbf{D}_{lm}(z) = \begin{pmatrix} e^{-k_{lm}z} & 0 \\ 0 & e^{k_{lm}z} \end{pmatrix}, \quad (20)$$

$$\mathbf{c}_{lm} = \begin{pmatrix} c_{1,lm} \\ c_{2,lm} \end{pmatrix}. \quad (21)$$

Next, consider a volume element along the z -axis with node $i-1$ and i at both ends of the element. Let $\mathbf{z}_{z,lm}(z_{i-1})$ and $\mathbf{z}_{z,lm}(z_i)$ be ${}_{i-1}\mathbf{z}_{z,lm}$ and ${}_i\mathbf{z}_{z,lm}$ for brevity, and apply a local coordinate to these state vectors. Hence, by setting ${}_{i-1}\mathbf{z}_{z,lm}$ to be a local origin, ${}_i\mathbf{z}_{z,lm}$ is written as a function of the distance between the two nodes. Then, the state vectors at each node are written as

$${}_{i-1}\mathbf{z}_{z,lm} = \mathbf{K}_{lm} \mathbf{D}_{lm}(0) \mathbf{c}_{lm} = \mathbf{K}_{lm} \mathbf{c}_{lm}, \quad (22)$$

$${}_i \mathbf{z}_{z,lm} = \mathbf{K}_{lm} \mathbf{D}_{lm}(l_z) \mathbf{c}_{lm} . \quad (23)$$

Then, combining the above equations yields

$${}_i \mathbf{z}_{z,lm} = \mathbf{K}_{lm} \mathbf{D}_{lm}(l_z) \mathbf{K}_{lm}^{-1} {}_{i-1} \mathbf{z}_{z,lm} = {}_{i,i-1} \mathbf{T}_{lm} {}_{i-1} \mathbf{z}_{z,lm} , \quad (24)$$

where ${}_{i,i-1} \mathbf{T}_{lm}$ indicates a transfer matrix between the nodes $i-1$ and i . Furthermore, applying the coordinate transformation to the state vector, $\mathbf{z}_{z,lm}(z)$, by $\mathbf{K}_{lm}$, a wave vector at $(l, m, *)$ mode group is defined as

$$\mathbf{w}_{z,lm}(z) = \begin{pmatrix} c_{1,lm} e^{-k_{lm} z} & c_{2,lm} e^{k_{lm} z} \end{pmatrix}^T = \mathbf{K}_{lm}^{-1} \mathbf{z}_{z,lm}(z) . \quad (25)$$

Next, consider the case where a noise source q_d acts at node 2 and N_c control sources act at node 1 as shown in Fig. 1. The state equation of a rectangular cavity is then written as

$${}_3 \mathbf{z}_{z,lm} = {}_{30} \mathbf{T}_{lm} {}_0 \mathbf{z}_{z,lm} + {}_{31} \mathbf{T}_{lm} \mathbf{v}_{c,lm} + {}_{32} \mathbf{T}_{lm} \mathbf{v}_{d,lm} , \quad (26)$$

where

$$\mathbf{v}_{c,lm} = \begin{pmatrix} 0 & \tilde{v}_{c,lm} \end{pmatrix}^T , \quad (27)$$

$$\tilde{v}_{c,lm} = c_q \sum_{r=1}^{N_c} q_{c,r} \tilde{p}_{x,l}(x_r) \tilde{p}_{y,m}(y_r) , \quad (28)$$

$$\mathbf{v}_{d,lm} = \begin{pmatrix} 0 & \tilde{v}_{d,lm} \end{pmatrix}^T , \quad (29)$$

$$\tilde{v}_{d,lm} = c_q q_d \tilde{p}_{x,l}(x_d) \tilde{p}_{y,m}(y_d) , \quad (30)$$

and c_q is a constant determined by the boundary conditions. Here, the boundary conditions at $z=0$ and $z=L_z$ are given by setting one out of two state variables in the state vector to be zero. Supposing that the i th state variable at the right end (node 3) is zero, and j th state variable at the left end (node 0) is non-zero, Eq. (26) expands to the following equation:

$$0 = {}_{30} t_{ij,lm} {}_0 z_{j,lm} + {}_{31} t_{i2,lm} \tilde{v}_{c,lm} + {}_{32} t_{i2,lm} \tilde{v}_{d,lm} , \quad (31)$$

where ${}_{op} t_{kl,lm}$ denotes the k th row and l th column variable in the transfer matrix ${}_{op} \mathbf{T}_{lm}$. Then, from the above equation, the non-zero state variable at the left end (node 0), that is, the initial state vector, is given by

$${}_0 z_{j,lm} = - \left({}_{31} t_{i2,lm} \tilde{v}_{c,lm} + {}_{32} t_{i2,lm} \tilde{v}_{d,lm} \right) / {}_{30} t_{ij,lm} . \quad (32)$$

Since the initial state vector is derived as written in the above equation, a state vector at an arbitrary position in a rectangular cavity can be calculated by using transfer matrices and external input vectors.

