A Structural Optimization Method Incorporating Level Set Boundary Expressions Based on the Concept of the Phase Field Method*

Takayuki YAMADA**, Shinji NISHIWAKI***, Kazuhiro IZUI***, Masataka YOSHIMURA*** and Akihiro TAKEZAWA†

** Graduate School of Engineering, Nagoya University
Furo-cho, Chikusa-ku, Nagoya 464–8603, Japan
E-mail: yamada@nuem.nagoya-u.ac.jp

*** Graduate School of Engineering, Kyoto University
Yoshida-hommachi, Sakyo-ku, Kyoto 606–8501, Japan

† Graduate School of Engineering, Hiroshima University
1–4–1 Kagamiyama, Higashi-hiroshima, Hiroshima 739–8527, Japan

Abstract

Topology optimization has been successfully used in many industries, especially those engaged in the design and manufacturing of mechanical devices, but numerical problems are often encountered, such as grayscale representations of obtained composites. A type of structural optimization method using the level set theory for boundary expressions has been proposed, in which the outlines of target structures are implicitly represented using the level set function, and optimal configurations are obtained by updating this function based on the shape sensitivities. Level set-based methods typically have a drawback, however, in that topological changes that increase the number of holes in the material domain are not allowed. To overcome the above numerical and topological problems, this paper proposes a new topology optimization method incorporating level set boundary expressions based on the concept of the phase field method, which we apply to a minimum mean compliance problem. First, a structural optimization problem is formulated based on a boundary expression, using the level set function. Next, a time evolutionary equation for updating the level set function is formulated based on the concept of the phase field method, and the minimum mean compliance problem is formulated using a level set boundary expression. An optimization algorithm for the topology optimization incorporating the level set boundary expression based on the concept of the phase field method is then derived. Several examples are provided to confirm the usefulness of the proposed structural topology optimization method.

Key words: Optimum Design, Structural Design, Structural Analysis, Finite Element Method

1. Introduction

This paper proposes a new topology optimization method based on level set boundary expressions and the concept of the phase field method. Topology optimization offers the greatest potential for design freedom among structural optimization methods because it allows topological changes, such as altering the number of holes in the target structure. Theoretically, layered structures consisting of infinitely fine ribs with infinite width or porous structures consisting of infinitely small cavities are allowed as optimal configurations but these singular material states are replaced by grayscale areas after the design space is relaxed.
Although grayscale areas are acceptable from a mathematical standpoint, such structures are meaningless from an engineering standpoint because they cannot be manufactured. In addition, topology optimization methods allow geometrically complex structures in the obtained optimal configurations. To overcome this issue, a perimeter constraint method\(^{(5)}\) and a density slope constraint method\(^{(6),(7)}\) have been proposed for use in topology optimizations. In the perimeter constraint method, however, the perimeter constraint parameter must be set by trial and error since certain values may cause instability in the optimization process, and in the density slope constraint method, the constraint tends to enhance the production of grayscale areas. Furthermore, with both of these methods, the relationship between the geometrical complexity of obtained optimal structures and the relevant parameter settings cannot be determined uniquely, so obtaining optimal structures that have a desired degree of geometrical complexity is elusive.

On the other hand, level set-based shape optimization methods have been proposed as a new type of structural optimization method.\(^{(8),(9)}\) In these methods, structural boundaries are represented using the iso-surface of a scalar function called the level set function. Changes in the shape of target structures are represented by changes in the distribution of the level set function. Although conventional topology optimization methods allow grayscale areas in optimal configurations, as discussed above, level set methods represent structural boundaries clearly, and design-dependent boundary conditions can therefore be imposed explicitly. However, since the formulation of level set optimization methods is based on the structural boundary advection concept, they are classified as shape optimization methods. Furthermore, finding appropriate parameter settings for initial configurations and boundary conditions for the level set function that will yield suitable optimal configurations is still difficult, and these methods also cannot control the geometrical complexity of the obtained optimal configurations. To overcome these issues, this paper proposes a new topology optimization method based on level set boundary expressions and the concept of the phase field method.\(^{(10),(11)}\) That is, a structural optimization problem is replaced by a time evolutionary problem, applied to the level set function and based on the concept of the phase field method, so that adjustment of the geometrical complexity of obtained optimal configurations becomes possible. In Section 2, the structural optimization problem incorporating level set boundary expressions is formulated. Based on this formulation and the concept of the phase field method, a time evolutionary equation for updating the level set function is derived. The minimum mean compliance problem with a volume constraint is then formulated. In Section 3, an optimization algorithm is constructed and a numerical implementation of the optimization algorithm is proposed. Finally, in Section 4, several numerical examples are provided to confirm the validity of the proposed topology optimization method.

