Influence of Cross-Sectional Configuration on the Longitudinal Vortex Excitation of the Upstream Cylinder in Cruciform Two-Cylinder System*

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Abstract
In earlier works by the present authors, it was shown that a large cross-flow excitation is induced to a circular cylinder by setting another cylinder downstream in a cruciform arrangement with a certain gap between them, caused by longitudinal vortices shedding periodically around the crossing. In this work, influence of the cross-sectional configuration of cylinders forming a cross on the longitudinal vortex excitation was investigated by wind tunnel experiments. Three systems, i.e. a two-circular-cylinder, a circular-cylinder/strip-plate and a square-cylinder/strip-plate system, with essentially equal structural parameters were compared. In the case of the circular-cylinder/strip-plate system, the trailing vortex excitation occurs over much wider velocity region, while the necklace vortex excitation is indefinite, as compared with the two-circular-cylinder system. In the case of the square-cylinder/strip-plate system, vortex excitation occurs in a certain velocity region of the galloping of single cylinder attributed to the necklace vortex.

Key words: Flow Induced Vibration, Flow Control, Flow Visualization, Wind Tunnel Experiment, Vibration Suppression, Vibration Generation

1. Introduction
Since the Kármán vortex excitation of a cylindrical body has caused many serious accidents in mechanical and structure engineerings, there are numerous experimental, numerical and theoretical works on this phenomenon, even when confined to the most elementary case of a circular cylinder. However, due to complicated interaction between body motion and vortex structure which works as a non-linear feed-back loop to the oscillation behavior, its mechanism is still unclear and left to be investigated. It is in general quite difficult to predict the oscillation in practical situations since there are so many parameters, such as Reynolds number, mass ratio, Scruton number and aspect ratio,
together with factors which are in reality beyond control such as non-uniformity of flow, turbulence intensity and end effect. From the viewpoint of practical application, various methods for controlling of the Kármán vortex excitation have been proposed so far, such as suction holes on the surface of the body, tripping wires near separation points, a splitter plate in the wake, and so on\(^4,5\).

Tomita et al. \(^6\) reported that the acoustic noise from a cylinder is suppressed by setting another cylinder downstream in cruciform arrangement with a certain gap between them as shown in Figure 1. Inspired by Tomita’s work, the present authors found that the Kármán vortex excitation of the upstream cylinder in the cruciform system is effectively suppressed when the gap-to-diameter ratio is less than around 0.5. At the same time, it was found that two types of longitudinal vortices, i.e. necklace vortex (Figure 1(a)) or trailing vortex (Figure 1(b)), shed periodically depending on the gap-to-diameter ratio and that these longitudinal vortices induce resonant excitation similar to the Kármán vortex excitation over respective velocity regions, several times higher than that of the latter\(^7,8,9\). Although the cruciform arrangement as shown in Figure 1 is most elementary as three dimensional ones and seen in many practical uses, reports on the flow around two cruciform cylinder system is rather few. Zdravkovich \(^10\) investigated the flow around two circular cylinders in contact forming a cross and showed that a trailing vortex with a foot on the upstream cylinder surface is formed at every corner of the cross. Fox investigated wake characteristics of fixed cruciform circular cylinder and square cylinder systems with the gap \(s\) between the two cylinders\(^11,12\). In the latter, he observed two longitudinal vortices similar to those shown in Figure 1 which shed periodically near the cross, and measured their shedding frequencies. However, no research work has been reported so far\(^13\) on the excitation induced by the longitudinal vortices shedding from the cruciform cylinder system, except those ones by the present authors’ group. Since this newly found vortex excitation can induce hazardous vibration when one sets another cylindrical body downstream of a cylinder otherwise stationary, it is required to investigate the conditions for the occurrence of this phenomenon. The specific aim of this work is to investigate the difference in the cross-flow oscillation of the upstream cylinder in a cruciform system caused by the longitudinal vortices when their cross sectional geometries are different. For this purpose, in addition to a circular cylinder, a square cylinder is tested as the upstream body since it has fixed separation points and is known to induce galloping, and a strip-plate as the downstream body since it is expected to have stronger interference effect with simple geometry. Thus, three systems with different configurations, (I) two-circular-cylinder, (II) circular-cylinder/strip-plate and (III) square-cylinder/strip-plate, having essentially equal structural parameters are tested in wind tunnel experiment.

