Active Noise Control Using High-Directional Parametric Loudspeaker*

Toshihiko KOMATSUZAKI** and Yoshio IWATA**

**Institute of Science and Engineering, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192 Japan
E-mail: toshi@t.kanazawa-u.ac.jp

Abstract
In general, the sound waves propagate spherically in free space, which accordingly makes the global attenuation difficult. Using ordinary loudspeaker as a control source may cause interfered sound field with both attenuated and amplified regions. On the other hand, the recent development of high-directional loudspeakers based on new sound production theory known as 'parametric array' has allowed sound transmission within a narrow range of acoustic space. In the present paper, a numerical model of parametric array based on the quasi-linear approach of Westervelt is studied, and the calculated results are compared with the measured fields. The spherically spreading noise in space is mitigated either by a normal or a parametric loudspeaker as control source, and it is known that the suggested ANC system can mitigate sound locally but cause less influence on circumferential field.

Key words: Active Noise Control, Ultrasound, Parametric Loudspeaker, Directivity

1. Introduction
Recent advances in digital signal processors have promoted the active noise control (ANC) technology where various kinds of research as well as application have been developed(1). Generally, the sound waves propagate spherically in free space(2), which accordingly makes the global attenuation difficult. Simply constituting SISO system using ordinary loudspeaker as a control source may cause interfered sound field of both attenuated and amplified regions. On the other hand, the recent development of high-directional loudspeakers based on a sound production theory known as 'parametric array' has allowed sound transmission within narrow range of acoustic space. Parametric array utilizes the high directive characteristics of ultrasonic sound, where the embedded difference component of two ultrasonic carrier waves is expanded by the nonlinear acoustic effect as audible sound while it travels directionally(3). Such 'spotlight-like' characteristic can be a measure to improve controlling sound field locally without adversely influencing vicinal space.

A number of studies have been reported on the theoretical description as well as the experimental investigations of parametric array. Westervelt(3) examined the case of the nonlinear interaction of two collimated sound waves traveling in the same direction, and derived an inhomogeneous wave equation where the difference frequency component appears as the secondary wave by the nonlinear interaction. This prediction has been supported experimentally by the earlier study of Bellin and Beyer(4). Shortly after the work of Westervelt, Berktau(5) developed an expression for the on-axis far-field time-dependent pressure of the transient parametric array to describe the self-demodulation of a pulsed carrier wave based on the frequency domain methods. A time domain formulation of the absorption limited transient parametric array has been presented by Stepanishen and
Koenigs\textsuperscript{6}, where the basic approach utilizes a time-dependent Green’s function and a source density function of Westervelt. The far-field time-dependent pressure which results from an amplitude modulated carrier pressure wave can be expressed as a convolution of spatial and time-dependent impulse response with the time-dependent source function. Zheng and Coates\textsuperscript{7} further developed the theoretical formula in temporal convolution form to discuss the off-axis far field amplitude and phase response characteristics of the secondary field. Recently, the numerical calculation of spectral components of finite amplitude sound radiated by a directive piston source has been proposed by Aanonsen et al.\textsuperscript{8} using the nonlinear equation known as the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation. Fourier series expansions and numerical methods are used to solve the governing equations of motion in the parabolic approximation. The extension of their analysis to the case of collinear sound beams radiated from the source is presented by Kamakura, et al.\textsuperscript{9} and Tjotta et al\textsuperscript{10}. With a view to apply the parametric array as control sources in the active control of noise, Brooks et al.\textsuperscript{11} investigated the feasibility of the commercially available parametric loudspeaker. They focused on studies of directivity and on-axis pressure characteristics mainly from analytical aspects based on Berktay’s approach. Despite the variety of analytical approach as mentioned above, the source seems to be harmonically defined in most cases. Considering the practical situation that the parametric loudspeaker is applied in the ANC scheme, spatio-temporal characteristic of propagating wave should be modeled and the feasibility of the parametric array should be discussed in the interfered sound field with transient noise which varies with time.

