研究論文

ON/OFF VALVE IMPLEMENT OF CONTINUOUS VSC TO
A PNEUMATIC CYLINDER*

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A pneumatic cylinder system has been difficult in position control due to the nonlinearity of the air, friction on packing seal, and parameter variations. Variable Structure Control (VSC) with a proportion valve has been used to overcome the difficult. However, VSC with an ON/OFF valve is not extensively investigated so far. Therefore, friction compensation using VSC with an ON/OFF valve is proposed in this research. VSC with an ON/OFF valve is designed based on VSC for the proportional one. The experimental results provided the accuracy with 0.1mm as same as using a proportional valve with chattering reduction. The results imply that the ON/OFF valve controlled by VSC is a possible replacement of proportional valve for conventional pneumatic cylinder control system.

Key words : Chattering Reduction, On-Off Control, Servo Valve Pneumatic cylinder

Nomenclature

m	: Payload
x	: Displacement of Load mass
v	: Velocity
a	: Acceleration
L	: Stroke length
L_0	: Initial displacement
F_p	: Cylinder Force
F_f	: Friction Force
A_1	: Piston sectional area
A_3	: Rod cross-sectional area
A_2	: Different Area between A_1 and A_3
A_{ei}	: Proportional-valve Effective area
$\hat{A}_{ei}$	: ON/OFF-valve Effective area
u	: Control action of proportional valve
$\hat{u}$	: Control action of ON/OFF valve
V_i	: Cylinder volume of i^{th} chamber
V_{0i}	: Cylinder dead volume of i^{th} chamber
Q_i	: Air Mass flow rate
P_i	: Absolute pressure of i^{th} chamber
P_a	: Atmosphere pressure
P_s	: Absolute Air supply pressure
P_{cr}	: Critical pressure ratio
R	: Gas constant
T	: Supply air temperature

k	: Specific heat ratio
C_*	: Fluid Constants
ζ	: Fictitious Disturbance
$\lambda, \psi, \phi, \phi$	: Positive Constant
σ	: Step function
Subscript	
i	: Chamber index 1, 2
d	: Desired reference value

1. Introduction

Though, in factory application, a pneumatic cylinder is widely used as an endpoint stop actuator so far, in a field of new robotics such as a disability assisting, it is expected to be controlled as adjustable stroke. However, the position control of a pneumatic cylinder has been difficult due to nonlinearity of the compressibility of air, external disturbances due to friction on packing seals, and so on. Nevertheless, the advantages have been driving researchers to seek the solution for making pneumatic actuators competitive servo systems¹⁾.

Many advanced controllers have been applied to control of pneumatic systems and in this research, Variable-Structure Control (VSC) is chosen as a controller due to the advantage of low sensitivity to plant parameter variations and disturbances, which eliminates the necessity of exact modeling. Some VSCs of pneumatic cylinders using proportional valves have been proposed with good performance, e.g.^{2) 3) 4)}. On the other hand, an ON/OFF valve is widely used in industrial applications because the cost is low and more robust to the environment change. Therefore, a pneumatic cylinder using ON/OFF valves is important in the

*Manuscript received August 17, 2007

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viewpoint of usefulness and price advantage in the market. However, VSC with ON/OFF valves is not extensively investigated so far. For example, the approximation of control laws using Variable Structure Methods in hydraulic servo systems control⁵⁾ has been evaluated, position control of a pneumatic actuator using ON/OFF solenoid valves by using PID control technique with novel PWM had been proposed for a servo system, and the results have an overshoot when the mass was changed⁶⁾.

In this paper, a simplified equation of chamber pressure for both cases when the piston extends and retracts is proposed. Based on the model, we design the VSC controller with full state feedback using ON/OFF valve based on proportional valve design method. Closed-loop stability is analyzed by Lyapunov's stability method. Chattering phenomenon reduction has been shown explored experimentally. The effectiveness of the proposed method is examined experimentally.

2. Pneumatic Cylinder Models

2.1 Equation of Motion

Fig. 1 Pneumatic Cylinder System

Figure 1 shows the schematic of the system: the pneumatic cylinder mounted to two 3/2-way SMC VEF Series valves for Proportional valve and 3/5-way SMC SY3000 Series for ON/OFF valve with a maximum operating pressure (gauge pressure) of 0.5 MPa, two pressure sensors for P_1 and P_2 , and magnetic-type position sensor for x with resolution of 0.1 mm. The equation of motion of the pneumatic cylinder system can be expressed as:

$$a = m^{-1} (A_1 P_1 - A_2 P_2 - A_3 P_a - F_f) \quad (1)$$

m is Piston+Rod+Load mass, x is piston position, F_f is static friction force, $P_{1,2}$ is the absolute pressure in pneumatic cylinder chamber, P_a is the ambient pressure, $A_{1,2}$ is the

piston effective area, and A_3 is the cross-sectional area of the rod.