3. Derivation of the active wave control laws

In this section, two control laws are derived; one is the reflected wave absorbing control (RWAC) and the other is the transmitted wave eliminating control (TWEC). As will be

Fig. 1 Three dimensional rectangular enclosure with disturbance and control sound sources

shown later, the former enables inactivation of vibration modes, and the latter generates a quiet zone in the target system.

First of all, the control law of the RWAC is derived. In this case, a reflected wave is defined as the wave propagating from a control plane (node 1) to a disturbance plane (node 2), that is, a positive traveling wave in Element 2 as shown in Fig. 1. The wave vector at an arbitrary node, a , in Element 2 is described as

$${}_a \mathbf{w}_{lm} = \mathbf{K}_{lm}^{-1} {}_a \mathbf{z}_{lm} = \mathbf{K}_{lm}^{-1} \mathbf{T}_{lm1} \mathbf{z}_{lm}, \quad (33)$$

where

$$\mathbf{K}_{lm}^{-1} \mathbf{T}_{lm1} = \frac{1}{2} \begin{pmatrix} e^{-k_{lm} z_a} & j\omega\rho_0 e^{-k_{lm} z_a} / k_{lm} \\ e^{k_{lm} z_a} & -j\omega\rho_0 e^{k_{lm} z_a} / k_{lm} \end{pmatrix}. \quad (34)$$

Furthermore, the state vector at node 1 expands to the following equation:

$${}_1 \mathbf{z}_{z,lm} = {}_{10} \mathbf{T}_{lm0} \mathbf{z}_{z,lm} + \mathbf{v}_{c,lm} = \begin{pmatrix} {}_{10} t_{1j,lm} {}_0 z_{j,lm} \\ {}_{10} t_{2j,lm} {}_0 z_{j,lm} + \tilde{\mathbf{v}}_{c,lm} \end{pmatrix}. \quad (35)$$

Substituting Eqs. (34) and (35) into Eq. (33), and setting the reflected wave component to be zero, the control law of the RWAC is derived as

$$\tilde{\mathbf{v}}_{c,lm} = -\tilde{\mathbf{g}}_{r,lm} \tilde{\mathbf{v}}_{d,lm} = \frac{\tilde{\mathbf{g}}_{dr,lm}}{\tilde{\mathbf{g}}_{cr,lm}} \tilde{\mathbf{v}}_{d,lm}, \quad (36)$$

where

$$\tilde{\mathbf{g}}_{cr,lm} = {}_{10} t_{1j,lm} {}_{31} t_{i2,lm} - j\omega\rho \left({}_{10} t_{2j,lm} {}_{31} t_{i2,lm} - {}_{30} t_{ij,lm} \right) / k_{lm}, \quad (37)$$

$$\tilde{\mathbf{g}}_{dr,lm} = {}_{10} t_{1j,lm} {}_{32} t_{i2,lm} - j\omega\rho {}_{10} t_{2j,lm} {}_{32} t_{i2,lm} / k_{lm}. \quad (38)$$

Then, assuming that the target mode groups are up to $(l_e, m_e, *)$, the following equation is derived from Eq. (28):

$$\mathbf{P}_{xy} \mathbf{q}_c = -\mathbf{g}_r \mathbf{q}_d, \quad (39)$$

where

$$\mathbf{P}_{xy} = \begin{pmatrix} \tilde{p}_{x,0}(x_1) \tilde{p}_{y,0}(y_1) & \cdots & \tilde{p}_{x,0}(x_{N_c}) \tilde{p}_{y,0}(y_{N_c}) \\ \vdots & \ddots & \vdots \\ \tilde{p}_{x,l_e}(x_1) \tilde{p}_{y,m_e}(y_1) & \cdots & \tilde{p}_{x,l_e}(x_{N_c}) \tilde{p}_{y,m_e}(y_{N_c}) \end{pmatrix}, \quad (40)$$

$$\mathbf{q}_c = (q_1 \cdots q_{N_c}), \quad (41)$$

$$\mathbf{g}_r = (\tilde{p}_{x,0}(x_d) \tilde{p}_{y,0}(y_d) \tilde{g}_{r,00} \cdots \tilde{p}_{x,l_e}(x_d) \tilde{p}_{y,m_e}(y_d) \tilde{g}_{r,l_e m_e}). \quad (42)$$

