Article

Stochastic Forest Management and Risk Aversion

Ken-ichi Akao*

ABSTRACT

This paper considers the timing of timber harvesting under uncertainty. The decision of a forest manager who maximizes the expected utility is formalized using a form of the closed loop control model. The stochastic optimization problem derived is very similar to the deterministic single stand problem known as the classical Faustmann formula, but there is a notable difference. Whilst optimal rotation periods are certain in the deterministic world, they are random variables in stochastic circumstances. Stochastic optimal rotation periods are difficult to include in forest economic analyses. Fortunately, under some technical assumptions, we can find a non-stochastic variable named "barrier" which corresponds to the optimal cutting stock size in the deterministic world. This paper shows that the von Neumann-Morgenstern forest manager cuts his/her trees when the stochastic stumpage values reach a certain amount, that is the barrier. Comparative statics on the barrier were conducted to evaluate the effect of planting costs and the degree of the manager's risk aversion. The results show that the barrier rises as planting costs increase or as the Arrow-Pratt index of absolute risk aversion becomes smaller.

Keywords: stochastic Faustmann formula, barrier, risk aversion, comparative statics

INTRODUCTION

For a forest manager, it is important to consider the uncertainties surrounding the forest, because there is a long period between planting and harvest. Literature in the field of forest economics has tried to explicitly introduce uncertainty into forest management models. The models used in the literature are classified into two categories: the open loop control model and the closed loop control model. In the former model, the harvest age is determined before harvesting, possibly at the time of planting. We can find this kind of model in REED(1984), AKAO(1987), BHATTACHARYYA and SNYDER(1988), CAULFIELD(1988) and KUROKAWA(1991). In contrast, the closed loop control model allows the cutting age to be changed, depending on future states which vary. The closed loop control model is found in MILLER and VOLTAIRE(1980,1983), MALLIARIS and BROCK(1982, pp.194-205), JOHANSSON and LÖFGREN(1985,

pp.261-264), BRAZEE and MENDELSON(1988), CLARKE and REED(1989), REED and CLARKE(1990) and THOMSON(1992).

Intuitively, in a stochastic model, decision making about a harvest is contingent upon the known present state as well as uncertain future states. Tomorrow provides new information which updates the decision maker's knowledge of the current state, and improves his/her knowledge about future states. Forest management models which incorporate uncertainty should allow some flexibility in decision making and be able to use new information coming in the future. Thus, closed loop control models are more appropriate than open loop control models for forest management under uncertainty. The open loop control model does not consider information which reaches the decision maker with the passage of time, whereas the closed loop control model does. Also the open loop control model may cause time inconsistency, which means that today's decision is inconsistent with tomorrow's one. AKAO(1987) observed this inconsistency in a feedback open loop control model. In general, the results from open loop control models do not coincide with ones from closed loop control models, although exceptions could exist, as shown by CLARKE and REED(1989). Furthermore, the differences between them

* School of Social Sciences, Waseda University,
1-6-1, Nishi-waseda, Shinjuku-ku, Tokyo 169
-50 Japan

may be great. BRAZEE and MENDELSON(1988) showed in a numerical example that the optimal expected reward from a closed loop control model is two or more times that of an open loop control model.

In this paper, we use the closed loop control model to formalize a timber harvesting problem. It is a simple version of the models developed by MALLIARIS and BROCK (1982) and by MILLER and VOLTAIRE(1983). The difference is that they constructed the model in terms of money, whereas the model proposed here is constructed in terms of utility, so that we can examine the effects of the forest manager's attitude towards risk. The remainder of this paper is constructed as follows. Section II gives the assumptions and formulates the model. Section III shows the results of comparative statics (sensitivity analysis) with respect to planting costs, the degree of risk aversion, and other parameters. Section IV discusses applications of the results to forest economics.