2.2 Equation of Chamber Pressure

The equation of pressure changing inside the chamber is determined with an ideal gas law, the conservation of mass equation, and the energy equation⁷⁾. Considering the control volume V , pressure P , temperature T , and assumed in isothermal system. For notational simplicity, the two models of the mass flow rate through an valve orifice area A_{ei} (Choked-flow and Subsonic-flow)³⁾ is denoted by adding the unit-step function of the effective area as shown below:

$$\begin{aligned} \dot{P}_i &= \frac{k}{V_i} (RTQ_i - P_i \dot{V}_i) \\ &= \alpha_i [K_q \{ \sigma((-1)^{(i+1)} A_{ei}) q(P_s, P_i) - \sigma((-1)^i A_{ei}) q(P_i, P_a) \} A_{ei} + (-1)^i P_i A_i \dot{x}], \quad i=1, 2 \end{aligned} \quad (2)$$

where

$$\alpha_i = \frac{k}{V_{0i} + A_i \left(\frac{L}{2} + (-1)^{i+1} x \right)}, \quad K_q = RC_f \sqrt{T}, \quad (3)$$

$$q(P_{up}, P_{dn}) = \begin{cases} C_a P_{up} & \text{for } \frac{P_{dn}}{P_{up}} \leq P_{cr} \\ C_b P_{up} K_p & \text{for } \frac{P_{dn}}{P_{up}} > P_{cr} \end{cases}, \quad (4)$$

$$K_p = \left(\frac{P_{dn}}{P_{up}} \right)^{(1/k)} \sqrt{1 - \left(\frac{P_{dn}}{P_{up}} \right)^{(k-1)/k}} \quad (5)$$

$$V_i = V_{0i} + A_i \left(\frac{L}{2} + (-1)^{i+1} x \right), \quad \dot{V}_i = (-1)^{i+1} A_i \dot{x}. \quad (6)$$

$\sigma(\cdot)$ represents the step function, $q(\cdot)$ represents the air flow entering and leaving the chamber. C_f is a non dimensional discharge coefficient, C_a and C_b are fluid constants³⁾, P_{up} is upstream pressure, P_{dn} is downstream pressure, and P_{cr} is critical pressure. Subscript $i=1, 2$ is the cylinder chamber index, V_{0i} is the inactive volume, A_i is the piston effective area, and L is the piston stroke.

Fig. 2 ON/OFF valve output $\hat{A}_{ei}(t)$ can be represented as Proportional valve output $A_{ei}(t)$ with a fictitious disturbance $\zeta(t)$.

2.3 Equation of Valve effective area

The model of effective area A_e of the Proportional valve can be obtained experimentally. The polynomial curve-fitting method is used to determine the model from experimental data. Resulting in

$$A_{ei}(u_i) = \begin{cases} a_1 + a_2 u_i + a_3 u_i^2 & \text{for } u > u_0 \\ 0 & \text{for } u = u_0 \\ a_4 + a_5 u_i + a_6 u_i^2 & \text{for } u < u_0 \end{cases} \quad (7)$$

$a_1 \sim a_6$ are valve constants, u_0 is equilibrium voltage. The ON/OFF valve can be modeled as :

$$\hat{A}_{ei} = \begin{cases} A_{ei}^+ & \text{for } u > 0 \\ 0 & \text{for } u = 0 \\ A_{ei}^- & \text{for } u < 0 \end{cases} \quad (8)$$

where $\hat{A}_{ei}$ is the effective area of the ON/OFF valve. The relation between $A_e(u_i)$ and $\hat{A}_e(u_i)$ can be expressed as :

$$\hat{A}_{ei}(u_i) = A_{ei}(u_i) + \zeta(t) \quad (9)$$

where $\zeta(t)$ is a fictitious disturbance as shown in Fig. 2 and this is represented as an additive disturbance in Fig. 3. Note that $\zeta(t)$ is only used in stability analysis in Section 4, and not used in control.

