Motion Control of a Hydraulic Parallel-Link Servomechanism with Three Degrees of Freedom on a Plane (1st Report)

—Control of profile trajectory and acceleration waveform using preview control—

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We present a motion control method for a hydraulic parallel-link servomechanism with three degrees of freedom on a plane. The system controls both the profile-tracing path and the acceleration waveform of the end effector. The concept of acceleration feedback method is introduced to the preview control system particularly to improve the acceleration waveform. The objective of the preview control system is to minimize profile-tracing error and to suppress the noise vibration of the acceleration waveform of the end effector. The performance of the proposed controller is verified through experiments.

Key words: Parallel link, Motion control, Preview control, Electrohydraulic servomechanism

1. Introduction

The parallel-link mechanism with multidegree of freedom is commonly used in industry, for example in flight simulators, vibration-testing machines, and heavy-duty manipulators, which require high power levels and accurate positioning.

In this research, a motion control method for a hydraulic parallel-link servomechanism with three degrees of freedom on a plane is studied.

Sato and Tanabe presented a digital preview controller with a disturbance observer or H-infinity control to realize profile tracing using a parallel-link servomechanism [2].

They reported that the preview control system is effective for controlling the profile-tracing path of the parallel-link servomechanism. However, they paid no attention to acceleration waveform. In some practical applications of the parallel-link mechanism such as a vibration simulator, it is required to control both the profile-tracing path and the acceleration waveform of the end effector. However, the acceleration waveform control of the parallel-link mechanism has never been studied. In this paper, we report on the preview controller with state variable feedback and compare the effect of the cylinder force feedback and the acceleration feedback.

Experiments are carried out to verify the performance of the preview control system with acceleration feedback under appropriate weighting factor of the performance index for the control of the profile tracing and the acceleration waveform of the end effector.

2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>posture matrix</td>
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<tr>
<td>C</td>
<td>(\text{diag}[M, M, J_e])</td>
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<tr>
<td>e</td>
<td>cylinder displacement error</td>
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<tr>
<td>(f = [f_x, f_y, f_z]^T)</td>
<td>cylinder force</td>
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<tr>
<td>(G_i, G_x)</td>
<td>servo cylinder parameter</td>
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<tr>
<td>(H, Q)</td>
<td>weighting factor</td>
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<tr>
<td>(J)</td>
<td>performance index</td>
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<td>(J_e)</td>
<td>moment of inertia of the end effector</td>
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<td>(K = [K_x, K_y, K_z]^T)</td>
<td>servo amplifier gain</td>
</tr>
<tr>
<td>(K_s = [K_x, K_y, K_z]^T)</td>
<td>potentiometer gain</td>
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<tr>
<td>(M)</td>
<td>mass of the end effector</td>
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<tr>
<td>(M_s)</td>
<td>preview step number</td>
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<tr>
<td>(r = [r_x, r_y, r_z]^T)</td>
<td>reference posture</td>
</tr>
</tbody>
</table>
\[ r = [r_1, r_2, r_3]^T \] reference of cylinder displacement
\[ s \] Laplace operator
\[ T \] sampling time
\[ u \] control input
\[ x \] state variable
\[ X_e = [x_d, y_d, \theta_d]^T \] posture of the end effector
\[ X_s = [x_s, x_\alpha, x_\beta, x_\gamma]^T \] displacement of servovalve
\[ y \] output variable
\[ y_e = [y_e, y_\alpha, y_\beta, y_\gamma]^T \] cylinder displacement
\[ \delta_\alpha, \gamma \] hydraulic cylinder parameters

3. Electrohydraulic parallel-link servomechanism with three degrees of freedom on a plane

3.1 Motion of the parallel-link servomechanism

Figure 1 shows the coordinate system of the parallel-link mechanism with three degrees of freedom used in this paper. The coordinate of the object at point \( G \) on the end effector is expressed as \( [x_G(t), y_G(t), \theta_G(t)] \).