THE MODEL

Notation and Assumptions

Suppose an even-aged forest whose current stumpage value is denoted by $x_t \in R$ (the set of real numbers), where t stands for the time. Denote by $t=0$ the present time and let $t \in R_+$ (the set of non negative real numbers). At time t , a certain stumpage value x_t is known, but until the time t comes, the value is uncertain. Denote by X_t the uncertain stumpage value to distinguish it from the realized certain value x_t . Suppose that X_t is a random variable defined on a probability space and that the probability process $\{X_t, t \geq 0\}$ is given by the Ito stochastic differential equation,

$$dX_t = f(x_t) dt + \sigma(x_t) dz, \quad dz = \varepsilon \sqrt{dt}, \quad \varepsilon \sim N(0, 1), \quad (1)$$

where f expresses the instantaneous mean growth rate of x_t , called the shift coefficient, and σ is the instantaneous standard deviation of x_t , called the diffusion coefficient. To ensure that the sample paths following (1) are continuous, we assume that

[A.1] f and σ are bounded and continuous functions defined on R ,

and

[A.2] f is non negative over R .

For simplicity, we restricted revenue to clear cutting and ignored thinning. Also we assumed that the costs for planting and silviculture are non-stochastic variables and constants. The total discounted cost evaluated at planting time is denoted by c .

Put the function $u: R \rightarrow R$. If the manager harvests the trees whose current value is x_t at time t , the present value of the utility is denoted by $u(x_t) \exp(-rt)$, where r is

his/her time discount rate. The current value monetary utility $u(x)$ might be interpreted as the value function of some dynamic optimization problem for the consumption schedule derived from the manager's instantaneous utility function under constraint of the budget including x . As for $u(x)$, we assumed that:

[A.3] $u(x)$ is at least twice continuously differentiable, strictly increasing, and concave.

[A.4] $u(0) = 0$.

Finally, we supposed that the aim of the forest manager is to maximize the sum of all future series of expected utilities evaluated at the present time. More exactly,

[A.5] The manager has additively separable utility and follows the VON NEUMANN -MORGENSTERN's expected utility hypotheses. The time horizon is infinite.

One rotation problem

We begin with the one rotation problem, called the Fisherian problem or the wine aging problem, which considers only the optimal harvest of the initial forest and ignores the costs of regeneration or revenues from harvests thereafter. This problem is preliminary to the infinite rotations problem considered later.

The manager must decide when it is the best time, T , to sell the trees. At any time, whether he/she should sell them depends on the current stumpage value. Since the value varies randomly, the best time T must also be a random variable. The manager's strategy is to choose a contingent time T , which is called the stopping time. If x_0 is the present stumpage value, the manager's strategy is formulated as:

$$\left. \begin{aligned} W(x_0) = \sup_{T \in [0, \infty)} E [u(X_T) \exp(-rT)], \\ \text{subject to } dX_t = f(x_t) dt + \sigma(x_t) dz, \text{ with } X_0 = x_0, \end{aligned} \right\} \quad (2)$$

where E is the expectation operator and the supremum is taken over the set of measurable stopping time T . Note that if the supremum were taken over R_+ , the problem is the choice of a certain optimal cutting time and becomes the open loop control model.

Although solving this problem looks complicated, MIROSHINICHENKO(1975, Lemma 1.) and MALLIARIS and BROCK(1982, Theorem 18.2.) showed that, under some conditions, the problem can be transformed into a more simple one as follows.

The real line of current stumpage values is divided conceptually into two sets called the continuation region

and the stopping region. If x_t falls into the stopping region, it is optimal to harvest the trees. If x_t remains in the continuation region, then it is optimal to keep the trees. Since the Ito process $\{X_t, t \geq 0\}$ used here is time independent, these two sets must be independent of time. Also since the decision making depends only on the realized current stumpage value at each time, the two sets are independent of the initial state x_0 . Therefore we can denote the stopping region by $\gamma = \{x \mid W(x) \leq u(x)\}$ and the continuation region by $C = \{x \mid W(x) > u(x)\}$.