2. Formulation

2.1. Structural Optimization Problem Based on the Level Set Method

First, we consider a structural optimization problem in a fixed design domain $D$ that consists of a material domain $\Omega$ and a void domain $D \setminus \Omega$. In the level set method, structural boundaries are represented by the iso-surface of the level set function $\phi$, as shown in Fig. 1. That is, the level set function $\phi$ has a positive value in the material domain $\Omega$, a negative value in the void domain $D \setminus \Omega$, and a value of zero on structural boundaries $\partial \Omega$, as follows:

\[
\begin{align*}
0 < \phi(x) &\leq 1 \quad \text{if} \quad \forall x \in \Omega \setminus \partial \Omega \\
\phi(x) & = 0 \quad \text{if} \quad \forall x \in \partial \Omega \\
-1 \leq \phi(x) &< 0 \quad \text{if} \quad \forall x \in D \setminus \Omega.
\end{align*}
\]  

(1)

Note that the level set function $\phi$ has upper and lower constraints imposed and that $\phi$ is updated based on the concept of the phase field method, as discussed below. The constraints do not affect shape representations because the level set function can be allowed an arbitrary distribution in the material domain when $\phi$ has a positive value.
Using the above level set boundary expressions, we propose the following structural optimization problem:

\[
\inf_{\phi} F(\Omega(\phi)) = \int_{\Omega} f(x) d\Omega \\
\text{subject to } G(\Omega(\phi)) = \int_{\Omega} d\Omega - V_{\text{max}} \leq 0,
\]

where \( F \) is an objective functional, \( f(x) \) is a distributed function representing the objective functional \( \Omega, \) \( G \) is the volume constraint and \( V_{\text{max}} \) is the upper limit of the volume constraint.

In this paper, a fictitious interface energy is integrated with the phase field method and the above structural optimization problem is replaced by a regularized optimization problem in which the objective functional is replaced by a sum of the fictitious interface energy and the original objective functional, as follows:

\[
\inf_{\phi} F(\Omega(\phi), \phi) = \int_{\Omega} f(x) d\Omega + \int_{\Omega} \frac{1}{2} \tau |\nabla \phi|^2 d\Omega \\
\text{subject to } G(\Omega(\phi)) = \int_{\Omega} d\Omega - V_{\text{max}} \leq 0,
\]

where \( \tau \) is a regularization parameter expressing the ratio of the fictitious interface energy and the objective functional. Excessively complex structures in the obtained optimal configurations are reduced by minimizing the fictitious interface energy. From an engineering standpoint, the regularization term, i.e., the second term in Eq. (4), plays an important role in obtaining meaningful and useful optimal configurations. If this regularization term were absent, the optimization problem would become unstable and the resulting configurations could have disconnected structures throughout the design domain. However, when an appropriate regularization term is included, the optimization process becomes numerically stable, and smooth and clear optimal structures are obtained. In addition, adjustments of the regularization parameter \( \tau \) allow the geometrical complexity of optimal configurations to be controlled. Note that the level set function \( \phi \) has upper and lower constraint values imposed in Eq. (1), since it must be defined so as to have the same property as the phase field variable in the phase field method.

Next, using the Lagrange multiplier method, the above regularized optimization problem is replaced by the following non-constraint optimization problem:

\[
\inf_{\phi} \bar{F}(\Omega(\phi), \phi) = F(\Omega(\phi), \phi) + \lambda G(\Omega(\phi)),
\]

where \( \bar{F} \) is the Lagrangian and \( \lambda \) is the Lagrange multiplier of the constraint, \( G.\)
2.2. Time Evolutionary Equation

In conventional level set-based structural optimization methods, the optimization problem (6) is replaced by a problem to solve the following Hamilton-Jacobi equation.