If the number of control sources is equal to or larger than that of the target mode groups, the control sources vector is derived using Moore-Penrose general inverse as follows;

$$\mathbf{q}_c = -(\mathbf{P}_{xy}^T \mathbf{P}_{xy})^{-1} \mathbf{P}_{xy}^T \mathbf{g}_r \mathbf{q}_d. \quad (43)$$

As shown in the above equation, the control sources vector is including the modal actuation at the x - y plane.

Next, the control law of the TWEC is derived. In this case, a transmitted wave is defined as the wave propagating from a control plane (node 1) to the left boundary (node 0), that is, a negative travelling wave in Element 1 as shown in Fig. 1. Wave vector at an arbitrary node, a , in Element 1 is described as

$${}_a \mathbf{w}_{lm} = \mathbf{K}_{lm}^{-1} {}_a \mathbf{z}_{lm} = \mathbf{K}_{lm}^{-1} \mathbf{T}_{lm0} \mathbf{z}_{lm}, \quad (44)$$

where $\mathbf{K}_{lm}^{-1} \mathbf{T}_{lm0}$ is in the same form as Eq. (34). Then, using Eq. (34) and setting the transmitted wave component to be zero, the control law of the TWEC is derived as

$$\tilde{\mathbf{v}}_{c,lm} = -\tilde{\mathbf{g}}_{t,lm} \tilde{\mathbf{v}}_{d,lm} = -\frac{{}_{32} t_{i2,lm}}{{}_{31} t_{i2,lm}} \tilde{\mathbf{v}}_{d,lm}. \quad (45)$$

Then, with the same procedure as derivation of the control law of the RWAC, the control sources vector is derived as

$$\mathbf{q}_c = -(\mathbf{P}_{xy}^T \mathbf{P}_{xy})^{-1} \mathbf{P}_{xy}^T \mathbf{g}_t q_d, \quad (46)$$

where

$$\mathbf{g}_t = (\tilde{p}_{x,0}(x_d) \tilde{p}_{y,0}(y_d) \tilde{g}_{t,00} \cdots \tilde{p}_{x,l_e}(x_d) \tilde{p}_{y,m_e}(y_d) \tilde{g}_{t,l_e m_e}). \quad (47)$$

4. Numerical simulations

In this section, the validity of the proposed method is verified from a viewpoint of numerical simulations. The dimension of the target cavity is 1.1m x 1.44m x 2.45m. All boundary conditions are set as rigid walls, and the first nine modal frequencies in this case are listed in Table 1. Furthermore, from the boundary condition of a rigid wall, the following equations are obtained:

$$\tilde{p}_{x,l}(x) = \cos \frac{l\pi x}{L_x}, \quad \tilde{p}_{y,m}(y) = \cos \frac{m\pi y}{L_y}, \quad (48)$$

$$c_q = \frac{2^{(\varepsilon_l + \varepsilon_m)}}{L_x L_y}, \quad (49)$$

where ε_x is a coefficient which is 0 for $x=0$ and is 1 for other cases.

The first four mode groups are set as control target in the simulations. Throughout this section, a noise source ($q_d=1000$) is set at $(x, y, z)=(1.01\text{m}, 0.1\text{m}, 2.35\text{m})$, and four control sources are located in an x - y plane at $(x, y)=(0.2\text{m}, 0.2\text{m})$, $(0.2\text{m}, 1.24\text{m})$, $(1.24\text{m}, 0.2\text{m})$ and $(1.24\text{m}, 1.24\text{m})$. To evaluate sound level in the cavity, the acoustic potential energy expressed as below is employed:

$$E_p = \frac{1}{4\rho c^2} \int_V |p(\mathbf{r}_e)|^2 d\mathbf{r}_e, \quad (50)$$

where ρ and $\mathbf{r}_e$ represent a medium density and an arbitrary position in a cavity, respectively.

