Direct numerical simulation of a turbulent mixing layer with a transversely oscillated inflow

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Abstract
Direct numerical simulation of a turbulent mixing layer with a transversely oscillated inflow is performed. The inlet flow is generated by two driver parts of turbulent boundary layers. The Reynolds number based on the freestream velocity on the low speed side, \( U_L \), the 99\% boundary layer thickness of the inflow, \( \delta \), and the kinematic viscosity, \( \nu \), is set to be \( Re = 3000 \). In order to compare the results with the experimental study of Naka et al. [Naka, Tsuboi, Kametani, Fukagata, and Obi, J. Fluid Sci. Technol., Vol. 5, pp. 156-168 (2010)], the angular frequency of the oscillation was set to be \( \Omega_c = 0.83 \) and 3.85 (referred to as Case A and Case B, respectively). From the three-dimensional visualization, large-scale spanwise vortical structures are clearly observed in the controlled cases. The momentum thickness and the vorticity thickness indicate that the mixing is enhanced in Case A, while it is temporarily suspended in Case B. In both cases, the Reynolds normal stresses are increased in the region right downstream of the forcing point due to the periodic forcing. Furthermore, in Case B, the Reynolds shear stress (RSS), \( -\overline{u'v'} \), is suppressed in the region downstream of the forcing point. The spatial development of the turbulent energy thickness, \( \delta_k \), and the Reynolds shear stress thickness, \( \delta_{rss} \), show that the Reynolds shear stress in Case B is decreased by the control despite the increase of the turbulent kinetic energy. From the spectral analysis, large-scale spanwise structures are found to be caused by the periodic forcing, while the spectra of the spanwise velocity fluctuations are nearly unchanged. Co-spectra of the Reynolds stresses show that the present forcings generally enhance the long wavelength component. In Case B, however, the long wavelength component of the Reynolds shear stress is not increased in the downstream region.

Key words: Turbulent mixing layer, Periodic forcing, Direct numerical simulation

1. Introduction

A mixing layer is one of the fundamental free shear flows generated by the Kelvin-Helmholtz instability due to a velocity gap (Hussain 1985; Brown and Roshko 2009). Its physics has been extensively studied; the mechanisms of the phenomena have been clarified theoretically, experimentally, and numerically.

For the inviscid instability, Michalke (1965) suggested a hyperbolic tangent velocity profile of the mixing layer; this profile is often used as the basic profile in the studies of mixing layers. Monkewitz (1988) performed an instability-wave analysis for an inviscid parallel mixing layer, and he described the interaction between the fundamental mode and its subharmonics. In order to understand the vortical dynamics in turbulent mixing layers, Brown and Roshko (1974) experimentally visualized the coherent structures. Huang and Ho (1990) experimentally studied an acoustically perturbed laminar mixing layer, and they observed small-scale turbulence produced due to the interaction of spanwise and streamwise structures after merging of the spanwise vortices.

Turbulent mixing layers appear in various practical applications, e.g., inside combustion chambers or around the exhaust of turbo engines. For more efficient utilization of these applications, control techniques for mixing enhancement or suppression have been investigated. One of the control strategies is to manipulate the flow in the upstream region of
the mixing layer, e.g., using perturbations. Ho (1982) performed an experimental study of the mixing layer in which the flow rates of the inflows were perturbed. He concluded that the spreading rate of the mixing layer can be manipulated at a very low forcing level if the mixing layer is perturbed near the subharmonic of the most-amplified frequency. Suzuki et al. (2004) conducted an experiment using an axisymmetric jet nozzle with electromagnetic flap actuators at its exit. They report that various modes of vortical structures can be introduced in the flow by changing the Strouhal number. Naka et al. (2010) reported a mixing layer periodically forced by using a flap-type actuator made of a piezo-plastic (Polyvinylidene fluoride: PVDF) film aiming at both enhancement and suppression of mixing. They showed that, in addition to mixing enhancement, mixing suppression can also be achieved at a higher frequency.

Owing to the progresses in the computer performance and the computational methods, direct numerical simulation (DNS) has become a powerful tool for investigating the mechanics of mixing layers. Rogers and Moser (1994) performed a DNS of a temporally developing turbulent mixing layer in order to describe the physics of turbulent mixing layer, such as the merging and the roller-blade structure. Similarly, Wang et al. (2007) performed a DNS of a spatially developing turbulent mixing layer to investigate the role of coherent fine-scale structures in the transition process. Sandham and Reynolds (1991) performed a DNS of a three-dimensional time-developing compressible mixing layer with an error function as the initial velocity profile and analyzed the instability. Such an instability analysis was extended to the compressible reacting the mixing layer based on the linear stability theory (Day et al., 1998) and nonlinear stability theory using a parabolic stability equation (Day et al., 2001). For a more practical configuration, Sandham and Sandberg (2013) performed a DNS of a turbulent mixing layer, where two boundary layers are split by a plate. They describe the physics of a turbulent mixing layer composed of a turbulent boundary layer and a laminar boundary layer. For controlled turbulent mixing layers, however, numerical investigations have not sufficiently been reported in contrast to the experimental studies.

