The global warming is serious problem, and the reduction of CO2 emissions, namely, the realization of low-carbon society is desired. Therefore, it is desired that the conventional servomotors widely used in the industrial community are replaced with the synchronous reluctance motors (SynRMs) whose efficiency is high even though they have no magnets. The rotor position information is necessary to use SynRMs as servomotors, and several rotor position sensorless estimation methods have been proposed. These methods, however, have difficulties at low speed when the voltage signal necessary to estimate rotor position is very small. The authors have proposed a new position estimation method using high-frequency extended e.m.f. (EEMF) in which high-frequency voltage and current are calculated using a disturbance observer. Simulations have shown the new method to be very useful. In this paper, the feedback control of the high-frequency currents is proposed for becoming able to estimate the rotor position even though the high-frequency currents are very small when the checking of the new proposed method by experiments. And the experimental results show that it is possible to drive the motor at low speeds with the coordinate transformation used the position \( \hat{\theta} \) estimated by the new proposed method.

**Key Words:** Synchronous reluctance motor, Sensorless, High-frequency currents, EEMF Disturbance observer, Low speed

### 1. Introduction

Synchronous reluctance motors (SynRMs) are generally treated as simple synchronous motors because they do not have slip rings or permanent magnets, and they can be driven directly by commercial-frequency power sources. However, SynRMs exhibit both low efficiency and a low power factor, and the control performance of SynRMs is worse than that of DC motors\(^1\).

On the other hand, vector control theory has enabled torque control of SynRMs using rotor position information. The SynRM has these advantages:

- Higher efficiency than the induction motor (IM) because heat is not generated by secondary currents as with IMs.
- Reduced material costs because expensive permanent magnets which include the rare earth are not required in contrast to the permanent magnet motor (PM).

And, SynRMs have practical application as machine tools motors because of low shaft thermal expansion, low degree of torque ripple and the ability to be driven at high speed\(^1\). Therefore, it is desired that IMs and PMs are replaced with the SynRMs for the realization of Low-Carbon Society and saving the world’s ecology.

As mentioned above, rotor position information is necessary for vector control of SynRMs. Rotor position sensors introduce problems, however, because of their complicated wiring, additional space requirements and restrictions placed on them by the environments in which motors are used. Consequently, sensorless control of the rotor position in SynRMs is desired.

Several sensorless control methods have been proposed by Senju et al\(^2\). A method using extended e.m.f. (EEMF) with a disturbance observer was also proposed by the authors\(^3\). However, position sensorless control at low speeds is difficult with these methods\(^2,3\), because the voltage signal used to estimate the rotor position is small. Moreover, the method proposed by Senju et al\(^2\) is problematic because it must use a first-order lag compensator instead of pure integration.
Therefore, the authors have proposed the new method using EEMF calculated from high-frequency voltage and current with a disturbance observer\(^3\), based on the previous method proposed by the authors\(^3\).

Moreover, with the new proposed method, the estimated rotor position \(\hat{\theta}\) is obtained directly because the rotor position estimator is constructed on the stator frame \((\alpha-\beta)\) axis. To "directly obtain" \(\hat{\theta}\) means that it is possible to estimate the rotor position accurately and simply. In the methods previously proposed\(^3\), only the rotor position error \(\Delta\theta\) can be estimated. In these methods, the rotor position \(\theta\) was estimated by a simple integration of \(\Delta\theta\) without adequate accuracy at acceleration and deceleration, or a complex estimator must be designed to improve accuracy.

The usefulness of the new method proposed by the authors\(^3\) has been demonstrated by simulation, and the experiments had not been carried out\(^4\).

In this paper, the feedback control of the motor is explained. To “directly obtain” \(\hat{\theta}\) means that it is possible to drive the motor at low speeds with the complex estimator must be designed to improve accuracy. Moreover, with the new proposed method, the authors have proposed the new method using EEMF calculated from high-frequency voltage and current with a disturbance observer\(^4\). Therefore, the authors have proposed the new method using EEMF calculated from high-frequency voltage and current with a disturbance observer\(^4\), based on the previous method proposed by the authors\(^3\).