![Arrangement of the two-circular-cylinder system and longitudinal vortices](image)

**Fig. 1** Arrangement of the two-circular-cylinder system and longitudinal vortices

**Nomenclature**

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\begin{align*}
d & \quad \text{characteristic length of the system, i.e. the diameter of circular cylinder, or the side of square cylinder (}= 26 \text{ mm}) \\
d' & \quad \text{diameter of downstream circular cylinder (}= 26 \text{ mm})
\end{align*}
\]
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Before investigating the vibration behavior, longitudinal vortices were observed by
flow visualization and their shedding frequencies were measured for fixed counterparts. To
observe configurations of the longitudinal vortices, the dye-streak technique was applied to
fixed systems with \( d = 10 \) or 22 mm in water tunnel with a 200 mm square cross section. For
the square-cylinder/strip-plate system, the tuft grid method was applied in the wind tunnel
experiment.

Arrangement of the wind tunnel experimental apparatus and the coordinate system are
shown in Figure 2. The test section was 1 m long with 0.32 m square cross-section. The
upstream cylinder passed through slots on the walls of the measuring section and was fixed
rigidly or supported elastically at both ends outside the measuring section. In the latter
setup, two twin-plate-springs were used as shown in Figure 2 so as to make the motion of
the upstream body pure cross-flow mode. Two single-plate-springs were also used in cases
where the effect of slight rotation of the body supported by a cantilever was insignificant
\(^{14}\). End plates were attached to the cylinder to remove influence of flow through slots \(^{15}\).
Effective span wise length of the upstream bodies defined as the distance between the end
plates was 318 mm. The downstream cylinder was mounted rigidly on a traversing table set
beneath the measuring section, which enable to adjust the gap \( s \) within a preciseness of 0.05
mm. The downstream bodies were spanned over the whole height of the measuring section.

Characteristics of the elastically supported test systems in wind tunnel experiment are given in Table 1. The characteristic length, $d$, of the three systems is fixed at a fixed value of 26 mm, since most of measurements in previous works were carried out for this value of $d$ and influence of the aspect ratio was shown insignificant \(^{(9)}\). The natural frequency $f_n$ and logarithmic damping factor $\delta$ were obtained by free damping oscillation in air otherwise at rest. The natural frequency was sensitive to the assembling conditions of the spring structure, and it was adjusted within a narrow region for the three systems. The damping factors of the three systems are well lower than the critical value. Thus, the differences in the oscillation behavior of the three systems are attributed to the influence of the configuration since the other structural parameters are considered to be essentially equivalent.

The free flow velocity $U$ was measured by the ring-type velocimeter developed in order to measure air flow velocity as low as 1 m/s within an error of 3 % with high repeatability \(^{(16)}\). The $x$-component of velocity, $u$, was detected by a hotwire probe placed at an appropriate location to detect the periodic shedding of vortices \(^{(8)}\). The displacement of the upstream cylinder, $Z$, was measured at the ends of the cylinder outside the measuring section by laser displacement sensors (Figure 2). The vortex shedding frequency and the vibration frequency were obtained from spectra of $u$ and $Z$, respectively. In some cases of System II, the velocity spectra $S_u$ did not show a definite peak corresponding the vortex shedding. To cover such cases, the lift force for the fixed system was also measured and its spectrum was used to determine the vortex shedding frequency.

<table>
<thead>
<tr>
<th>Table 1 Characteristics of three test systems in wind tunnel experiment</th>
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<tr>
<td>Test systems (Upstream body/Downstream body)</td>
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<tr>
<td>I Two-circular-cylinder</td>
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<tr>
<td>II Circular-cylinder/strip-plate</td>
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<tr>
<td>III Square-cylinder/strip-plate</td>
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3. Experimental results

3.1. Flow visualization

Photographs of the two types of longitudinal vortices around a fixed two-circular-cylinder system are reproduced in Figure 3. In this system, the necklace and the trailing vortices are most clearly observed at $s/d = 0.28$ and 0.08, respectively, although the values of $Re$ are much lower than those corresponding with the velocity measurement in wind tunnel experiment. When the downstream cylinder is replaced by the strip-plate, i.e. in the case of the circular-cylinder/strip-plate system, both the necklace and the trailing vortices are observed to form as shown in Figure 4. However, the configurations of each vortex are considerably different between the two systems. A remarkable difference is that the periodic shedding of the necklace vortex is not definitely observed by the visualization in the case of the circular-cylinder/strip-plate system although its contour is more clearly observed in Figure 4 (a) than in Figure 3 (a).