If $x \in \gamma$, then if dt is small enough,

$$u(x) \geq E[u(X(dt)) \exp(-r dt)]$$

holds. Expanding the right hand side by invoking ITO's lemma, we have

$$u(x) \geq u(x) + [-ru(x) + u'(x)f(x) + (1/2)u''(x)\sigma^2(x)] dt.$$

Therefore, if $x \in \gamma$,

$$-ru(x) + u'(x)f(x) + (1/2)u''(x)\sigma^2(x) \leq 0.$$

This implies that the set

$$A = \{x \mid -ru(x) + u'(x)f(x) + (1/2)u''(x)\sigma^2(x) > 0\}$$

must be contained in C . Furthermore if A is connected, then C must be connected. Otherwise, a connected component of C must exist such that the intersection of the component and A is an empty set. Denote this connected component of C by C_1 and let the stopping time for $x \in C_1$ be $T(x)$. $A \cap C_1 = \emptyset$ implies

$$-ru(x_t) + u'(x_t)f(x_t) + (1/2)u''(x_t)\sigma^2(x_t) \leq 0$$

for every feasible x_t from x at $t \in (0, T(x))$. Thus, we have

$$\begin{aligned} W(x) &= E[u\{X[T(x)]\} \exp(-rT(x))] \\ &= E[\int_0^{T(x)} \{-ru(x_t) + u'(x_t)f(x_t) + (1/2)u''(x_t)\sigma^2(x_t)\} dt + u(x)] \end{aligned}$$

However, $x \in C_1 \subset C$ implies $W(x) > u(x)$. A contradiction appears.

Thus, if a unique real number x^* exists such that $A = (-\infty, x^*)$ and $A^c = [x^*, \infty)$, then the boundary between the stopping region and the continuation region is unique. Then the problem (2) can be transformed into

$$W(x_0) = \max_x u(x) E[\exp(-rS(x_0, x))],$$

where $S(x_0, x)$ is the first hitting time from x_0 to x . This is a stochastic Fisherian problem.

Notice that the solution is the boundary between the continuation region and the stopping region. If the initial stumpage value is less than the boundary, the harvest occurs when the stumpage value touches the boundary. This is the reason why the solution is called "barrier". Also notice that the problem is to choose the non-stochastic optimal stumpage value. It is much easier to handle than the problem (2).

The FAUSTMANN problem

Now consider the infinite rotations problem. Let X_{T_i} ($T \geq 0, i=1, 2, \dots$) be the stumpage value for the trees planted at i th whose age is T_i . Suppose that so far, the manager has iterated planting and cutting, and now he/she stands on the just planted land. The problem is to determine the contingent cutting ages $(T_1, T_2, \dots)$. Let x_{01} be the stumpage value at the present time and denote by $W(x_{01})$ the sum of infinite series of the present value of expected utilities attained in the optimal strategy. In order to apply the foregoing results to the infinite rotations problem, let us assume that

[A.6] There exists $W(x_{01}) < \infty$.

In the stochastic but time independent circumstances supposed here, at the time when the first harvest just has been done, the problem must be the same as one at the present time. Therefore $x_{01} = x_{02}$ and the harvest strategy T_2 must be same to T_1 . Put $x_0 = x_{01} (= x_{02} = \dots = x_{0i} = \dots)$. Then we have the stochastic Bellman equation.

$$\left. \begin{aligned} W(x_0) &= \sup_{T_1 \in [0, \infty)} E[\{u(X_{T_1} - c) + W(x_0)\} \exp(-rT_1)], \\ &\text{subject to } dX_{T_1} = f(x_{T_1}) dt + \sigma(x_{T_1}) dz, \text{ with } X_{01} = x_0, \end{aligned} \right\} \quad (3)$$

where the supremum is taken over the set of measurable stopping times. In order to apply the previous results, let us assume that

[A.7] There exists a continuation region and a stopping region for problem (3) and the boundary is unique.