In this study, an approach for the active control of sound in free space is presented using high directional parametric loudspeaker. A numerical model of parametric array based on the quasi-linear approach of Westervelt along with the expression of the amplitude modulated primary carrier pressure suggested by Yoneyama et al.\textsuperscript{12} is derived, where the far-field time-dependent pressure is calculated by the convolution of a spatial and time-dependent impulse response with an carrier pressure in a source density function. The model is also used to evaluate the features of attenuated and amplified regions of sound field due to the interference between omni-directional and parametric sources, and the calculated sound fields are compared with the measured fields formed by a commercial loudspeaker. The spherically spreading noise in space is mitigated either by a normal or a parametric loudspeaker as a control source. It is known that the suggested numerical model can represent the consistent sound field with classical model. The interfered sound fields also show that the parametric loudspeaker can mitigate sound locally but cause less influence on circumferential space.

2. Theoretical description of parametric array

In this section, an inhomogeneous wave equation derived by Westervelt where the difference frequency component appears as the secondary wave by the nonlinear interaction is first explained. Description of modified version of the model by Berktay is then followed, in which the aperture effect of cross section of collimated waves is considered. Finally, a numerical model of parametric array based on the quasi-linear approach of Westervelt with the expression of the amplitude modulated (AM) primary carrier pressure is derived, where the far-field time-dependent pressure is calculated by a convolution of spatial and time-dependent impulse response with an AM carrier pressure in a source density function.

2.1 Westervelt’s theory

Westervelt\textsuperscript{3} analytically discovered that if two high-level ultrasonic waves which are different in frequency are collinearly emitted, the difference component of two carrier sources forms an array along the sound axis due to nonlinear interaction with the medium. A
highly directional beam is therefore created. The theoretical model of parametric array suggested by Westervelt is described by the following Eqs. (1) and (2).

\[
\nabla^2 p_s - \frac{1}{c_0^2} \frac{\partial^2 p_s}{\partial t^2} = \rho_0 \frac{\partial q}{\partial t}
\]

(1)

\[
q(x, t) = \frac{1}{\rho_0 c_0^2} \left\{ \frac{\rho_0}{2c_0} \left( \frac{\partial p}{\partial \rho} \right)^2 \right\} \frac{\partial p_s^2}{\partial t} = \frac{\beta}{\rho_0^2 c_0^4} \frac{\partial p_s^2}{\partial t}
\]

(2)

In Eqs. (1) and (2), \( p_s \) denotes the primary sound pressure which should contain two source frequencies, and \( p_s \) the secondary sound pressure. Equation (1) represents well known wave equation, while Eq. (2) expresses equivalent secondary source strength \( q \) due to the interaction between two waves. \( \rho_0 \) and \( c_0 \) are the density and the sound speed of the medium. \( \beta \) is called nonlinear parameter. For adiabatic expansion, the second derivative of pressure \( p \) with respect to \( \rho \) in Eq. (2) can be written as follows:

\[
\left( \frac{\partial^2 p}{\partial \rho^2} \right)_{\rho=\rho_0} = \gamma - 1 \frac{c_0^2}{\rho_0^2}
\]

(3)

In the above Eq. (3), \( \gamma \) denotes specific heat ratio which equals to 1.4 in air. The virtual source strength \( q \) forms an array along longitudinal axis, which then generates difference secondary sound. Since the source strength is proportional to time derivative of the product of the carrier wave pressure, \( \partial p_s^2/\partial t \), the sum and difference components of the carrier frequencies are consequently created in the scattered field. This can be examined by a simple consideration, assuming the primary pressure as follows:

\[
p_s = P_1 \exp(-\alpha_1 x) \cos(\omega_1 t - k_1 x) + P_2 \exp(-\alpha_2 x) \cos(\omega_2 t - k_2 x)
\]

(4)

In Eq. (4), \( P \) denotes the pressure amplitude, \( \alpha \) the absorption coefficient, \( k \) the wave number and the suffix number each carrier which differs in frequency, respectively. Then, on condition that the higher frequency components attenuate quickly in comparison to the difference frequency and neglecting them in the product of Eq. (4),

\[
p_s^2 = P_1 P_2 \exp\{-\alpha_1 \alpha_2 x\} \cos(\Omega - K x),
\]