In this paper, the feedback control of the motor is explained. To “directly obtain” \(\hat{\theta}\) means that it is possible to drive the motor at low speeds with the complex estimator must be designed to improve accuracy.

2. Mathematical Models of SynRM

In this section, SynRM circuit equations\(^3\) used as mathematical models for sensorless control are discussed for two kinds of coordinates: the \(d-q\) rotating coordinate and the \(\alpha-\beta\) fixed coordinate. These coordinates are defined in Fig. 1.

![Fig. 1 Coordinates of SynRM](image)

The circuit equation for SynRMs on the \(d-q\) rotating coordinate is given by Eq. (1) as

\[
\begin{bmatrix}
  v_d \\
  v_q \\
  i_d \\
  i_q
\end{bmatrix} =
\begin{bmatrix}
  R + pL_d & -\omega L_q & 0 \\
  -\omega L_d & R + pL_q & 0 \\
  \omega L_d & -\omega L_q & \rho
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q \\
  \omega
\end{bmatrix}
\]

(1)

where

\[
\begin{align*}
  [v_d, v_q]^T & \quad \text{voltage on rotor frame,} \\
  [i_d, i_q]^T & \quad \text{current on rotor frame,} \\
  R & \quad \text{stator resistance,} \\
  L_d & \quad \text{inductance of the \(d\)-axis,} \\
  L_q & \quad \text{inductance of the \(q\)-axis,} \\
  \rho & \quad \text{differential operator,} \\
  \omega & \quad \text{angular velocity at electrical angle,} \\
  \theta & \quad \text{rotor position at electrical angle.}
\end{align*}
\]

Transforming Eq. (1) for the \(\alpha-\beta\) fixed coordinate, Eq. (2) is derived as

\[
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix} =
\begin{bmatrix}
  R + pL_a & -\omega L_b & -\omega L_c & 0 & 0 & 0 \\
  -\omega L_a & R + pL_b & -\omega L_c & 0 & 0 & 0 \\
  -\omega L_a & -\omega L_b & R + pL_c & 0 & 0 & 0 \\
  0 & 0 & 0 & \rho & 0 & 0 \\
  0 & 0 & 0 & 0 & \rho & 0 \\
  0 & 0 & 0 & 0 & 0 & \rho
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c \\
  \omega \\
  \phi_a \\
  \phi_b
\end{bmatrix}
\]

(2)

where

\[
\begin{align*}
  [v_a, v_b, v_c]^T & \quad \text{voltage on fixed frame,} \\
  [i_a, i_b, i_c]^T & \quad \text{current on fixed frame,} \\
  L_a & \quad \text{is the \(L_a\) of Eq. (6),} \\
  L_b & \quad \text{is the \(L_b\) of Eq. (6),} \\
  L_c & \quad \text{is the \(L_c\) of Eq. (6),} \\
  L_{\phi} & \quad \text{is the \(L_{\phi}\) of Eq. (6),} \\
  L_r & \quad \text{is the \(L_r\) of Eq. (6),} \\
  L_s & \quad \text{is the \(L_s\) of Eq. (6),} \\
  L_{\omega} & \quad \text{is the \(L_{\omega}\) of Eq. (6),} \\
  L_{\theta} & \quad \text{is the \(L_{\theta}\) of Eq. (6),}
\end{align*}
\]

Rewriting Eq. (2), we have Eq. (3) given by

\[
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix} =
\begin{bmatrix}
  R + pL_d & -\omega L_q & 0 & 0 & 0 & 0 \\
  -\omega L_d & R + pL_q & 0 & 0 & 0 & 0 \\
  0 & 0 & \rho & 0 & 0 & 0 \\
  0 & 0 & 0 & \rho & 0 & 0 \\
  0 & 0 & 0 & 0 & \rho & 0 \\
  0 & 0 & 0 & 0 & 0 & \rho
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q \\
  \omega \\
  \phi_a \\
  \phi_b \\
  \phi_c
\end{bmatrix}
\]

(3)