\[ \frac{\partial \phi}{\partial t} + V_N |\nabla \phi| = 0, \] (7)

where \( t \) is fictitious time and \( V_N \) is the normal component of advective velocity that is defined using shape sensitivities derived from the formulation of the optimization problem. Since the Hamilton-Jacobi equation is derived from the boundary advection concept, these methods are classified as shape optimization methods, so topological changes creating new holes in the material domain during the optimization process are not allowed. To overcome this issue, a method using a technique whereby holes can be created in the material domain in an arbitrary manner based on topological derivatives has been proposed, however the obtained optimal configurations crucially depend on parameter settings.\(^{(15)}\) Although several numerical techniques for solving the Hamilton-Jacobi equation have been proposed, obtaining optimal solutions stably is difficult because the equation has the property of an advection equation. Furthermore, in thermoelastic problems and electro-thermoelastic problems, obstacles remain concerning numerical stability and poor convergence of the objective functional.\(^{(17),(18)}\)

To overcome these issues, this paper focuses on the concept of the phase field method, which represents movement of the interface by using the interface diffusion. Based on this concept, a time evolutionary equation for the level set function is derived. That is, we assume that the gradient of the level set function in the direction of a fictitious time is proportional to the gradient of the Lagrangian \( \bar{F} \), as follows:

\[ \frac{\partial \phi}{\partial t} = -K(\phi) \frac{\delta \bar{F}}{\delta \phi} \] (8)

where \( K(\phi) > 0 \) is a coefficient of proportionality and \( \frac{\delta \bar{F}}{\delta \phi} \) is the functional derivative of the Lagrangian \( \bar{F} \), which is defined as topological derivatives. Substituting Eq. (6) into Eq. (8), the following equation is derived.

\[ \frac{\partial \phi}{\partial t} = -K(\phi) \left( C \frac{\delta \bar{F}}{\delta \phi} - \tau \nabla^2 \phi \right) \] (9)

Here, to normalize the above equation, normalizing parameter \( C \) is introduced as follows:

\[ \frac{\partial \phi}{\partial t} = -K(\phi) \left( C \frac{\delta \bar{F}}{\delta \phi} - \tau \nabla^2 \phi \right) \] (10)

To provide for a representation free of influence from the exterior domain, the boundary conditions of the equation are defined using Neumann and Dirichlet boundary conditions as follows:

\[
\begin{align*}
\frac{\partial \phi}{\partial t} &= -K(\phi) \left( C \frac{\delta \bar{F}}{\delta \phi} - \tau \nabla^2 \phi \right) \\
\frac{\partial \phi}{\partial n} &= 0 \quad \text{on } \partial D \setminus \partial D_N \\
\phi &= 1 \quad \text{on } \partial D_N,
\end{align*}
\] (11)

where \( \partial D_N \) is the non-design boundary. Although conventional methods based on Eq. (7) do not preserve the smoothness of the level set function, the proposed method does because the above time evolutionary equation is a reaction-diffusion equation.

Next, the Lagrange multiplier \( \lambda \) is derived. If the volume constraint on Eq. (5) is active, the following equation must be satisfied.

\[
\frac{dG(\phi)}{dt} = \int_{\partial D} \frac{\partial \phi}{\partial t} d\Omega = 0
\] (12)
Using Eq. (6), Eq. (10) and Eq. (12), Lagrange multiplier $\lambda$ is obtained as the following equation.

$$
\lambda = -\frac{\int_{\Omega} K(\phi)\left(C \frac{\partial F}{\partial \phi} + \tau \nabla^{2} \phi \right) d\Omega}{\int_{\Omega} K(\phi) d\Omega}
$$

(13)

If the volume constraint on Eq. (5) is non-active, Lagrange multiplier $\lambda$ is zero, as follows:

$$
\lambda = 0.
$$

(14)

The level set function is updated using Eq. (11) after calculating Lagrange multiplier $\lambda$ based on the above equations. When Lagrangian $\bar{F}$ is converged, the level set function represents a candidate optimal solution.