(5)

where \( \Omega = \omega_1 - \omega_2 \) and \( K = k_1 - k_2 \). By substituting Eq. (5) into Eq. (2), the source density function \( q \) can be written as follows.

\[
q(x, t) = -\frac{\beta \rho_0 P_1 P_2}{\rho_0^2 c_0^4} \exp\{-\alpha_1 \alpha_2 x\} \sin(\Omega t - K x)
\]

(6)

The solution of the difference pressure \( p_s \) in Eq. (1) is obtained by the volume integral,

\[
p_s(r, t) = -\frac{\rho_0}{4\pi} \int \frac{\exp(-\alpha_1 |\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} \frac{\partial}{\partial t} \left[ q\left( \mathbf{r}', t - \frac{|\mathbf{r} - \mathbf{r}'|}{c_0} \right) \right] dV'.
\]

(7)

In the above equation, \( \mathbf{r} \) and \( \mathbf{r}' \) denote position vectors for the observation and the sound source, respectively, and \( \alpha_1 \) the absorption coefficient of difference frequency component.

---

**Fig. 1 Geometry used for primary wave distribution**
Equation (7) can be simplified using the geometry in Fig. 1,

\[ p_s(R, \theta, t) = -\frac{\rho S_0}{4\pi} \left( \frac{\exp(-\alpha_r r)}{r} \right) \frac{\partial}{\partial t} \left[ q(x, t - \frac{r}{c_0}) \right] dx, \]  

(8)

if two carrier sources propagate collinearly and distribute along the x axis. Here, \( S_0 dx \) represents the elementary volume of the source density. Assuming that \( r \approx R - x \cos \theta \) and \( 1/r \approx 1/R \), the secondary pressure can be obtained from Eqs. (6) and (8) as follows.

\[ p_s(R, \theta, t) = \frac{\beta P_2 S_0}{4\pi \rho c_0^2 R} \exp(-\alpha_r R) \times \frac{\cos(\Omega t - KR - \phi)}{\left[ A^2 + 4K^2 \sin^4(\theta/2) \right]^{1/2}} \]  

(9)

In Eq. (9), \( A = \alpha_1 + \alpha_2 - \alpha_3 \), and \( \tan \phi = A/[2K \sin(\theta/2)] \).

### 2.2 Berktay’s model

As an extension to Westervelt’s model, Berktay considered the aperture effect of cross section of end-fire array formed by two collimated planar primary waves (Fig. 2). The cross section of the column is assumed to be rectangular with size \( 2b \times 2d \), on which the source strength is uniform. Due to the finite cross section of the volume distributed virtual sources, the radiated field at a point in the far field is proportional to the following factor,

\[ \frac{(2d)(2b)}{(dK \cos \gamma)(bK \sin \theta \sin \gamma)} \cdot \frac{(dK \cos \gamma)}{(bK \sin \theta \sin \gamma)}. \]  

(10)

If the observer is restrained on the plane \( z = 0 \) in the far field, then \( \gamma = \pi/2 \). The amplitude of the scattered pressure field can be obtained by replacing cross section term \( S_0 \) in Eq. (9) by Eq. (10), and is

\[ p_s(R, \theta) = \frac{\beta P_2 S_0}{4\pi \rho c_0^2 R} \exp(-\alpha_r R) \times \frac{\sinh(bK \sin \theta)}{(bK \sin \theta) \left[ A^2 + 4K^2 \sin^4(\theta/2) \right]^{1/2}}. \]  

(11)

The amplitude of the on-axis difference pressure is obtained by setting \( \theta = 0 \) in either of Eq. (11) or (9), as

\[ p_s(R, 0) = \frac{\beta P_2 S_0}{4\pi \rho c_0^2 RA} \exp(-\alpha_r R). \]  

(12)

### 2.3 Time-dependent numerical model of amplitude modulated parametric source

Analytical models on the basis of Westervelt’s inhomogeneous equation and the source density method can express the approximate solution of the parametric array, however, the representation of the difference wave is limited to harmonic cases and the models are not suited for the transient waves as observed in practical situations in noise control applications. Therefore, a numerical model of parametric array is presented in this
subsection, where the far-field time-dependent pressure is calculated by a convolution of a spatial and time-dependent impulse response with an amplitude-modulated carrier pressure in a source density function. The primary carrier pressure is expressed by an amplitude-modulated monotonic wave with frequency $\omega_0$, whereby the embedded audible sound within an envelope function $g(t)$ is self-demodulated in the radiated field. The amplitude modulation scheme suggested by Yoneyama et al.\(^{(12)}\) is introduced here and a time domain formulation of the transient parametric array is demonstrated.