4.1 Reflected wave absorbing control (RWAC)

First of all, the control effects of the RWAC are verified. Figure 2 shows the acoustical potential energy in the controlled region (Element2) with and without control when the control plane (node 1) is set at $z_c=0$. As shown in the figure, there are 9 acoustic modes in the frequency range of interest. In contrast, if the RWAC is applied at $z=0$, all peaks except at the $(0, 1, 0)$, $(1, 0, 0)$ and $(1, 1, 0)$ modal frequencies disappear. The reason why three peaks remains after control is logically explained recalling the process of the mode excitation and the viewpoint of the RWAC inactivating those. The process of the mode excitation is simply as follows;

A. When a disturbance is applied to a target system, the input energy propagates as the

Table 1 Modal frequencies of the rigid-walled rectangular enclosure

Modal No.	Modal index	Frequency (Hz)	Cut-on frequency
1	(0,0,0)	0	○
2	(0,0,1)	69.39	
3	(0,1,0)	118.06	○
4	(0,1,1)	136.94	
5	(0,0,2)	138.78	
6	(1,0,0)	154.55	○
7	(1,0,1)	169.41	
8	(0,1,2)	182.2	
9	(1,1,0)	194.48	○

progressive wave in the system.

B. Then, interference between the progressive wave and the reflective wave due to a boundary condition generates a standing wave.

C. If and only if the standing wave is tuned to the modal shape, the mode excitation (resonant phenomenon) occurs in the target system

The process described above in other words suggests that preventing the emergence of standing wave results in inactivation of vibration mode, on which the RWAC is developed based. However, there exists no standing wave in z direction for the remainders of three peaks, since those frequencies are identical to the cut-on frequencies as shown in Table 1 and Fig. 2. As a result, those modes excitation in the x and/or y direction cannot be avoided, but even so, the peak levels are reduced by approximately 10dB with the RWAC.

Figure 3 shows the frequency characteristics of the control laws in this case. In this figure, g_1 , g_2 , g_3 and g_4 indicates the control laws for the $(0, 0, *)$, $(1, 0, *)$, $(0, 1, *)$ and $(1, 1, *)$ mode groups, respectively. The gain levels of each control law are approximately 0dB in the frequency ranges more than each cut-on frequency. This is because the control plane is set at the almost symmetric point of noise source location in the z direction. Furthermore, in terms of the phase characteristics, only the control law for the $(0, 0, *)$ mode group has a phase linearity. Hence, this controller can be constructed by pure time delays. This is because the progressive waves corresponding to the $(0, 0, *)$ mode group do not have the dispersive nature. In contrast, since the progressive waves corresponding to other mode groups have the dispersive nature, it is necessary to approximate controllers

Fig. 2 Potential energy in the controlled region (Element 2) with and without the reflected wave absorbing control when the control plane is set at $z_c=0$

Fig. 3 Frequency characteristics of the reflected wave absorbing control laws when the control plane is set at $z_c=0$

using causal filters.

Although sound waves are not dispersive in general, the existence of the dispersive nature is clarified by Eqs. (10), (11), (14) and (48). If $l=0$ and $m=0$, then $\alpha_l=0$ and $\beta_m=0$, so that the wavenumber in the z direction is proportional to the frequency. Therefore, the progressive wave is non-dispersive in this case. However, in other mode groups, α_l and/or β_m are non-zero, and thus the corresponding progressive waves have the dispersive nature. Nonetheless, as shown in Eq. (14), if the frequency becomes sufficiently higher, then the effects of α_l and β_m become relatively smaller, so that the dispersion can be ignored in this case.

Then, the control effect of the RWAC is evaluated from a viewpoint of sound pressure distribution. Figures 4 and 5 show the absolute sound pressure distribution at the (1, 0, 1) modal frequency and the corresponding time history along the line at $x=0.24$ and $y=1.26$ with and without control when the control plane (node 1) is set at $z=0$. As shown in the figures, the standing wave is formed, and nodal plane normal to the z axis appears. In this case, the acoustic mode is excited since the form of the standing wave is coincident with that of the modal shape, and the maximum sound pressure is 64.55Pa. In contrast, when the RWAC is applied, the nodal plane which exists before control disappears. Furthermore, wave propagation toward the control plane is observed in the corresponding time history, indicating that the acoustic mode becomes inactive by the RWAC. The maximum sound pressure is reduced to 4.18Pa in this case, which is 6.48% of the uncontrolled amplitude.