The geometrical equation is expressed as
\[
y_e(t) = [x_G(t) - \frac{l_2}{2} \cos \theta_G(t)]^2 + [y_G(t) - \frac{l_2}{2} \sin \theta_G(t)]^2
\]
\[
y_e(t) = [x_G(t) - \frac{l_2}{2} \cos \theta_G(t) - \frac{l_1}{2}]^2 + [y_G(t) - \frac{l_2}{2} \sin \theta_G(t)]^2
\]
\[
y_e(t) = [x_G(t) + \frac{l_2}{2} \cos \theta_G(t) - \frac{l_1}{2}]^2 + [y_G(t) + \frac{l_2}{2} \sin \theta_G(t)]^2
\]

The relationship between the piston velocity \( y_e \) and the velocity of the end effector \( \dot{X}_e \) is approximated by
\[
\dot{y}_e(t) = A (X_{es}, y_e) \dot{X}_e(t).
\]
where
\[
A = \begin{bmatrix}
x_\alpha - l \cos \theta_G & y_\alpha - l \sin \theta_G & x_\alpha \sin \theta_G - y_\alpha \cos \theta_G & l \\
x_\beta - l \cos \theta_G & y_\beta - l \sin \theta_G & x_\beta \sin \theta_G - y_\beta \cos \theta_G & l \\
x_\gamma - l \cos \theta_G & y_\gamma - l \sin \theta_G & x_\gamma \sin \theta_G - y_\gamma \cos \theta_G & l \\
x_\alpha + l \cos \theta_G & y_\alpha + l \sin \theta_G & x_\alpha \sin \theta_G + y_\alpha \cos \theta_G & l
\end{bmatrix}
\]

The equation of motion of the end effector is expressed as
\[
\frac{d^2}{dt^2} x_G(t) = A^T (X_{es}, y_e) f(t).
\]

The electrohydraulic servo cylinder exhibits nonlinear characteristics of properties such as the flow rate through the servovalve, variable cylinder volume, and unsymmetrical cylinder area. The dynamic motion of the servo cylinder is approximated by the following linear form
\[
\dot{f}(s) = G_e(s) [G_e x_e(s) - G_f y_e(s)].
\]
\[ G_e(s) \] means the effect of compressibility of oil and is assumed as follows.
\[
G_e(s) = \text{diag} \left[ \frac{\delta_x}{s + \gamma_x} \right]
\]
The displacement of the servovalve \( x_e(t) \) controlled by the proportional controller is expressed as
\[
x_e(t) = K_e K_r^{-1} [r_e(t) - y_e(t)].
\]

Figure 2 shows a block diagram of parallel-link servomechanism.

3.2 State space equation of parallel-link servomechanism

Considering computer sampling time delay, the state space equation and the output equation of discrete-time system are expressed as
\[
x(k+1) = A_n x(k) + B_n u(k-1)
\]
\[
y(k) = C_n x(k)
\]
where
\[
A_n = e^{A n} T, B_n = \int_0^T e^{A n} d n T B_v, C_n = C_v
\]
4. Controller Description

4.1 Error system of preview control system and performance index

The displacement error and first-order differential operator $\Delta$ are defined as follows

$$ e(k) = r(k) - y(k) $$

$$ \Delta e(k+1) = e(k+1) - e(k) $$

$$ \Delta x(k+1) = A_c \Delta x(k) + B_c \Delta u(k) $$

then

$$ \Delta e(k+1) = \Delta r(k+1) - C_c \Delta x(k+1) $$

$$ = \Delta r(k+1) - C_c A_c \Delta x(k) - C_c B_c \Delta u(k) $$

The error system including $\Delta u(k-1)$ as a state variable can be expressed as

$$ \begin{bmatrix} e(k+1) \\ \Delta x(k+1) \\ \Delta u(k) \end{bmatrix} = \begin{bmatrix} \Phi & G \\ 0 & A_c \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e(k) \\ \Delta x(k) \\ \Delta u(k-1) \end{bmatrix} + \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \Delta r(k+1). $$

Select the state variable $X_e(k)$

$$ X_e(k) = \begin{bmatrix} e(k) \\ \Delta x(k) \end{bmatrix}, \quad \Phi = \begin{bmatrix} I_n & -C_c A_c \\ 0 & A_c \end{bmatrix}, $$

$$ G = \begin{bmatrix} -C_c B_c \\ B_c \end{bmatrix}, \quad C_c = [I_c 0 0]. $$

The performance index $J$ is defined as follows

$$ J = \sum_{i=1}^n \left[ X_e(k) \right]^T Q X_e(k) $$

$$ + \sum_{i=1}^n \left[ \Delta u(k) \right]^T H \Delta u(k). $$

(14)

4.2 Design of the state-feedback controller

An integral servo control with state variable feedback is applied to the parallel-link servomechanism.