The primary pressure at a point $x$ at time $t$ is expressed in the form below,

$$p(x, t) = P_0 [1 + mg(t - x / c_0)] \exp(-\alpha_0 x) \sin(\omega_0(t - x / c_0)) .$$  \hspace{1cm} (13)

Here, $P_0$, $\alpha_0$ and $\omega_0$ denote pressure amplitude, absorption coefficient and the frequency of primary carrier source. $m$ signifies modulation index and $g(t)$ an audio signal function to be demodulated. Using Eq. (13), virtual source strength $q$ is written as follows.

$$q(x, t) = -\frac{mP_0^2 \rho c_0^2}{\rho_0 c_0^4} \exp(-2\alpha_0 x) \frac{\partial}{\partial t} \left[ mg\left(t - \frac{x}{c_0}\right) + \frac{1}{2} m^2 g^2 \left(t - \frac{x}{c_0}\right) \right] .$$  \hspace{1cm} (14)

The second term in the rhs of Eq. (14) signifies the distortion of demodulated wave. Assuming that $m < 1$ and neglecting higher order term, the equation becomes,

$$q(x, t) = -\frac{mP_0^2 \rho c_0^2}{\rho_0 c_0^4} \exp(-2\alpha_0 x) \frac{\partial}{\partial t} \left[ g\left(t - \frac{x}{c_0}\right) \right] .$$  \hspace{1cm} (15)

By substituting Eq. (15) into Eq. (8), we obtain

$$p_s(R, \theta, t) = -\frac{mP_0^2 \rho c_0^4 S_0}{4\pi \rho_0 c_0^4 R} \exp(-2\alpha_0 x) \frac{\partial}{\partial t} \left\{ \frac{r}{x} \right\} \left[ g\left(t - \frac{x}{c_0}\right) - \frac{R - 1 - \cos \theta}{c_0} x \right] dx .$$  \hspace{1cm} (16)

Based on the far field assumption, $r \approx R - x \cos \theta$ for the phase and absorption term and $1/r \approx 1/R$ for the range term, the above equation can be expressed as follows.

$$p_s(R, \theta, t) = -\frac{mP_0^2 \rho c_0^4 S_0}{4\pi \rho_0 c_0^4 R} \exp(-2\alpha_0 x) \frac{\partial}{\partial t} \left\{ \frac{r}{x} \right\} \left[ g\left(t - \frac{x}{c_0}\right) - \frac{R - 1 - \cos \theta}{c_0} x \right] dx .$$  \hspace{1cm} (17)

If we let $u = [(1 - \cos \theta) / c_0] x$ and also approximate $-2\alpha_0 + \alpha_0 \cos \theta \approx -2\alpha_0$, the Eq. (17) is simplified as,

$$p_s(R, \theta, t) = C \frac{1}{D} \exp(-1 / D) \frac{\partial^2}{\partial x^2} \left[ g\left(t - \frac{R - 1 - \cos \theta}{c_0} x \right) \right] du .$$  \hspace{1cm} (18)

where $C = -mP_0^2 \rho c_0^4 S_0 / 8\pi \rho_0 c_0^4 R$ and $D = \sin^2(\theta / 2) / \alpha_0 c_0$. If one assumes an impulse response function $h(t, \theta)$ of the parametric array,

$$h(t, \theta) = \begin{cases} 1 / D \exp(-1 / D), & t \geq 0 \\ 0, & t < 0 \end{cases} ,$$  \hspace{1cm} (19)

the spatial integration in Eq. (18) is replaced by the temporal convolution of the source function $g(t)$ with the impulse response function $h(t, \theta)$ of the parametric array,

$$p_s(R, \theta, t) = C h(t, \theta) \ast \frac{\partial^2}{\partial t^2} \left[ g\left(t - \frac{R - 1 - \cos \theta}{c_0} x \right) \right] .$$  \hspace{1cm} (20)