Then (3) can be transformed into

$$W(x_0) = \max_x [u(x - c) + W(x_0)] \Phi(x), \quad (4)$$

where $\Phi(x) = E[\exp(-rS(x_0, x))]$.

Let $p(s; x_0, x)$ be the probability density with which the first hitting time from x_0 to x is s . Then $\Phi(x)$ can be expressed as the LAPLACE transformation of $p(s; x_0, x)$, that is

$$\Phi(x) = \int_0^\infty \exp(-rs) p(s; x_0, x) ds. \quad (5)$$

Notice that both of $\exp(\cdot)$ and $p(\cdot)$ in the right hand side in (5) are nonnegative. Also notice that $\int_0^\infty p(\cdot) ds \leq 1$ for any $x \in R$ and $\exp(\cdot) \geq 1$ for any $s \in R_+$. From these observations, we know $\Phi(x) \in [0, 1]$. Let x^* be the solution of (4). $\Phi(x^*) = 0$ implies that the optimal strategy is to

keep the trees eternally with probability one. On the other hand, $\Phi(x^*)=1$ implies that the optimal strategy is to cut the trees at the moment the land is planted. Since both of these polar cases are non realistic, we assume that

$$[A.8] \quad \Phi(x^*) \in (0, 1).$$

From (4), the following inequality holds.

$$W(x_0) \geq [u(x-c) + W(x_0)] \Phi(x), \text{ any } x \in R.$$

Equality holds if and only if $x=x^*$. [A.8] implies that $W(x_0) > [u(x_0-c) + W(x_0)] \Phi(x_0)$. Since $\Phi(x_0)=1$, $u(x_0-c)$ must be negative. Therefore we can rearrange above inequality to

$$W(x_0) \geq u(x-c) \Phi(x) / [1-\Phi(x)], \text{ any } x \in R.$$

Equality holds if and only if $x=x^*$. As a result, we can rewrite the problem (4) to

$$\max_x u(x-c) \Phi(x) / [1-\Phi(x)], \quad (6)$$

which is the stochastic FAUSTMANN formula.

There are two important points about this formula. First, when $\Phi(x)$ is displaced by $\exp[-rT(x)]$ where T is the function which maps the stumpage values to the age of trees in a deterministic model, we get the deterministic FAUSTMANN formula. Therefore the stochastic FAUSTMANN formula is very similar to the deterministic one. Second, for any sample path starting from x_0 , the principle of optimality must be satisfied. Therefore if x^* is the solution for the stochastic FAUSTMANN formula starting from x_0 , then x^* also must be the solution for the problem starting from any initial forest whose stumpage value is less than x^* .

COMPARATIVE STATICS

The first order condition

In this section, we conduct comparative statics on the barrier x^* , i.e. the solution of (6), with respect to planting costs, the degree of forest manager's risk aversion, and other parameters. To do this, let us assume the differentiability of $\Phi(x)$. That is, for $p(s; x_0, x)$ (the probability density with which the first hitting time from x_0 to x is s), we assume that

$$[A.9] \quad p(s; x_0, x) \text{ is twice continuously differentiable in } x.$$

Under [A.9], we can have at the optimal barrier x^* , $d\{u(x^*-c) \Phi(x^*) / [1-\Phi(x^*)]\} / dx = 0$.

From this equation, the first order condition for the optimal barrier x^* are derived as

$$u_x(x^*-c) \Phi(x^*) + \Phi_x(x^*) [u(x^*-c) + W(x_0)] = 0,$$

where $u_x = u'$ and $\Phi_x(x^*) = \partial \Phi(x^*) / \partial x$.