3. Variable Structure Controller Design

3.1 VSC design with Proportional Valve

By following the design method in Bonchis's⁵⁾, we have designed the control law for the case when piston extends and retracts using Eqs. (1), (2), and (7). From equation of motion as shown in Eq. (1), denoting : velocity $\dot{x} = v$, acceleration $\dot{v} = a$, then

$$\dot{a} = \frac{1}{m} (A_1 \dot{P}_1 - A_2 \dot{P}_2) \quad (10)$$

Substituting the equation of chamber pressure Eq. (2) into Eq. (10), we can rewrite Eq. (10) in nonlinear state equation as :

$$\dot{a} = f(x, v, P_1, P_2) + b(x, P_1, P_2, A_{ei}) \cdot A_e(u) \quad (11)$$

where

$$f(\cdot) = -\frac{v}{m} (\alpha_1 A_1^2 P_1 + \alpha_2 A_2^2 P_2) \quad (12)$$

$$b(\cdot) = \frac{K_g}{m} [\alpha_1 A_1 \{ \sigma(A_e) q(P_s, P_1) - \sigma(-A_e) q(P_1, P_a) \} - \alpha_2 A_2 \{ \sigma(-A_e) q(P_s, P_2) - \sigma(A_e) q(P_2, P_a) \}] \quad (13)$$

where $A_e(u) = A_{ei}(u) = -A_{e2}(u)$. Now, define $e_x = x_d - x$ be the error in the piston position x with desired value represented by subscript d . For the system Eq. (11) the switching surface $s = 0$ of VSC is defined^{10) 11)} :

$$s = \left(\frac{d}{dt} + \lambda \right)^{n-1} \cdot e_x \quad (14)$$

where λ is a strictly positive constant that effects to desired rising time of the closed-loop system, $n=3$ for our third order system. Then Eq. (14) becomes :

$$s = e_a + 2\lambda e_v + \lambda^2 e_x \quad (15)$$

$$\dot{s} = \dot{e}_a + 2\lambda \dot{e}_v + \lambda^2 \dot{e}_x \quad (16)$$

where $e_v = \dot{x} = v_d - v$ represents the tracking error of the velocity and e_a represents the acceleration error and these are obtained by a numerical differentiation method of position data. The control action can be determined from the condition of $\dot{s}=0$, taking Eqs. (1) and (11) into Eq. (16), resulting in

Fig. 3 Variable Structure Controller Block Diagram with ON/OFF valve

$$A_e(u) = b^{-1} \left(-f + \dot{a}_d + 2\lambda \dot{a}_d + \lambda^2 e_v - \frac{2\lambda F_p}{m} \right) + b^{-1} \frac{2\lambda}{m} F_f \quad (17)$$

$$F_p = A_1 P_1 - A_2 P_2 - A_3 P_a \quad (18)$$

Note that A_e is invertible as shown in Eq. (7). To eliminate the effect of the static friction F_f , it can be replaced by signum function of switching surface (14) as shown bellows :

$$F_f \rightarrow \phi \operatorname{sgn}(s) \quad (19)$$

$$\phi > |F_f| > 0 \quad (20)$$

where, ϕ is positive constant of the friction boundary. Now, we obtained the control action u as :

$$u = A_e^{-1} \left(u_{eq} + b^{-1} \frac{2\lambda}{m} \phi \operatorname{sgn}(s) \right) \quad (21)$$

where

$$u_{eq} = b^{-1} \left(-f + \dot{a}_d + 2\lambda \dot{a}_d + \lambda^2 e_v - \frac{2\lambda F_p}{m} \right) \quad (22)$$

3.2 VSC design with ON/OFF valve

To design the controller by using ON/OFF valve based on Proportional one, the control action in Eq. (11) will be replace by Eq. (9) and then Eq. (11) becomes :

$$\dot{a} = f + b \cdot (A_e(u) + \zeta) \quad (23)$$

The VSC (21) becomes :

$$u = A_e^{-1} \left(u_{eq} + b^{-1} \frac{2\lambda}{m} \phi \operatorname{sgn}(s) - \phi \operatorname{sgn}(s) \right) \quad (24)$$

where

$$\phi > |\zeta| > 0 \quad (25)$$

The control action for the pneumatic cylinder with ON/OFF valve becomes :

$$\hat{u} = \begin{cases} u^+ & \text{for } u > 0 \\ 0 & \text{for } u = 0 \\ u^- & \text{for } u < 0 \end{cases} \quad (26)$$

$\hat{u}$ is the control signal for the ON/OFF-type of valve, u^+ is fully control action voltage when the piston extends, and u^- when the piston retracts. Figure. 3 shows the Variable Structure Controller Block Diagram.