It is difficult to directly extract position \(\theta\) from Eq. (3), because the \(2\theta\) terms make the solution quite difficult. However, we can eliminate the \(2\theta\) terms in Eq. (3). Since it is known that there is no second harmonic component in the motor current, we can consider eliminating the \(2\theta\) terms by a purely mathematical method. One reason for the appearance of the \(2\theta\) terms is the asymmetry of the impedance matrix in Eq. (1). If we rewrite the impedance matrix symmetrically as

\[
\begin{bmatrix}
  v_d \\
  v_q \\
  i_d \\
  i_q
\end{bmatrix} =
\begin{bmatrix}
  R + pL_d & -\omega L_q & 0 & 0 \\
  -\omega L_d & R + pL_q & 0 & 0 \\
  0 & 0 & \rho & 0 \\
  0 & 0 & 0 & \rho
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q \\
  \omega
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  0
\end{bmatrix}
\]

(4)

then the circuit equation for the \(\alpha-\beta\) coordinate can be derived as Eq. (5), in which there are no \(2\theta\) terms, given by

\[
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix} =
\begin{bmatrix}
  R + pL_a & -\omega L_b & -\omega L_c & 0 & 0 & 0 \\
  -\omega L_a & R + pL_b & -\omega L_c & 0 & 0 & 0 \\
  -\omega L_a & -\omega L_b & R + pL_c & 0 & 0 & 0 \\
  0 & 0 & 0 & \rho & 0 & 0 \\
  0 & 0 & 0 & 0 & \rho & 0 \\
  0 & 0 & 0 & 0 & 0 & \rho
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c \\
  \omega \\
  \phi_a \\
  \phi_b
\end{bmatrix}
\]

(5)

Here, \(i_d\) in the second term on the right side of Eq. (5) means the differential of \(i_q\). This second term on the right side of Eq. (5) is defined as the extended e.m.f. (EEMF) \(e\) by Eq. (6), below. If the EEMF can be estimated, then the rotor position can be obtained from its phase.

\[
e = e_p = (L_d - L_q)\omega i_q - i_d \frac{-\sin \theta}{\cos \theta}
\]

(6)

In Eq. (6), \(\omega\) becomes small at low speed. Moreover, \(i_d\) may be zero in many cases because \(i_q\) is differentiated. Therefore, it is not possible to estimate the rotor position because the EEMF is very small even if current \(i_q\) flows at low rotor speeds. To solve this problem, the use of high-frequency currents is proposed. The high-frequency voltages and currents on the stator coordinates \((\alpha-\beta)\) are defined as \(v(a,\omega)\) and \(i(a,\omega)\), respectively. Moreover, the high-frequency currents on the rotor coordinates \((d-q)\) are defined as \((i(a,\omega))\).

The mathematical model given by Eq. (5) is then transformed to the high-frequency mathematical model of Eq. (7) using high-frequency voltages and currents. The EEMF of Eq. (6) is also transformed to
high-frequency EEMF $e_h$ in Eq. (8).

\[
\begin{bmatrix}
v_{\alpha} \\
v_{\beta}
\end{bmatrix} = \begin{bmatrix}
R + pL & o(L_d - L_q)\\
-o(L_d - L_q) & R + pL
\end{bmatrix} \begin{bmatrix}
i_{\alpha} \\
i_{\beta}
\end{bmatrix} + \begin{bmatrix}
-(L_d - L_q)w_{\alpha} - i_{\beta} \\
(L_d - L_q)w_{\beta} - i_{\alpha}
\end{bmatrix} \begin{bmatrix}
-\sin\theta \\
\cos\theta
\end{bmatrix}.
\tag{7}
\]

By extracting components of $v_h$, $i_h$ using a band-pass filter (BPF), we can estimate the rotor position using Eq. (7) and Eq. (8) based on high-frequency voltages and currents, because $i_{h\alpha}, i_{h\beta}$ are not zero even if $\omega i_{h\alpha}$ is close to zero, and $i_{h\alpha}, i_{h\beta}$ are close to zero.

### 3. Disturbance Observer

Regard the high-frequency EEMF $e_h$ in Eq. (8) as a disturbance, it can be estimated by a disturbance observer at standstill and at low speeds. The observer is based on a linear state equation so that linear control theory can be used in its design to guarantee stability.