To make further investigation on the RWAC, let us set the control plane at $z_c=1.225$, that is, a middle point in the z direction. The acoustical potential energy in the controlled region under this condition is shown in Fig. 6. As is obvious from the figure, not only the

Fig. 4 Absolute sound pressure distribution at the (1, 0, 1) modal frequency and the corresponding time history along the line at $x=0.24$ and $y=1.26$ when no control is applied

Fig. 5 Absolute sound pressure distribution at the (1, 0, 1) modal frequency and the corresponding time history along the line at $x=0.24$ and $y=1.26$ when the reflected wave absorbing control is applied at $z_c=0$

Fig. 6 Potential energy in the controlled region (Element 2) with and without the reflected wave absorbing control when the control plane is set at $z_c=1.225$

Fig. 7 Frequency characteristics of the reflected wave absorbing control laws when the control plane is set at $z_c=1.225$

acoustic modes corresponding to the cut-on frequency remain as peaks on after control, but also those with the odd indices in z direction. The reason is clarified by the frequency characteristics of the control laws shown in Fig. 7. It is confirmed that there exists several peaks on the control gain whose frequencies correspond to the acoustic modes with its nodal lines at the control plane. Consequently, the excessive power of the control gain is required to conduct the RWAC and it significantly affects the non-target mode groups. It is however assumed that the produced peak levels of the after control never exceed that of before control as long as the target frequency range is sufficiently low where the non-target acoustic modes hardly contribute to the system.

4.2 Transmitted wave eliminating control (TWEC)

Next, the control effects of the TWEC are verified. The control plane is set at $z_c=1.225$, and the acoustical potential energy in the controlled region (Element1) is shown in Fig. 8. As can be seen from the figure, the acoustic modes with the odd indices in the z direction remain as peaks because of the same reason as discussed in previous section on the RWAC. However, it should be noted that the control performance at other frequencies is as high as the depression of the asymptote. This result indicates that elimination of the transmitted wave certainly leads to that of the reflective wave, and hence a quiet zone is generated in the controlled region.

A coincidence between the location of the control plane and the nodal line of the acoustic mode is the sufficient condition that new resonant phenomena appear in the uncontrolled region, and its necessary and sufficient condition has not been found. Taking a

Fig. 8 Potential energy in the controlled region (Element 1) with and without the transmitted wave eliminating control when the control plane is set at $z_c=1.225$

Fig. 9 Potential energy reduction vs. the control plane position of the transmitted wave eliminating control

flexible beam as an example, it was reported that the peak frequency shifts into high frequency as the control plane approaches to a disturbance point⁽⁶⁾. From the fact above, removing the peaks into higher frequency range is possible by setting the control plane close enough to a disturbance point. Figure 9 shows the potential energy reduction in the controlled region (Element 1) versus the position of the control plane (node 2). As shown in the figure, the potential energy is increased by applying the TWEC in the case of $z=1.5$ and $z=1.9$. These deteriorations result from acoustic modes newly produced in the uncontrolled region (Elements 2 and 3). Furthermore, in the case of $z=1.9$, the control effect of the TWEC becomes less around the (1, 0, 0) and (1, 1, 0) modal frequencies. This is because the frequencies of the peaks caused by the newly produced modes are coincident with those of the uncontrolled acoustic modes. Thus, the deteriorations of the control effect are caused by the modes newly produced in the uncontrolled region. However, those peak frequencies are determined by the volume of the uncontrolled region, so that it is possible to shift the peak frequencies out of the frequency range of interest. For example, if the controlled plane is set at $z=2.1$ or $z=2.3$, the potential energy reduction more than 25dB is obtained in the frequency range up to 200Hz.

Then, the control effect of the TWEC is evaluated from a viewpoint of sound pressure distribution. Figure 10 shows the absolute sound pressure distribution at the (1, 0, 1) modal frequency and the corresponding time history along the line at $x=0.24$ and $y=1.26$ with the TWEC when the control plane (node 1) is set at $z=2.1$. As shown in the sound pressure distribution on the boundary surfaces, relatively strong sound pressure is observed around the noise source. This result means that the potential energy is confined in the uncontrolled region. Furthermore, as shown in the time history, the sound pressure distribution in the controlled region is in the form of a near-field. This result indicates that non-targeted mode

Fig. 10 Absolute sound pressure distribution at the (1, 0, 1) modal frequency and the corresponding time history along the line at $x=0.24$ and $y=1.26$ when the transmitted wave eliminating control is applied at $z_c=2.1$

groups which have the cut-on frequencies in the higher frequency range are affected by the control inputs while the transmitted waves corresponding to the target mode groups are completely cancelled out. Therefore, at the distant region from the control plane, the quiet zone is significantly realized.