4. Stability Analysis of VSC with ON/OFF-valve

To analyze the stability of close-loop control, by using the Lyapunov's function^{8) 9)} candidate,

$$v = 0.5s^2 \quad (27)$$

a stability condition is given by

$$\dot{v} = s\dot{s} \leq 0. \quad (28)$$

Taking Eqs. (1), (11), and (24) into Eq. (16) and assumed that the desired value is step-type function. The treatment of the another functions of desired value are similar. Now, the first time derivative of Eq. (27) can be obtained as

$$\dot{v} = s \left(-\frac{2\lambda}{m} (\phi \operatorname{sgn}(s) - F_f) + b(\phi \operatorname{sgn}(s) - \zeta) \right) \quad (29)$$

Taking Eqs. (13), (15), (20), and (25) into consideration, the condition of (28) is satisfied if

$$\left| \frac{2\lambda}{m} (\phi \operatorname{sgn}(s) - F_f) \right| \geq |b(\phi \operatorname{sgn}(s) - \zeta)| \geq 0. \quad (30)$$

Note that, inequality (30) with $\zeta = \phi = 0$, which implies that the proportional valve case makes the inequality (30) always be satisfied, is identical to the stability condition of VSC with a proportional valve to eliminates the effect of the static friction⁸⁾.

5. Experimental Results

5.1 VSC with Proportional Valve

The valves in Table.1 are used in the experiment.

Table 1 Proportional VS ON/OFF Valve

Valve	Ae [mm ²]	RT* [ms]	Price [USD]
Proportional	1.0	30	180
ON/OFF	1.0	10	80

* RT : Response Time in catalog

5.1.1 Control Parameter Determination

Two major parameters λ and ϕ of the controller in Eq. (21) are determined under the stability condition of Eq. (30) with $\phi = \zeta = 0$. The value of $\phi = 30$ can be determined by expression of Eq. (20), with experimental consideration of F_f . The value of $\lambda = 50$ can be obtained by trial and error technique in tuning phase that depends on desire rising time of the closed-loop system. The reference position of closed-loop system control is a Square-wave with 0.05 m amplitude and 0.25 Hz frequency and Table 2 shows the experimental parameters. The designed controller (24) provided position response curve with chattering phenomenon as shown in Fig. 4. The chattering phenomenon¹⁰⁾ is in bound 20 (see Fig. 4 bottom), and this value of switching surface will be discuss in next section.

Fig. 4 Position Response using Proportional-valves without chattering reduction

5.1.2 Chattering Reduction

In order to reduce the chattering phenomenon, the signum function in control action Eq. (24) can be replaced by one of four functions as follows :

- Smoothing Function

$$\operatorname{sgn}(s) \rightarrow \frac{s}{|s| + \delta} \quad (31)$$

- Dead-Zone Function

$$\operatorname{sgn}(s) \rightarrow \begin{cases} +1, & \text{for } s > \delta \\ 0, & \text{for } -\delta \leq s \leq \delta \\ -1, & \text{for } s < -\delta \end{cases} \quad (32)$$

- Hyperbolic Tangent Function

$$\operatorname{sgn}(s) \rightarrow \tanh\left(\frac{s}{\delta}\right) \quad (33)$$

Table 2 Experimental Parameters

Parameters	Value	Unit
A_1	314	mm ²
A_2	235	mm ²
A_3	78.5	mm ²
m	3.0	kg
L	0.2	m
V_{01}	314	mm ³
V_{02}	235	mm ³
R	287	N·m/kg·K
C_f	0.23	—
C_a	0.04	$\sqrt{1/K}$
C_b	0.16	$\sqrt{1/K}$
T	293	K
k	1.4	—
P_a	0.1	MPa
P_s	0.6	MPa
P_{cr}	0.5	—

Fig. 5 Chattering Reduction Methods with same δ

• Saturation Function

$$\text{sgn}(s) \rightarrow \text{sat}\left(\frac{s}{\delta}\right) = \begin{cases} +1, & \text{for } s > \delta \\ \frac{s}{\delta}, & \text{for } -\delta \leq s \leq \delta \\ -1, & \text{for } s < -\delta \end{cases} \quad (34)$$

Figure 5 shows the output of above chattering reduction methods with same value of δ . Now, let us investigate the performance of each chattering reduction method using δ is 20, and Fig. 6 shows the results.