#### 3.1 High-Frequency Linear State Equation

From the new model given by Eq. (7), the SynRM can be described by a linear state equation and an output equation as given in Eq. (9). Here, the state variables are the high-frequency stator current $i_h$ and the high-frequency EEMF $e_h$. The system input is high-frequency voltage $v_h$ and its output is high-frequency current $i_h$. Assuming the time constant of the electrical system is sufficiently smaller than the mechanical one, velocity $\omega$ can be regarded as a constant parameter. Then,

\[
\begin{bmatrix}
i_h \\
e_h
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
0 & A_{22}
\end{bmatrix} \begin{bmatrix}
i_h \\
e_h
\end{bmatrix} + \begin{bmatrix}
B_1 \\
0
\end{bmatrix} v_h
\]

\[
i_h = C \begin{bmatrix}
i_h \\
e_h
\end{bmatrix}
\tag{9}
\]

where,

\[
A_{11} = -(R/L_d)I + (\omega(L_d - L_q)/L_d)J, A_{12} = (-1/L_d)I = a_{11}I, A_{22} = a_{22}J, B_1 = (1/L_d)I, C = [I \ 0],
\]

\[
J = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

#### 3.2 Position and Velocity Estimation Using Disturbance Observer

To estimate high-frequency EEMF $e_h$ described by Eq. (9), a reduced-order observer is constructed in Eq. (10) as

\[
\begin{align*}
\dot{i}_h &= A_{11}i_h + A_{12}e_h + B_1v_h, \\
\dot{e}_h &= A_{22}\dot{e}_h + G_i i_h - i_h
\end{align*}
\]

\[
\dot{e}_h = A_{11}G_i \dot{e}_h + (A_{12}G + A_{22})\dot{e}_h + GBv_h - G_i i_h,
\tag{10}
\]

where,

- $i_h$: estimated state variable,
- $G = g_1I + g_2J$: feedback gain.

Eq.(10) is the disturbance observer based on high-frequency current for estimating high-frequency EEMF $e_h$. From Eq. (9) and Eq. (10), the error equation is given by Eq. (11) as

\[
\dot{e}_h = \dot{e}_h - \dot{e}_h = (A_{11} + A_{12}G)\dot{e}_h - \dot{e}_h
\]

\[
\dot{e}_h = (A_{11} + A_{12}G)\dot{e}_h - \dot{e}_h
\tag{11}
\]

Here, $\alpha$ and $\beta$ are the poles of the observer and have the following relation to observer gain $G$:

\[
\alpha = g_1a_{11}, \quad \beta = a_{22} + g_2a_{12}.
\]

To avoid differentiation of current $i_h$, an intermediate variable $\dot{e}_h$ is introduced in Eq. (12) as

\[
\dot{e}_h = \dot{e}_h - Gi_i
\tag{12}
\]

Substituting Eq. (10) into Eq. (12) yields Eq. (13) as

\[
\dot{\xi}_h = (A_{11}G + A_{12})\dot{\xi}_h + B_1v_h + G(A_{11}I - A_{12}G - A_{22})i_h,
\]

\[
\dot{\xi}_h = \dot{\xi}_h - Gi_i
\tag{13}
\]

Eq. (13) is the equivalent disturbance observer shown in Fig. 2 where the transfer function of filter $H(s)$ is given by Eq. (14) as

\[
H(s) = \frac{a_{11} - \beta I}{(s + \alpha I + \beta I)^2}
\tag{14}
\]

Using the high-frequency EEMF, the position is calculated by Eq. (15) as

\[
\hat{\theta} = \tan^{-1}(\dot{\theta}h_{AB}/\dot{\theta}h_{KB}).
\tag{15}
\]

4. Position Sensorless Control

Fig. 3 shows the configuration used for position sensorless control using a disturbance observer based on high-frequency current. The motor currents are detected by sensors and sent into a digital signal processor (DSP) through A/D converters. In the DSP program, the three-phase current signals ($u-v-w$ coordinate) are converted into two-phase coordinate signals ($\alpha-\beta$ coordinate). Taking the difference between mechanical velocity command $\omega^*$ and velocity $\hat{\omega}$ of the adaptive velocity estimation etc., the result is used to generate current command $i^*$ at the rotor reference frame ($d-q$ coordinate) by the proportional plus integral (PI) velocity controller, shown in Fig. 3. In this paper, the velocity $\omega$ of the sensor is used instead of the estimated one $\hat{\omega}$. From $i^*$ and detected $i$, voltage command $v^*$ for driving the motor is calculated by the current controller. This result is then converted to the stator reference frame ($\alpha-\beta$ coordinate) using the rotor position $\hat{\theta}$ estimated by the disturbance observer.