5. Conclusions

This paper has proposed active wave control for a rectangular cavity, clarifying the validity of the control system. The main results of this work are summarized as follows:

- (1) The transfer matrix method was proposed for a rectangular cavity, and the two active wave control laws were analytically derived.
- (2) It was clarified that waves do not propagate under the corresponding cut-on frequencies in a rectangular cavity. Furthermore, it was found that progressive sound waves in a cavity have a dispersive nature unlike those in free space.
- (3) The reflected wave absorbing control (RWAC) enable inactivation of acoustic modes except at the cut-on frequencies. In this case, the peaks of the potential energy in the controlled region disappear. Furthermore, the RWAC reduces the peaks by approximately 10dB at the cut-on frequencies.
- (4) The transmitted wave eliminating control (TWEC) generates the quiet zone in the controlled region. In this case, the sound pressure distribution in the controlled region depends on the near-field corresponding the non-targeted mode groups that have the cut-on frequencies in the higher frequency range. Thus, the quiet zone is significantly achieved at the region which has a decent distance from the control plane.

References

- (1) Ministry of the Environment Government of Japan ed., Annual Report on the Environment and the Sound Material-Cycle Society in Japan 2007, (2007), p.117, Ministry of the Environment Government of Japan.
- (2) Nelson, P.A. et al., The Active Minimization of Harmonic Enclosed Sound Fields, Part 1: Theory, *Journal of Sound and Vibration*, Vol.117, No.1 (1987), pp.1-13.
- (3) Pan, J., Hansen, C.H., and Bies, D.A., Active Control of Noise Transmission through a Panel into a Cavity, *Journal of the Acoustical Society of America*, Vol.87, No.5 (1990), pp.2098-2108.

- (4) Park, Y.C., and Sommerfeldt, S.D., Global Attenuation of Broadband Noise Fields Using Energy Density Control, *Journal of the Acoustical Society of America*, Vol.101, No.1 (1997), pp.350-359.
- (5) Cazzolato, B.S., and Ghan, J., Frequency Domain Expression for the Estimation of Time-Averaged Acoustic Energy Density, *Journal of the Acoustical Society of America*, Vol.117, No.6 (2005), pp.3750-3380.
- (6) Tanaka, N., and Kikushima, Y., "Active Wave Control of a Flexible Beam (Proposition of the Active Sink Method)", *JSME International Journal, Series 3*, Vol.34, No.2 (1991), pp.159-167.
- (7) Gibbs, G.P. et al., "Active Control of Flexural and Extensional Power Flow in Beams Using Real Time Wave Vector Sensors", *Proc. of 2nd conference on Recent Advances in Active Control and Vibration*, (1993), 909.
- (8) Iwamoto, H. and Tanaka, N., Active Wave Feedforward Control of a Flexible Beam Using Wave Filter Constructed with Point Sensors, *Transaction of the Japan Society of Mechanical Engineers, Series C*, Vol.69, No.685 (2003), pp.2233-2239.
- (9) Beatty, L.G., Acoustic Impedance in a Rigid-Walled Cylindrical Sound Channel Terminated at Both Ends with Active Transducers, *Journal of the Acoustical Society of America*, Vol.36, Issue 6 (1964), pp.1081-1089.
- (10) Guicking, D. and Karcher, K., Active Impedance Control for One-Dimensional Sound, *Transaction of the ASME, Journal of Vibration, Acoustics, Stress and Reliability in Design*, Vol.106, (1984), pp.393-396.
- (11) Orduna-Bustamante, F., Nelson, P.A., An adaptive controller for the active absorption of sound, *Journal of the Acoustical Society of America*, Vol.91, Issue 5 (1992), pp.2740-2747.
- (12) Song, B.H., and Bolton, J.S., A Transfer-Matrix Approach for Estimating the Characteristic Impedance and Wave Numbers of Limp and Rigid Porous Materials, *Journal of the Acoustical Society of America*, Vol.107, No.3 (2000), pp.1131-1152.
- (13) Yasuda, K., *Mechanical Acoustics* (in Japanese), (2004), p.126, Corona Publishing Co., LTD..