Using Eq. (20), we can numerically obtain the secondary demodulated field of arbitrary audio signal.
3. Numerical and experimental investigation of the parametric array model

Numerical investigations are performed for the parametric array models described in § 2, and those are compared with measured sound fields. The directivity pattern and the on-axis sound pressure distributions are mainly focused. Parameters used in the analysis are shown in Table 1. Schematic of experimental setup is also shown in Fig. 3. The audio signal is embedded into amplitude-modulated ultrasonic carrier with a base frequency 40kHz by a modulator, which is then radiated from a commercial parametric loudspeaker. A horizontal plane of size 1.0 (m) x 2.0 (m) incorporating the central axis the loudspeaker is set as a measurement plane. A microphone is scanned at every 0.05 (m) grid. The measurement is performed in a semi-anechoic chamber.

3.1 Directivity testing

Directivity of the sound is evaluated by the 3dB beam width, when the pressure amplitude drops as much as 3dB relative to the pressure at an axis of the radiated field. The gradient can be analytically derived by Eq. (9), for $A / 2K < 1$, as

$$2\theta_d = 4\sqrt{A / 2K} .$$

The 3dB beam widths calculated from respective analytical models are plotted in Fig. 4, along with the measured result for 500, 800 and 1kHz monotonic pressure. Analytical results fairly coincides each other, as long as the size of the radiating surface is small so that the aperture effect will not be important. The beam width in the experimental result seems not related to the demodulated frequency, but is strongly dependent on the measurement accuracy. The results in Fig. 5 show the directivity pattern of the radiated sound beams for the difference frequencies of 500, 1k, 1.5k and 2kHz. The sound is measured at a radius of 0.5m distant from the radiating plane. The sound from the parametric source shows strong directivity in front of the radiating plane. Here it is also known that the analytical models agree well each other and also with measured beam patterns, however, the profile of experimentally observed beam pattern is rather stretched than those of numerical results.

Table 1 Parameter used in analytical calculation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier pressure amplitude $P_c$</td>
<td>10 [Pa]</td>
</tr>
<tr>
<td>Density of air $\rho_0$</td>
<td>1.2 [kg/m$^3$]</td>
</tr>
<tr>
<td>Sound speed $c_0$</td>
<td>344.0 [m/s]</td>
</tr>
<tr>
<td>Absorption coefficient (Carrier) $\alpha_1, \alpha_2$</td>
<td>0.087 [dB / m]</td>
</tr>
<tr>
<td>Absorption coefficient (Difference) $\alpha_0$</td>
<td>0.0087 [dB / m]</td>
</tr>
<tr>
<td>Nonlinear parameter $\beta$</td>
<td>1.2</td>
</tr>
<tr>
<td>Size of aperture $2b \times 2d$</td>
<td>15 x 15 [cm]</td>
</tr>
</tbody>
</table>

Fig. 3 Experimental setup for directivity measurement
3.2 On-axis sound pressure

The on-axis pressure distribution is investigated, where the numerically obtained results from both Westervelt's model and the amplitude-modulated numerical model are compared with measured results. Fig. 6 shows the on-axis pressure for 500, 1k and 1.5kHz demodulated sound pressure. It is known from the numerical results that the overall sound pressure level varies depending on the difference frequency $\Omega^2$ in Eq. (12). The distance decay signified by the gradient of each curve seems to be constant for both numerical models. On the other hand, the pressure amplitude and the distance decay in measured results are quite not as simple as numerical ones. Relatively low pressure within a distance of 0.5m implies that a finite distance exists for the distorted carrier wave to be demodulated, which cannot be explained by the far-field approximation by these models. Moreover, the measured sound pressure level seems to be independent of the demodulated frequency, which may due to the equalizing function of the modulator obeying $1/\omega^2$. 