Compared with the above two cases, flow around the square-cylinder/strip-plate is very
complicated as shown in Fox’s report\(^{(12)}\). Hence, the longitudinal vortex was visualized in the wind tunnel by tuft-grid method when the vortex shedding was synchronizing with the cylinder oscillation as described later. In Figure 5, the cross section of longitudinal vortex is clearly observed at the antiphase instants, Figure 5 (a) and (b), in a period of cylinder displacement \(Z\). It is clearly seen that the pattern is symmetric about the \(x-z\) plane both in photos Figure 5 (a) and (b) and the vortex patterns at an instant is observed only one side of positive or negative \(Z\) region. However, it cannot be discerned whether the longitudinal vortex in Figure 5 is the necklace or the trailing vortex.

### 3.2. Vortex shedding frequency from fixed system

The hot-wire probe was set at a position where peaks in spectra \(S_c\) corresponding with the vortices as observed in Figure 5 appeared most clearly, and the gap \(s\) was varied while the velocity \(U\) was kept constant. Figure 6 (a) shows change of \(S_c\) thus obtained for System I. In the figure, \(S_c\) has a sharp peak at an almost fixed frequency when \(0.25 < s/d < 0.5\). When \(s/d < 0.25\), \(S_c\) has a definite but a lower peak at a considerably lower and unstable frequency. These results infer that the flow regime near the cross of two-circular cylinder system changes abruptly from the necklace vortex to the trailing vortex at \(s/d = 0.25\) when the gap is decreased.

In the case of System II, the velocity spectra \(S_c\) showed no definite peak resulting in...
scattered data for the vortex shedding frequency. Hence, spectra of the lift for varying $s/d$ at a fixed flow velocity are shown instead in Figure 6 (b). In the figure, the behavior of $S_L$ in the region of $s/d < 0.33$ resembles that of $S_u$ in Figure 6 (a), although the peaks in spectra are very low and it is difficult to determine the peak frequency at a definite value. When $s/d > 0.33$, $S_L$ is practically flat. Note that the maximum value of $s/d$ in (a) and (b) is 0.6 and there observed no peak corresponding to the Kármán vortex shedding near the cross at such small values of gap.

Compared with the other two systems, the behavior of $S_u$ for System III with varying gap is largely different as seen in Figure 6 (c). Note that $s/d$ ranges from 0 to 6 in Figure 6 (c) which is much larger than those of Figure 6 (a) and (b). In Figure 6 (c), two peaks are observed in a spectrum $S_u$ for $s/d > 0.5$. Since the frequency of the lower peak is double the higher peak frequency, the latter is regarded as the vortex shedding frequency, $f_{v0}$. When $s/d$ is larger than 6, $S_u$ has a sharp peak at the Kármán vortex frequency $f_{vK}$. The peak becomes

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Fig. 6 Spectra of velocity for Systems I and III and that of lift force for System II from fixed systems for various $s/d$ in wind tunnel
less sharp and the value of the peak frequency $f_{v0}$ is lower when the gap is smaller in the range of $1.8 < s/d < 6$. When the gap is still smaller, the peak becomes sharp again but at a frequency considerably lower than that in the larger gap range.

The peak frequencies in the spectra, $f_{v0}$, shown in Figure 6 are reduced into the Strouhal number $S_{St0}$ and plotted against the non-dimensional gap $s/d$, as in Figure 7. For System I, there is an abrupt change of $S_{St0}$ at $s/d = 0.25$ as seen in Figure 7(a), corresponding with the transition of vortex type from the necklace vortex ($0.25 \leq s/d \leq 0.5$) to the trailing vortex ($0 \leq s/d \leq 0.25$) as observed in the visualization experiment. In the case of System II, similar transition seems to occur at around 0.25, but values of $S_{St0}$ in the region $s/d > 0.2$ is not reliable since both the spectra of velocity and lift, $S_u$ and $S_L$, had indefinite peaks as shown in Figure 6 (b).