As shown in the following equation, we can confirm that when level set function $\phi$ is updated using Eq. (10), Lagrangian $\bar{F}$ decreases monotonically.

$$
\frac{d\bar{F}}{dt} = \int_{D} \frac{\delta \bar{F}}{\delta \phi} \frac{\partial \phi}{\partial t} dD
$$

$$
= \int_{D} \left( -K(\phi) \frac{\delta \bar{F}}{\delta \phi} \right) dD
$$

(\therefore \text{(8)})

$$
= -\int_{D} K(\phi) \left( \frac{\delta \bar{F}}{\delta \phi} \right)^{2} dD < 0
$$

(15)

2.3. Minimum Mean Compliance Problem

It is assumed that the material domain is filled with a linearly elastic material where the displacement $u$ is fixed on boundary $\Gamma_{u}$, traction $t$ is applied on boundary $\Gamma_{t}$ and a body force $b$ is applied in the material domain $\Omega$. The minimum mean compliance problem with a volume constraint can then be formulated as

$$
\inf_{\Omega} F(\Omega) = l(u)
$$

(16)

subject to

$$
a(u, v) = l(v)
$$

for $\forall v \in U \quad u \in U$

$$
G(\Omega) \leq 0,
$$

(17)

(18)

where the above notations are defined as follows.

$$
a(u, v) = \int_{\Omega} \epsilon(u) : E : \epsilon(v) d\Omega
$$

(19)

$$
l(v) = \int_{\Gamma_{t}} t \cdot v d\Gamma + \int_{\Omega} b \cdot v d\Omega
$$

(20)

$$
G(\Omega) = \int_{\Omega} d\Omega - V_{\max},
$$

(21)

where $\epsilon$ is the strain tensor, $E$ is the linear elastic tensor, and

$$
U = \{ v = v_{i}e_{i} : \quad v_{i} \in H^{1}(D) \text{ with } v = 0 \text{ on } \Gamma_{u} \}.
$$

(22)

Next, the KKT conditions and the sensitivities are derived. Based on the above formulations, the Lagrangian $\bar{F}$ is derived as follows:

$$
\bar{F} = l(u) + a(u, v) - l(v) + \lambda G.
$$

(23)

Using Eq. (23), the KKT conditions are derived as follows:

$$
\delta \bar{F} = 0, \quad a(u, v) - l(v) = 0,
$$

$$
\lambda G = 0, \quad \lambda \geq 0, \quad G(\Omega) \leq 0.
$$

(24)
Although a level set function $\phi$ that satisfies the KKT conditions is a candidate optimal solution, it cannot be obtained directly. Therefore, an appropriate initial level set function is given and the sum of the objective functional and the fictitious interface energy is decreased by updating the level set function using Eq. (11).

Next, the sensitivity of the objective functional is derived using the adjoint variable method. The sensitivity of the Lagrangian $\bar{F}$ is derived as follows:

$$\bar{F}' = l(u') + a(u', v, \Omega) + a(u, v', \Omega) - l(v') + \lambda G(\Omega').$$

(25)

The adjoint variable $v$ is defined as

$$a(v, u) = l(u) \quad \text{for } \forall u \in U, \quad v \in U.$$

(26)

Substituting Eq. (24) and Eq. (26) into Eq. (25), the sensitivity of the Lagrangian is given as

$$\bar{F}' = a(u, v, \Omega') + \lambda G(\Omega').$$

(27)

3. Numerical Implementations

3.1. Optimization Algorithm

Fig. 2 shows the flowchart of the optimization procedure. First, the level set function representing an appropriate initial configuration is given. Second, based on this level set function, the objective functional and constraint functional are calculated using the FEM. If the objective functional is converged, the optimization is terminated, otherwise the level set function is updated using Eq. (11). Then, if the constraint functional is not satisfied, the level set function is modified using the numerical technique discussed below and the procedure returns to the second step. Note that if the initial configuration does not satisfy the volume constraint, the volume constraint is extended during the optimization process, to decrease variation in the value of the constraint functional.
3.2. Scheme for the Time Evolutionary Equation

First, using the Finite Difference Method, Eq. (11) is discretized in the time direction as

\[
\begin{align*}
\frac{\phi(t + \Delta t)}{\Delta t} - K(\phi(t))\nabla^2 \phi(t + \Delta t) &= -K(\phi(t))CF'(x, t) + \frac{\phi(t)}{\Delta t} \\
\phi &= 1 & \text{on } \partial D_N \\
\frac{\partial \phi}{\partial n} &= 0 & \text{on } \partial D/\partial D_N,
\end{align*}
\]

(28)

where \( \Delta t \) is the time increment. Next, the weak form of Eq. (28) is derived as

\[
\begin{align*}
\int_{\Omega} \frac{\phi(t+\Delta t)}{\Delta t} \delta \phi dD + \int_{\Omega} \nabla^T \phi(t + \Delta t)(\tau K(\phi(t))\nabla \delta \phi) dD &= \int_{\Omega} (-K(\phi(t))CF'(x, t) + \frac{\phi(t)}{\Delta t}) \delta \phi dD \\
\phi &= 1 & \text{on } \partial D_N,
\end{align*}
\]