![Fig. 4 3dB beam width of sound pressure radiated by parametric source](image)

(a) 500Hz  (b) 1kHz  
(c) 1.5kHz  (d) 2kHz

![Fig. 5 Directivity pattern for respective source frequency](image)

![Fig. 6 On-axis sound pressure distribution](image)
4. The interfered sound field

The second investigation deals with the measurement of the interfered sound field formed by two sound sources. One assumes a noise source produced by a normal loudspeaker, while the other a control source where the sound is generated by either the normal or a parametric loudspeaker. The purpose of the present investigation is based on the expectation that the parametric loudspeaker as a control source can control locally the narrow range of the target field without amplifying the rest of the space. The measurement system for the interfered field is identical to the system shown in Fig. 4, and the procedure is the same as the previous directivity testing. The difference sound source is chosen to be sinusoidal, 1kHz. In the measurement plane, the control source is fixed at (0.5m, -0.1m), whereas the noise source is located in three ways: i) at (0.5, 2.1) (m), the opposite side of the plane so that two loudspeakers face each other, and, ii) at (1.1, 0.5) (m) so that the sound axes of two loudspeakers become orthogonal, and also iii) at (0.5, -0.3) where the sound axes and the wave propagating directions of both loudspeakers coincide.

The sound field is formed by an adaptive controller based on the least mean square algorithm, where the interfered sound pressure measured at error microphone is minimized. Evaluation microphone is placed at (0.5, 1.0) (m) for the case i), and also at (0.5, 0.5) (m) for the case ii) and also at (0.5, 1.0) (m) for the case iii), respectively.

4.1 Interfered field of opposed placement of loudspeakers (placement A)

The measurement results for the opposed placement of two sound sources are shown in Fig. 7. Striped pattern seen in the sound field is formed by reversely traveling sound waves which in consequence generates the standing wave. As can be predicted by the previous investigations, it is known from the figure the high-directional characteristic of the parametric loudspeaker causes less influence on the rest of the field, in contrast with the case of normal control loudspeaker (Fig. 7 (b)). Also specifically it is clearly seen that the normal control source deadly amplifies the pressure level of the circumferential space. Furthermore, sound fields calculated by the numerical model well coincide qualitatively with the actual interfered field.

![Figure 7 Interfered sound field obtained for placement A (left: analytical, right: experiment)](image-url)
4.2 Interfered field of orthogonal placement (placement B)

In Fig. 8, the calculated and the measured fields of orthogonal placement of loudspeakers are shown. Same as the previous case, the patterns of the interfered field formed by the numerical model well agree with experimental results. It should be noted that the beamlike pressure distribution is formed behind the evaluation point when using parametric loudspeaker as a control source, hence the position and the wave traveling direction of the parametric array must be carefully determined. Nevertheless, the overall influence on the rest of the field is still relatively less in this case.

4.3 Interfered field of coaxial placement (placement C)

In this case, only the interfered field of a normal and a parametric array source is demonstrated. The calculated and measured fields are shown in Fig. 9. As seen from these figures, silent zone is formed in the sound wave traveling direction along the sound axis, whose width is almost identical to the width of the parametric array sound beam. This last case is thought to be the most feasible placement for the practical application of the parametric array to the active noise control.

Fig. 8 Interfered sound field obtained for placement B (left: analytical, right: experiment)

Fig. 9 Interfered sound field obtained for placement C (left: analytical, right: experiment)
5. Conclusions

In the present study, a numerical model of parametric array based on the quasi-linear approach of Westervelt with the expression of the amplitude modulated primary carrier pressure is derived, where the far-field time-dependent pressure is calculated by a convolution of a spatial and time-dependent impulse response with an amplitude modulated carrier pressure. The investigations include directivity testing, the interfered acoustic field of both noise and control sources. The results obtained for the parametric loudspeaker are consistently compared with those of the normal loudspeakers.

It is known that the suggested numerical model can represent the consistent sound field with classical model given by Westervelt. The directivity and the distance decay characteristics of the actual loudspeaker can partly be modeled due to the far-field assumption in the model, however, the numerical results qualitatively conforms to the measured characteristics. The results of the interfered field also show that the parametric loudspeaker can cancel the noise locally without disturbing the rest of the field which is attributed to the characteristics mentioned above. These properties that the parametric loudspeaker has can be useful for controlling very local, a distant sound field without adversely affecting circumferential space.

References