$$u(x^*-c) < (>) 0 \text{ implies } W(x_0) < (>) 0 \text{ since}$$

$W(x_0) = u(x^*-c) \Phi(x^*) / [1-\Phi(x^*)]$ and $0 < \Phi(x^*) < 1$. Note that if $u(x^*-c) < 0$, the manager would cease management of the forest after the first harvest because regeneration costs exceed revenue from the first harvest. As a result, the problem is reduced to the one rotation problem. The case that $u(x^*-c) = 0$ is the same. Therefore so long as we can consider the FAUSTMANN problem, $u(x^*-c)$ must be positive. That is,

$$u(x^*-c) > 0. \quad (7)$$

Then,

$$\begin{aligned} \Phi_x(x^*) &= -u_x(x^*-c) \Phi(x^*) / [u(x^*-c) + W(x_0)] \\ &= -[1-\Phi(x^*)] \Phi(x^*) u_x(x^*-c) / u(x^*-c) < 0. \end{aligned} \quad (8)$$

In order to conduct comparative statics simply, we define the function for problem (6) as

$$w(x(a), a) = \max_x u(x-c) \Phi(x) / [1-\Phi(x)]. \quad (9)$$

Here a stands for a shift parameter in what follows (e.g. planting costs c) and $x(a) = x^*$ (the solution for (6)). The total differential of the first order condition

$$\partial w(x(a), a) / \partial x = 0$$

gives

$$w_{xx}(x(a), a) dx + w_{xa}(x(a), a) da = 0.$$

From [A.7], the solution $x(a)$ is unique. This implies $w_{xx}(x(a), a) < 0$. Hence we have

$$dx(a) / da = -w_{xa}(x(a), a) / w_{xx}(x(a), a)$$

and

$$\text{sign}[dx(a) / da] = \text{sign}[w_{xa}(x(a), a)].$$

This result shows that for comparative statics we only have to examine the sign of $w_{xa}(x(a), a)$.

The effect of planting costs

For planting costs, calculate $w_{xc}(x(c), c)$ to get

$$w_{xc}(x(c), c) = [-u_{xx} \Phi(1-\Phi) - u_x \Phi_x] / (1-\Phi)^2.$$

Substituting (8) into the numerator, we have

$$w_{xc}(x(c), c) = [u_x \Phi(1-\Phi)] [(-u_{xx}/u_x) + u_x/u] / (1-\Phi)^2 > 0. \quad (10)$$

The strict inequality follows from [A.3], [A.8], and (7)

Thus we can conclude that as the planting costs rise, the barrier also rises. Even if the manager is a risk lover, this inequality holds as long as the Arrow-Pratt index of absolute risk aversion ($-u_{xx}/u_x$) is less than u_x/u .

The effect of degree of risk aversion

One measure of a manager's attitude towards risk is the Arrow-Pratt index of absolute risk aversion $-u_{xx}(x)/u_x(x)$. This index depends on both the first and second order derivatives of the utility function. However, the first order condition (8) is concerned only with the first order derivative of the utility function. In order to use the Arrow-Pratt index of absolute risk aversion, we must

restrict the utility functions considered here to those in which the second order derivative of the utility function is uniquely determined by the first order derivative.

Using some real number $\lambda \in (0, 1)$ and some function $v: R \rightarrow R$, let us express the utility function as

$$u(x) = \lambda x + (1 - \lambda) v(x). \quad (11)$$

Note that $v(x)$ is (strictly) concave if $u(x)$ is (strictly) concave. We use λ as the shift parameter for comparative statics. It is easily verified that as λ increases, the Arrow-Pratt index of absolute risk aversion approaches zero monotonically. In other words, as λ increases, the manager's attitude towards risk becomes more neutral.

By calculating $w_{x\lambda}$, we get

$$w_{x\lambda} = \{ [1 - v_x(x^* - c)] \Phi (1 - \Phi) + [x^* - c - v(x^* - c)] \Phi_x \} / (1 - \Phi)^2.$$

Substituting (8) into the numerator of right hand side and rearranging it, we have

$$w_{x\lambda} = \{ [\Phi (1 - \Phi) / u] / (1 - \Phi)^2 \} [v(x^* - c) - (x^* - c) \times v_x(x^* - c)].$$

Notice that $\{ [\Phi (1 - \Phi) / u] / (1 - \Phi)^2 \} > 0$. Then we check the sign of $[v(x^* - c) - (x^* - c) \times v_x(x^* - c)]$.