In the case of System III, Figure 7(b), the Kármán vortex shedding frequency at $s/d = 6$ is equal to that of the isolated counterpart and becomes smaller with decreasing gap till $s/d$ attains at 1.5. Then, $S_{St0}$ suddenly decreases, followed by a continuous increase with decreasing gap. By comparing with the tuft visualization and the oscillation behavior described later, the lower values of $S_{St0}$ at $s/d < 1.5$ are attributed to the periodic longitudinal vortices, although its type was not identified by the visualization experiment. The relationship between $S_{St0}$ and $s/d$ agrees well with that of square-cylinder/square-cylinder system reported by Fox (12) except that no jump was seen in the latter.

The relationship between the Strouhal number $S_{St0}$ and the Reynolds number $Re$ of the longitudinal vortices for the three fixed systems are compared in Figure 8. For System I, $S_{St0}$ of necklace vortex ($s/d = 0.28$) is nearly constant around 0.04 in the range of $Re$ from 3,000 to 40,000. While, $S_{St0}$ for the trailing vortex ($s/d = 0.08$) increases with $Re$ and jumps discontinuously at $Re = 6,000$. Similar tendency of $S_{St0}$ of the trailing vortex is seen in the
case of System II when $4,000 < Re < 12,000$, although the data are largely scattering. The low values of $St_0$ in the higher $Re$ range are due to flat profiles of $Su$ and $SL$ which are attributed to the fact that the periodic vortex shedding is indefinite. The fact that $St_0$ for System III with $s/d = 1.2$ is virtually constant at around 0.06 infers that the vortex is the necklace vortex.

### 3.3. Oscillation behavior of elastically supported cylinders

Oscillation of elastically supported upstream cylinders and the vortex shedding frequency were measured with an increasing and then decreasing flow velocity for the three systems at values of $s/d$ which give the periodic vortex shedding most definitely.

In Figure 9, the oscillation amplitude expressed by the RMS value of $z$-displacement, $Z_{rms}$, and the vortex shedding frequency $f_v$ are plotted against the flow velocity $U$ for System I with $s/d = \infty$ (single cylinder), 0.28 and 0.08. In the case of single cylinder, the well-known Kármán vortex excitation is observed over a range of $U$, where $f_v = f_n$ caused by the lock-in phenomenon. The two longitudinal vortices induce large excitation accompanied with respective wide lock-in regions around $U = U_0$ ($U_0$ is the velocity at which natural vortex shedding frequency $f_v$ is equal to $f_n$ of the structure). It should be noted that the shape of $Z_{rms} \sim U$ curves and the width and the location of the lock-in regions relative to $U_0$ are largely different among the three vortex excitations. A definitely different behavior is seen between the increasing and the decreasing $U$ in the case of $s/d = 0.28$, showing the hysteresis nature of this vortex excitation.

Figure 10 shows similar plot for System II with $s/d = 0.08$, compared with the results on the single circular cylinder. In this case, the trailing vortex synchronizes with the cylinder oscillation, and consequently causes large oscillation, over very wide region of $U$. While, the necklace vortex excitation was not observed in spite that certain values of $s/d$ larger than 0.25 were tested in the same experimental procedure. In Figure 10, it seems peculiar that large trailing vortex excitation occurs over such a wide velocity range in spite of the fact that periodic vortex shedding from the fixed counterpart is less definite than the case of System I as seen in Figure 6 (b).

Figure 11 shows results on System III with $s/d = 1.2$, compared with the case of a single square cylinder ($s/d = \infty$) as given by the broken lines. In the latter, the Kármán vortex excitation and the galloping are clearly observed in separated velocity regions. In the case of single square cylinder, the vortex shedding frequency $f_v$ coincides with $f_n$ under the Kármán vortex excitation, and $f_v$ is much higher than $f_n$ in the galloping region since $f_v$ increases proportionally with $U$ outside the Kármán vortex excitation region. In System III with $s/d = 1.2$, the gap at which the longitudinal vortex in Figure 5 is most clearly observed, both the Kármán vortex excitation and the galloping disappear and a new excitation is induced over a velocity region of $U = 6.5 \sim 8$ m/s, as seen in Figure 11. Since this velocity range of the new excitation expands around $U_0$, the velocity where $f_v$ estimated from $St_0$ in Figure 7 (b) is equal to $f_n$, and there observed clear "lock-in" over this velocity range, it is inferred that the oscillation is caused by the longitudinal vortex as seen in Figure 5.

### 3.4. Oscillation behavior with varying gap

To see how the downstream body induces the longitudinal vortex excitation, the flow velocity was set at a certain constant value, and the downstream cylinder was made to approach from a position with $s/d$ essentially infinity. The results are shown in Figure 12, where the velocity $U$ is set at values for each longitudinal vortex excitation to be appropriately large in Figures 9, 10 and 11.