(29)

where \( \Phi \) is the functional space of the level set function defined as follows:

\[
\Phi = \{ \phi \in H^1(D) \text{ with } \phi = 1 \text{ on } \partial D_N \}.
\]

(30)

Using the FEM, the weak form of Eq. (29) is discretized as

\[
\begin{align*}
T \Phi(t + \Delta t) &= Y \\
\phi &= 1 & \text{on } \partial D_N,
\end{align*}
\]

(31)

where \( \Phi(t) \) is the nodal value vector of the level set function at fictitious time \( t \). Matrix \( T \) and vector \( Y \) are given as follows:

\[
\begin{align*}
T &= \bigcup_e \int_{V_e} \left\{ \frac{1}{\Delta t} N^T N + \nabla^T N K(\phi(t)) \nabla N \right\} dV_e \\
Y &= \bigcup_e \int_{V_e} \left\{ -K(\phi(t))CF'(x, t) + \frac{\phi(x, t)}{\Delta t} \right\} N dV_e,
\end{align*}
\]

(32)

(33)

where \( N \) is the shape function of the FEM and \( \bigcup_e \) represents the union set of the elements.

3.3. Approximation of the Finite Element Analysis

In this paper, to update the level set function based on an Eulerian coordinate system, the void domain is approximated as a domain filled with weak material. That is, equilibrium equation (24) is replaced by the following equation.

\[
\int_D \epsilon(u) : E : \epsilon(v) H_e(\phi) d\Omega = \int_D b \cdot v H_e(\phi) d\Omega + \int_{\Gamma_i} t \cdot v d\Gamma,
\]

(34)

where \( H_e(\phi) \) is defined as follows:

\[
H_e(\phi) = \begin{cases} 
\frac{d}{2} + \frac{15}{16} \phi^3 - \frac{5}{16} \phi^5 + \frac{3}{16} \phi^4 (1 - d) & (\phi < -t) \\
\frac{d}{2} & (-t < \phi < t) \\
\frac{1}{2} & (\phi > t),
\end{cases}
\]

(35)

where \( d \) is a small positive parameter representing the weak material and \( t \) is a small positive parameter to implicitly represent the width of the intermediate domain. In addition, the volume constraint, \( G \), is also approximated as follows:

\[
G(\phi) = \int_D H_e(\phi) d\Omega - V_{\text{max}}.
\]

(36)

3.4. Numerical Technique

Parameter \( K(\phi) \) represents the ratio of the effects of boundary advection and topological change, since it is a function of the level set function \( \phi \). That is, if \( K(\phi) \) is set to a large value in the neighborhood where \( \phi = 0 \), the boundary advection effect will be large compared with the topological change effect. In this research, parameter \( K(\phi) \) is set to \( K_s(\phi) = 1 \),
and $K_2(\phi) = \exp(-\phi^2)$, and the influence of these different $K(\phi)$ settings will be examined in the next section. Note that when $K(\phi)$ is set to $K_1(\phi)$, the effects of boundary advection and topological change are comparable, however when $K(\phi)$ is set to $K_2(\phi)$, the boundary advection effect is huge compared to the topological change effect.

The proposed optimization method has the following issues. The numerical error of the volume constraint is exacerbated during the optimization procedure, since the volume constraint is imposed using the Lagrange multiplier method. Therefore, the level set function is modified to satisfy the volume constraint at every iteration. Using a bisection method algorithm, a tiny value, $\Delta \phi$, is found to satisfy the following equation.

$$G(\phi(x) + \Delta \phi(x)) \leq 0 \quad (37)$$

4. Numerical Examples

Fig. 3 shows the design domain and boundary conditions of design problem 1. The fixed design domain is discretized using a structural mesh whose length is $5 \times 10^{-4}$ m. The upper limit of the volume constraint functional is 40% of the fixed design domain. The fixed design domain is fixed on the left side and a traction is imposed at the center of the right side in a downward direction.

Fig. 4 shows the design domain and boundary conditions of design problem 2. Here, the fixed design domain is discretized using a structural mesh whose length is $4 \times 10^{-4}$ m and the upper limit of the volume constraint functional is 50% of the fixed design domain. The fixed design domain is fixed at the small left and right segments at the bottom, and a traction in a downward direction is imposed at the center of the bottom.