We know that $x^* - c > 0$ from [A.4] and (7), otherwise, the problem is reduced to the Fisherian problem. Also we know that v is concave and $v(0) = 0$ since u is concave and $u(0) = 0$. Thus from the property of concave functions, $v(x^* - c) - v(0) = v(x^* - c) \geq (x^* - c) \times v_x(x^* - c)$, (12) holds. From the inequality (12), we conclude $w_{x\lambda} \geq 0$. In particular, if u is a strictly concave function, v is also strictly concave and then (12) holds with strict inequality. In this case, we can claim that as the Arrow-Pratt index of absolute risk aversion approaches zero from the right side, i.e. as the attitude towards risk becomes more neutral, the barrier rises. If the manager is a risk lover, with a strictly convex utility function, the barrier falls as his/her attitude towards risk becomes more risk neutral.

The effects of other parameters

There remain three parameters to examine. They are discount rates r , shift coefficients f , and diffusion coefficients σ . All of them are in Φ . Thus we rewrite Φ to $\Phi(x, a)$ where a stands for a shift parameter. Take the partial derivative of $w(x(a), a)$ with respect to a to get

$$\begin{aligned} w_a &= [u \Phi_a + u \Phi_a \Phi / (1 - \Phi)] / (1 - \Phi) \\ &= (\Phi_a / \Phi) w + [\Phi_a / (1 - \Phi)] w \\ &= \{ \Phi_a / [\Phi (1 - \Phi)] \} w. \end{aligned} \quad (13)$$

Taking the partial derivative of (13) with respect to x and substituting the first order condition, we have

$$w_{ax} = [\partial \{ \Phi_a / [\Phi (1 - \Phi)] \} / \partial x] w. \quad (14)$$

This equation shows that the effect of the shift parameter a on the optimal barrier is only because of its effect on Φ and not on w , and thus it is independent of the manager's attitude towards risk.

In general, it is difficult to analyze the bracket $[\cdot]$ in the right hand side in (14). Here we show the results for

a special case which was analyzed by MILLER and VOLTARE (1983). They considered the model where the shift coefficient f and the diffusion coefficient σ are constants and strictly positive. In this case, we can reduce Φ to

$$\Phi = \exp[-\eta(x - x_0)],$$

where $\eta = \sigma^{-2} [f + (f^2 + 2r\sigma^2)^{(1/2)}]$.

Calculate $w_{\eta x}$ to get

$$w_{\eta x} = -\{ \exp[-\eta(x - x_0)] - 1 - \eta(x - x_0) \} \exp[-\eta(x - x_0)] / \{ 1 - \exp[-\eta(x - x_0)] \} < 0.$$

The inequality is derived by using the MACLAURIN series of $\exp[-\eta(x - x_0)]$. Taking the partial derivatives of η with respect to r , f , and σ , respectively, we have

$$\partial \eta / \partial r = (f^2 + 2r\sigma^2)^{(-1/2)} > 0,$$

$$\partial \eta / \partial f = \sigma^{-2} [1 + f(f^2 + 2r\sigma^2)^{(-1/2)}] > 0,$$

$$\partial \eta / \partial \sigma = \eta^2 \sigma / (f - \eta \sigma^2) < 0.$$

(The inequality of last line follows from

$$f - \eta \sigma^2 = -(f^2 + 2r\sigma^2)^{(1/2)} < 0.)$$

From these inequalities, we conclude that in this special case, the optimal barrier rises if discount rates decrease, or shift coefficients decrease, or diffusion coefficients increase.

Although these results are similar to results from the deterministic Faustmann formula, this special case is not very realistic for forest management problems. Therefore, further analysis under more appropriate assumptions for the probability process is required.