In the case of System I, $U$ was set at 5.5 m/s and 12 m/s as the respective representative values for the trailing vortex and the necklace vortex excitations, as seen in Figure 9. The vortex shedding frequency data at the closest value of $Re$ from the fixed counterparts
normalized by the structure frequency $f_n$ are also added in the figure. When $U = 5.5$ m/s, the oscillation is large in the region of $s/d < 0.25$ and very small at $s/d > 0.25$. This is because the trailing vortex shedding synchronizes with the cylinder motion since natural shedding frequency $f_v$ of the trailing vortex is close to $f_n$ in this region of $s/d$ as seen by the plot of $f_v/f_n$. On the other hand, the oscillation is large in the region of $s/d$ from 0.25 to 0.6 when $U = 12$ m/s. In this region of $s/d$, the necklace vortex synchronizes with the cylinder motion since $f_v/f_n$ is close to unity, but the trailing vortex excitation does not occur since $f_v$ in the region $s/d < 0.25$ is too much higher than $f_n$ for the synchronization.

In Figure 12 (b) for System II, $U = 6.0$ m/s and 12.0 m/s are adopted since at these velocities $Z_{rms}$ is considerably large in Figure 10 and $St_n = 0.065$ and 0.033 correlating with
the higher and the lower levels of $St_0$ in Figure 7(a). The fact that $Z_{rms}/d$ in the region $s/d > 0.3$ is insignificant at the both velocities infers that the oscillations at $s/d < 0.3$ are caused by the periodic vortices $St_0$ of which are shown in Figure 7(a). Since the vibration amplitude remains small when $s/d > 0.3$ for the two values of flow velocity, it is inferred that the necklace vortex does not form, or does not shed periodically, in the circular-cylinder/strip-plate geometry.

For the case of System III, $U$ was set at 7.6 m/s at which the new oscillation peak is maximum in Figure 11. Note that the oscillation of single square cylinder due to galloping

Fig. 12 Longitudinal vortex frequencies and excitation induced by approaching the downstream body
is considerable at this velocity. Hence, Zrms is normalized by the galloping amplitude and plotted against s/d in Figure 12 (c), together with the vortex shedding frequency normalized by fn. In this figure, it is seen that the galloping of the square cylinder is essentially suppressed by setting the plate downstream with a gap 2 < s/d < 4 or s/d < 0.8. The high peak value of Zrms at s/d around 1.4 is induced by the longitudinal vortex, as confirmed by the fact that f/fn = 1 over the region of s/d where the oscillation is large. This synchronization phenomenon was directly observed by the tuft visualization as seen in Figure 5.

4. Conclusions

In this work, wind tunnel experiment was carried out to investigate the influence of cross-sectional configuration on the longitudinal vortex excitation which had been found for cruciform two-circular cylinder systems by the present authors. Three systems composed of different cross sectional configuration cylinders but with essentially equal structural parameters were tested, i.e. (I) two-circular-cylinder, (II) circular-cylinder/strip-plate and (III) square-cylinder/strip-plate. Tuft-grid method in wind tunnel and dye-streak technique in water tunnel were applied to visualize the longitudinal vortices, although the Reynolds number was much lower in the latter as compared with the wind tunnel experiment.

The conclusions are summarized as follows.

1. The longitudinal vortex excitations are induced for all the three configurations by setting the cruciform arrangement downstream cylindrical body with non-dimensional gap s/d properly determined for the three systems.
2. In the case of two-circular-cylinder system, the necklace and the trailing vortices shed periodically depending on the non-dimensional gap s/d, and cause resonant oscillations in different velocity ranges. In contrast, only the trailing vortex excitation is observed over much wider velocity range in the case of the circular-cylinder/strip-plate system, but the necklace vortex excitation is not observed.
3. In the case of square-cylinder/strip-plate system, the necklace vortex excitation is observed in the range of s/d considerably larger than that of the two-circular-cylinder system, while the trailing vortex excitation is not observed.

The results obtained in this work suggest that unexpected vibration of a cylindrical body in flow can occur if another cylinder is set downstream in cruciform arrangement. Hence, further investigation is required to clarify the conditions of the occurrence of the longitudinal vortex excitation and the estimation of the exciting force by the longitudinal vortices.

Acknowledgements

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