Assuming that the reference input is constant in Eq. (13), then $\Delta r(k+1) = 0$. Solving the optimal regulator problem of the error system for the performance index $J$, the control input $\Delta u(k)$ is given by

$$ \Delta u(k) = \left[ F_{o} F_{o1} \right] \begin{bmatrix} X_e(k) \\ \Delta u(k-1) \end{bmatrix} $$

$$ = F_{o1} e(k) + F_{o1} \Delta x(k) + F_{o1} \Delta u(k-1). $$

(15)

where

$$ F_{o} = -\left[ H + G' P G \right]^{-1} G' P \Phi G = F_{o1} \Phi G $$

$$ F_{o1} = -\left[ H + G' P G \right]^{-1} G' P \Phi $$

$P$ is a positive definite matrix.

$$ P = Q + \Phi' P \Phi - \Phi' P \left[ H + G' P G \right]^{-1} G' P \Phi $$

(16)

4.3 Design of the preview feedforward controller

When reference input values at the future step number $1 \sim M_k$ for time $k$ are known, they can be used to improve the performance of the system by applying the preview control method. The control input $\Delta u(k)$ can be expressed as

$$ \Delta u(k) = \left[ F_{o} F_{o1} \right] \begin{bmatrix} X_e(k) \\ \Delta u(k-1) \end{bmatrix} $$

$$ + \sum_{i=1}^{M_k} F_{o} \Delta r(k+1) $$

$$ = F_{o1} e(k) + F_{o1} \Delta x(k-1) + \sum_{i=1}^{M_k} F_{o} \Delta r(k+1) $$

(13)

$$ \Delta u(k) $$

$$ = F_{o1} X_e(k) + F_{o1} \Delta u(k-1) + \sum_{i=1}^{M_k} F_{o} \Delta r(k+1) $$

$$ = F_{o1} X_e(k) + F_{o1} \Delta u(k-1) + \sum_{i=1}^{M_k} F_{o} \Delta r(k+1) $$

Figure 3 Block diagram of the optimal preview control system
\[ F_d(x) = F_{d0} + F_{d1}x(k) + F_{d2}\Delta u(k-1) + \sum_{j=1}^{M_{d}} F_{d}(j) \Delta r(k+j). \]

where, \( F_d(j) \) is given by Eq. (18) to minimized the performance index \( J \).

\[ F_d(j) = -[H + G^T PG]^{-1}G(\xi_j)^{-1}PG \xi (j \geq 1) \]

\[ \xi = \Phi + GF. \]

Figure 3 shows a block diagram of the optimal preview control system which includes the integral servo system with state variable feedback.

### 4.4 Preview control system with state variable feedback

As shown in Figure 3, the preview control system has state variable feedback. As the feedback variable, it is possible to choose the cylinder force or the acceleration of the end effector. Because the relationship between the cylinder force and the acceleration of the end effector is linear as expressed by Eq. (3), the cylinder force feedback and the acceleration feedback should have the same effect. However, the experimental results are different for the cylinder force feedback system and the acceleration feedback system.

From the experimental results of the system with cylinder force feedback, noise vibration is observed in the acceleration waveform of the end effector, as shown in Figures 6(b), 7(b) and 8(b). The spring effect of the mechanical parts is assumed as a factor of the noise vibration in the acceleration waveform. However, the spring effect is neglected in the modeling of the mechanism to avoid the complexity for practical system design. Assuming that the acceleration signal is more sensitive to noise vibration than the cylinder force signal, the concept of the preview control system with acceleration feedback is applied to the system to suppress the noise vibration in the acceleration waveform.