For each design problem, the fixed design domains are discretized using four-node plane stress isoparametric elements when solving the equilibrium equation, and four-node isoparametric elements when updating the level set function. The material domain is assumed to be filled with an isotropic material whose Young’s modulus and Poisson ratio are 210 GPa and 0.3, respectively. The parameters in Eq. (35) are set to $d = 1 \times 10^{-6}$ and $t = 0.1$, respectively. The normalizing parameter $C$ is set to $C = 1/\max(F’)$.
4.1. Effect of Parameter $K(\phi)$ Variation

First, using design problem 1, we examine the effect that different parameter $K(\phi)$ settings have upon obtained optimal configurations. The initial configuration has only the upper two thirds of the fixed design domain filled with material. The regularization parameter $\tau$ is set to 0.07. Fig. 5 and Fig. 6 show the initial, intermediate and optimal configurations when parameter $K(\phi)$ is set to $K(\phi) = K_1(\phi)$ and $K(\phi) = K_2(\phi)$, respectively.

![Fig. 5 Configurations of design problem 1 with the parameter $K_1(\phi) = 1$: (a) Initial configuration; (b)-(e) Numerical results at intermediate steps; (f) Optimal configuration](image5)

![Fig. 6 Configurations of design problem 1 with the parameter $K_2(\phi) = \exp(-\phi^2)$: (a) Initial configuration; (b)-(e) Numerical results at intermediate steps; (f) Optimal configuration](image6)

In these figures, the gray domain and white domain represent the material and void domains, respectively. Comparing Fig. 5 and Fig. 6, despite differences in the intermediate configurations, the obtained optimal configurations are nearly the same. Therefore, the effect of different $K(\phi)$ parameter settings upon the obtained optimal configurations is extremely low. In addition, the obtained optimal configurations are clear and smooth.
4.2. Effect of Different Initial Configurations

Second, using design problem 1, we examine the effect of the initial configuration upon the obtained optimal configuration. Parameter $\tau$ and $K(\phi)$ are set to $\tau = 0.07$ and $K(\phi) = 1$, respectively. In case 1, the initial configuration of the fixed design domain is filled with material. In case 2, the initial configuration of the fixed design domain is filled with material that includes two holes. In case 3, the initial configuration is filled with material only in the upper two thirds of the fixed design domain. Fig. 7 shows the initial, intermediate and optimal configurations. As shown in Fig. 7, the obtained optimal configurations are nearly the same.

Therefore, we can confirm that the effect of the initial configuration upon the obtained optimal configuration is extremely low.

4.3. Effect of Settings for Regularization Parameter $\tau$

Finally, using design problem 1 and 2, we examine the effect that different regularization parameter $\tau$ settings have upon obtained optimal configurations. The initial configuration has the fixed design domain filled with material. Parameter $K(\phi)$ is set to $K(\phi) = 1$. In design problem 1, the regularization parameter $\tau$ for Case 1, Case 2 and Case 3 are set to 0.5, 0.05 and 0.03, respectively. In design problem 2, the regularization parameter $\tau$ for Case 1, Case 2 and Case 3 are set to 0.01, 0.005 and 0.0001, respectively. Fig. 8 and Fig. 9 show the obtained optimal configurations for design problem 1 and design problem 2, respectively. As shown in these figures, the obtained optimal configurations are clear and smooth. In addition, in each case, the optimal configurations express significant differences in geometrical complexity. Therefore, the geometrical complexity of the optimal configurations can be qualitatively controlled by adjusting the regularization parameter $\tau$.

5. Conclusions

This paper proposed a new topology optimization method based on level set boundary expressions and the concept of the phase field method. The authors achieved following.
Based on level set structural boundary expressions and the concept of the phase field method, a topology optimization problem was formulated as a minimization problem including a fictitious interface energy. In addition, the topology optimization problem was replaced by a reaction-diffusion problem.

A minimum mean compliance problem was formulated based on the proposed method.

An optimization algorithm for the proposed topology optimization method, and a specific scheme for the reaction-diffusion equation were constructed.

The validity and utility of the topology optimization method were verified using numerical examples. The obtained optimal configurations were smooth and clear in all cases. Furthermore, we showed that the geometrical complexity of optimal configurations can be controlled by adjusting the regularization parameter $\tau$.

References