Table 3 Comparison of Chattering Reduction Methods

Methods	Settling time [s]	error [mm]
Smooth Function	0.36	0.4
Dead-Zone Function	0.31	4.0
Hyperbolic Tangent	0.39	0.2
Saturation Function	0.30	0.1

Observing that, all chattering reduction methods can reduce the chattering phenomenon. The differences in position accuracy can be discussed as follows: the position response curve using *Smooth* and *Hyperbolic tangent* function when the switching surface is out of boundary layer neighboring the switching surface¹⁰ are less than one (see Fig. 5), that have been an effect to the control action is lower than properly. For the *dead-zone* and *saturation* function has no

Fig. 6 Position Response using Proportional-valves with chattering reduction

such an effect as described above but the *dead-zone* function is zero when s is in the bound of switching surface that also degrade the control action and provide the largest steady-state error. The comparison of VSC with chattering reduction methods are shown in Table 3. Consequently, the VSC with *saturation* function is an appropriate improving method to reduce the chattering phenomenon since it provides lower rise time, smaller error, and smoother motion than other three methods.

5.2 VSC with ON/OFF Valve

5.2.1 Control Parameter Determination

Three major parameters of the controller in Eq. (24) are also determined under the stability condition of Eq. (30) with the same plant parameters as VSC with proportional valve with chattering reduction method of *saturation* function. For λ and φ are same as VSC with proportional valve, and $\varphi = 1.01$ can be determined by expression (25) with consideration of maximum value of the effective area of the valve with 1.0 mm².

5.2.2 Comparison to Proportional Valve

The experimentals with ON/OFF valves are carried out by using the same reference position of Square wave as VSC with proportional valve in Fig. 6. The Displacement [m] result are shown in Fig. 7.

Fig. 7 Position Response using ON/OFF valves with chattering reduction of saturation function

VSC with ON/OFF valve provided good performance of position controls same as the using of proportional valves. The comparison of both results between proportional and ON/OFF valves in detail are shown in Fig. 8. Observing that, the VSC with both type of valves provided almost the same settling time and the steady-state error is 0.1 mm for VSC with proportional valve, 0.6 mm for VSC with ON/OFF valve.

5.2.3 Robustness Investigation

The robustness investigation is carried out as follows. Figure 9 above ($m = 3.0\text{kg}$) shows the experimental results

Fig. 8 VSC with Proportional and ON/OFF valve with chattering reduction of saturation function

Fig. 9 VSC with ON/OFF valve Robustness In vestigation in Variation of references input and payload

with square-wave position reference signal of 0.05m, 0.01m amplitude and 0.25 Hz frequency. Dash-dot line represents the position response of 0.01m reference input and Dash-dot-dot line represent the position response of 0.05m reference input. For both of position response curves, the VSC with ON/OFF valves still provided good performance in position control, and the steady-state error is in bound of the allowable to larence (2% of reference) as shown in Table 4

Table 4 Comparison of Variation of Reference Input

Reference Position [mm]	Steady-state error [mm]
10	0.1
50	0.6

And also the robustness of designed VSC in payload and input variation are investigated. The nominal mass of the system (piston, rod, linear slide, and payload) is $m=3\text{kg}$. The remove of the payload decreased the mass by 10 times to $m=0.3\text{kg}$. Figure 9 below ($m=0.3\text{kg}$) shows position response with the same reference signal as in Fig. 9 above ($m=3\text{kg}$). Observing the responses, the designed VSC provided similar rise time to nominal one and the overshoot

phenomenon⁶⁾ is not appeared. Even though the steady-state error of VSC with ON/OFF valve is bigger than proportional valve, the VSC with ON/OFF valve having a useful on price advantage (Table. 1) .

6. Conclusion

The Variable Structure Control (VSC) with ON /OFF valve for eliminate the effect of the static friction have been proposed based on VSC with Proportional valve. A fictitious disturbance in control action have been also proposed to examine robustness theoretically as in Eq. (30). Experimentally identified model with ordinary equations has been carried out. By using the models, the VSC with ON/OFF valve has been designed based on proportional valve method. Closed-loop stability analysis of VSC with ON/OFF valve using Lyapunov's function has been carried out. The chattering phenomenon reduction by using the saturation function has been introduced by comparing to other methods experimentally. The comparison of VSC using proportional and ON/OFF valve with the same of control parameters have been carried out and both have comparable performances. The robustness of designed VSC with ON/OFF valve for the variation of reference input and payload have been investigated and experimental results provided similar response to nominal ones, which implies good robustness of the proposed method. Therefore, by using the proposed VSC with ON/OFF valve, the ON/OFF valves can be a replacement of the proportional valves in pneumatic cylinder systems. A further issue is detailed interpretation of the robustness condition in Eq. (30).

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