High-frequency current command $i_h^*$ is generated
and used by both high-frequency current controller and high-frequency coordinate transformation to calculate voltage command \( v^* \) \((\alpha - \beta)\text{coordinate}\) for injecting high-frequency current \( i^* \) into the motor. Note that the high-frequency currents are controlled using feedback shown by the red line of Fig.3 to enable to estimate the rotor position even though the high-frequency currents are very small, unlike the method proposed by the authors\(^5\).

Adding \( v^* \) to \( v \), voltage command \( v \) of the motor is calculated and converted into three-phase signals. Voltage command \( v \) is sent to the inverter to drive the motor. High-frequency currents \( i^* \) and voltages \( v^* \) are converted into two-phase signals and sent to the disturbance observer through BPF.

5. Experimental Results of Position Sensorless Control At Low Speeds

For this study, an experiment was carried out for the rotor position sensorless control at low speeds using the proposed method, provided that the actual velocity \( \omega \) is used instead of the estimated one \( \hat{\omega} \). Table 1 shows the specifications of the SynRM employed in the experiment.

![Fig. 3 Configuration of position sensorless control](image)

**Fig. 3 Configuration of position sensorless control**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated electric current</td>
<td>6.0[A]</td>
</tr>
<tr>
<td>Rated output</td>
<td>1010[W]</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>1.08[\Omega]</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>( L_s: 19.00[\text{mH}], L_d: 40.08[\text{mH}] )</td>
</tr>
<tr>
<td>Inertia</td>
<td>0.00416[\text{kg} \cdot \text{m}^2]</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>DC source voltage</td>
<td>135[V]</td>
</tr>
</tbody>
</table>

High-frequency current command \( i^* \) was set to 200Hz, 0.06A. Sixth-order Butterworth BPFs with a selectivity and center frequency of \( Q=1 \) and \( f_0 =200\text{Hz} \), respectively, were used to extract the high-frequency voltages \( v^* \) and currents \( i^* \) of the motor. The poles of the disturbance observer were \( \alpha =1000\text{rad/s} \) and \( \beta = \omega \text{rad/s} \). The DSP system was the PE-Expert2 of Myway Corporation and the DSP was a TMS320VC33. Voltage and current signals were provided to the DSP by 12-bit A/D converters. A pulse width modulation (PWM) inverter with a switching frequency of 20 kHz was used. The calculation period of the disturbance observer was 50 \( \mu \text{s} \). The calculation period of the velocity controller was 1000 \( \mu \text{s} \). The estimated position \( \hat{\theta} \) at low speeds is calculated from the phase of high-frequency EEMF \( \phi^* \) using Eq. (15). Fig.4 and Fig.5 show the position \( \theta \) and estimated one \( \hat{\theta} \) with the velocity command \( \omega^* \) and the actual one \( \omega \) at both no load and low speeds. Fig.4 and Fig.5 show the cases when the velocity command \( \omega^* \) is changed as \( 0 \rightarrow 10\text{rpm} \) and \( 30 \rightarrow 10\text{rpm} \), respectively. The estimated position \( \hat{\theta} \) agrees well with real one \( \theta \) at low speeds even though \( i^* \) was set to 0.06A because the high-frequency currents were controlled using feedback.

6. Conclusion

In this paper we proposed a new method of position sensorless control for a SynRM using a disturbance observer based on high-frequency currents. It is shown, experimentally, that it is possible to drive the motor at low speeds with the coordinate transformation used the position \( \theta \) estimated by the new proposed method. The experiments verify the feasibility of the proposed method. The proposed method expects to contribute to spreading the use of the SynRMs and the realization of Low-Carbon Society.

![Fig.4 Position and estimated one( \( \omega^* : 0 \rightarrow 10\text{rpm} \))](image)

![Fig.5 Position and estimated one( \( \omega^* : 30 \rightarrow 10\text{rpm} \))](image)

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References