DISCUSSION

Under some technical assumptions, this paper derived a stochastic Faustmann formula and investigated the properties of the optimal barrier under uncertainty. The results imply that most of the properties of the deterministic Faustmann formula for determining optimal cutting stock size, i.e. the optimal barrier, are carried over in stochastic circumstances. This suggests that forest economic analyses conducted under deterministic settings such as the assumption of perfect foresight are reliable.

By introducing uncertainty into a forest management model explicitly, we found that a forest manager's propensity for risk aversion promotes harvest in early stages. It is often observed in Japan that a manager who owns large forests tends to adopt longer rotation periods than a small forest owner. This might be explained by differences of their attitudes towards risk. It seems plausible that a small forest owner is more risk averse for his/her forest than a large forest owner since a large forest owner has several stands and could make a better portfolio than a small forest owner. Another aspect of forestry in Japan is that the cutting ages have become older since about 1965. The deterministic Faustmann formula suggests that this phenomena has been caused by the rise of wage rates of forestry and the stagnation of timber prices, which accompanied the economic growth in Japan. Our analysis of

the effect of planting costs indicates that this would still hold under uncertainty. Besides this analysis, the tendency might be explained partly by the manager's attitude towards risk. As the economy in Japan has developed, the money, asset, and insurance markets have matured. As a result, forest managers have been able to make better portfolios than previously. This indicates that forest managers in Japan have become less risk averse for their forests. If so, our results show that a change in the forest manager's propensity for risk aversion causes the optimal cutting stock size of the forest to rise.

LITERATURE CITED

- AKAO, K., (1988) : A theoretical study on a forest management under uncertainty (I). *Ringyo Keizai Kenkyu* **113**: 86-92 (in Japanese)
- BHATTACHARYYA, R. N. and SNYDER, D. L., (1988) : Stumpage price uncertainty and the optimal rotation of a forest: an application of Sandmo model. *Journal of Environmental Systems* **17**: 305-313
- BRAZEE, R. and MENDELSON, R., (1988) : Timber harvesting with fluctuating prices. *For. Sci.* **34**: 359-372
- CAULFIELD, J. P., (1988) : A stochastic efficiency approach for determining the economic rotation of a forest stand. *For. Sci.* **34**: 441-457
- CLARKE, H. R. and REED, W. J., (1989) : The tree-cutting problem in a stochastic environment. The case of age-dependent growth. *Journal of Economic Dynamics and Control* **13**: 569-595
- JOHANSSON, P. -O. and LÖFGREN, K.G., (1985): The economics of forestry and natural resources. Basil Black Well, Oxford, 292pp
- KUROKAWA, Y., (1991) : Study on the optimization of the silvicultural investment under uncertainty: a case study in Hyogo prefecture. *Trans. 102th Mtg. Jpn. For. Soc.*: 97-100 (in Japanese)
- MALLIARIS, A.G. and BROCK, W. A., (1982): Stochastic methods in economics and finance. North-Holland, Amsterdam, New York & Oxford, 303pp
- MILLER, R. A. and VOLTAIRE, K., (1980) : A sequential stochastic tree problem. *Economics Letters* **5**:135-140
- MILLER, R. A. and VOLTAIRE, K., (1983) : A stochastic analysis of the tree paradigm. *Journal of Economic Dynamics and Control* **6**: 371-386
- MIROSHINICHENKO, T. P., (1975) : Optimal stopping of the integral of a Wiener process. *Theory of Probability and Its Applications* **20**: 387-391
- REED, W. J., (1984) : The effects of the risk of fire on the optimal rotation of a forest. *Journal of Environmental Economics and Management* **11**: 180-190
- REED, W. J. and CLARKE, H. R., (1990) : Harvest decisions and asset valuation for biological resources exhibiting size-dependent stochastic growth. *International Economic Review* **31**: 147-169
- THOMSON, T. A., (1992) : Optimal forest rotation when stumpage prices follow a diffusion process. *Land Economics* **68**: 329-342

(Received 11 July 1996)

(Accepted 30 September 1996)