### 5. Experiment

The experimental setup is shown as Picture 1.

Figure 4 and Picture 1 show the construction of the experimental setup which consists of a hydraulic pressure source and three servo cylinders with position feedback. Three servo cylinders, which are driven by signal \( r, \) connect the base and the end effector, and manipulate the end effector.

The controller (computer) : 2.4 GHz Pentium 4 PC, Sampling time \( t = 1 \) msec. The mechanical properties of the experimental apparatus : \( M = 56 \) kg, link length \( y_c = 0.468 \sim 0.706 \) m, \( y_{c1} = 0.439 \sim 0.648 \) m, \( y_{c2} = 0.422 \sim 0.614 \) m, \( h_c = 0.50 \) m, \( l_c = 0.70 \) m, \( l = 0.20 \) m, \( K_c = 3.00 \times 10^{-3} \) m/V, \( G_c = 4.32 \times 10^4 \) m/s²/Pa, \( G_c' = 21.18 \) m²/Pa, Flow rate of servovalve = 2.5 × 10⁻³ m³/sec, \( A_t = 314 \) mm², \( A_c = 235 \) mm², Supply pressure = 6.87 MPa, \( K_c' = 2.52 \times 10^{-7} / 2.87 \times 10^{-7} / 3.13 \times 10^{-7}. \)

For a given preview step \( M_e \) and weighting factors \( Q \) and \( H \) of the preview control system, the optimal control theory yields the optimal values of \( F_{d0}, F_{d1}, F_{d2} \) and \( F_{d3} \) for minimizing the performance index \( J \).

To examine the effect of the preview step number, the value of the performance index of the designed controller is calculated as shown in Figure 5. Because
the acceleration waveform is very sensitive to $Q$ and $H$. An appropriate value should be chosen in the system design. In this research, $Q$ and $H$ are chosen as follows.

$Q = \text{diag}(0, 0.01, 0.01, 0, 0.01, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

$H = \text{diag}(1, 1, 1)$

From the value of $J$ shown in Figure 5, the preview step number is chosen as $M_p = 100$.

We applied the design method explained in section 4 with cylinder force feedback or acceleration feedback to the motion control system of the end effector. Circular, square and triangular paths with $r_s = 0$ are used as test patterns.

The cylinder force is calculated using the cylinder pressure signals detected by the pressure sensors attached to the cylinder. The acceleration signals are detected directly by three accelerometers attached to the end effector.

The performance of the preview control system is compared via two different systems: Case I with the cylinder force feedback and Case II with the acceleration feedback.

Figures 6, 7 and 8 show the experimental results of

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**Figure 6** Experiment (Circular path)

(a) Trajectory (Case I, II)

(b) Acceleration waveform (Case I)

(c) Acceleration waveform (Case II)

**Figure 7** Experiment (Square path)

(a) Trajectory (Case I, II)

(b) Acceleration waveform (Case I)

(c) Acceleration waveform (Case II)
Comparing the experimental results of the acceleration waveform shown in Figures 6(b), 7(b) and 8(b) with Figures 6(c), 7(c) and 8(c), respectively, the noise vibration magnitude of the acceleration waveform in Case II is smaller than that in Case I. The results in this experiment clearly show that the noise vibration was suppressed by applying the acceleration feedback method.

6. Conclusions

We proposed a preview control method with state variable feedback to control both the profile tracing and the acceleration waveform of the parallel-link servomechanism with three degrees of freedom on a plane. From the experimental results, the spring effect of mechanical parts is assumed to induce the noise vibration in the acceleration waveform, which is neglected in the modeling of the mechanism to avoid complexity of practical design. Comparing the cylinder force feedback method with the acceleration feedback method, the latter is more effective in suppressing the noise vibration in the acceleration waveform. The state variable acceleration feedback method with a simple mechanical model, neglecting the spring effect of the parts under appropriate values of the weighting factor \( Q \) and \( H \), can be applied to the practical design of the preview control system for the parallel-link mechanism. The performance of the preview control method was investigated through experiments.

References