In the present study, direct numerical simulation (DNS) of a turbulent mixing layer produced by two turbulent boundary layers is performed. The mixing layer is perturbed at the inlet by transversely oscillating the inflow velocity field. The degree of mixing enhancement or suppression is evaluated by the momentum thickness and the vorticity thickness. The amplitude and the frequency of the periodic forcing are determined by following the experiment of Naka et al. (2010), where mixing enhancement or suppression was confirmed. The mechanism of the control effects is discussed in detail by analyzing the turbulence statistics.

2. Direct numerical simulation of an incompressible turbulent mixing layer

Direct numerical simulation (DNS) of a turbulent mixing layer (i.e., the main part) is performed. In order to provide realistic turbulent inflows, which fluctuate in time and space, DNSs of two turbulent boundary layers (i.e., the driver parts)
Table 2 Forcing conditions.

<table>
<thead>
<tr>
<th>Case</th>
<th>Uncontrolled</th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular freq.</td>
<td>0</td>
<td>0.83</td>
<td>3.85</td>
</tr>
<tr>
<td>Frequency, ( f_c )</td>
<td>-</td>
<td>0.182</td>
<td>0.615</td>
</tr>
<tr>
<td>Wavelength, ( \lambda )</td>
<td>-</td>
<td>11.4</td>
<td>2.44</td>
</tr>
<tr>
<td>Period, ( T )</td>
<td>-</td>
<td>7.58</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Fig. 2 Inflow velocity condition: (a) time trace of the origin of the mixing layer (black, Case A; red, Case B); (b) mean streamwise velocity, \( U \); (c) streamwise turbulence intensity, \( u_{rms} \).

are simultaneously performed. Figure 1 shows the schematic of the computational domain used in the present study. The inlet flows are assumed to be split by an infinitesimally thin plate.

The governing equations are the incompressible continuity and Navier-Stokes equations:

\[
\frac{\partial u_i}{\partial x_i} = 0,
\]

\[
\frac{\partial u_i}{\partial t} = -\frac{\partial u_i u_j}{\partial x_j} - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_i \partial x_j},
\]

where \( x_i \) \((i = 1, 2, 3)\) denote the Cartesian coordinates and \( u_i \) are the corresponding velocity components. All variables are non-dimensionalized by the freestream velocity on the low-speed side, the 99% boundary layer thickness of the inflow turbulent boundary layer on the low-speed side, denoted as \( U_L \) and \( \delta \), respectively. The Reynolds number, \( Re = U_L \delta / \nu \), where \( \nu \) denotes the kinematic viscosity, is set to be 3000.

The DNS code for the main part, i.e., the mixing layer, is basically the same as that of Hreppfner et al. (2011), which is based on the channel flow code of Fukagata et al. (2006): the spatial discretization is done by the energy-conservative second-order finite difference scheme (e.g., Ham et al. 2002); the time integration uses the low-storage third-order Runge-Kutta/Crank-Nicolson scheme (e.g., Spalart et al. 1991). The inflow velocity fields are generated in the driver parts by
Fig. 3 Instantaneous vortical structures: transparent white, iso-surface of pressure, \( (\rho' = -0.01) \); color, iso-surface of the second invariant of the velocity-gradient tensor \( (Q = -30) \) colored by \( \rho' \). a) Side view, b) top view.

simultaneously performing DNSs of spatially developing turbulent boundary layers similarly to Kametani and Fukagata (2011), in which the recycle method of Lund et al. (1998) is used. The freestream velocity on the high speed-side is set twice faster than that on the low speed-side, i.e., \( U_H = 2U_L \).

In both the main and the driver parts, the convective boundary condition is applied at the outlet of the computational domain, i.e.,

\[
\frac{\partial u_i}{\partial t} + U_c \frac{\partial u_i}{\partial x} = 0,
\]

where \( U_c \) denotes the average of the freestream velocities in the high-speed side and the low-speed side, i.e., \( U_c = 1.5 \).

The pressures at the inlet and the outlet boundaries are given by the Navier-Stokes characteristic boundary condition (NSCBC) of Miyauchi and Tanahashi (1996), i.e.,

\[
\frac{\partial p}{\partial t} + U_c \frac{\partial p}{\partial x} = \frac{1}{2Re} \omega_z^2,
\]

where \( \omega_z \) denotes the spanwise vorticity. It is known that this boundary condition considerably suppresses the unphysical pressure reflections from the boundaries.

The dimensions of the main part are \( 0 \leq x \leq 6 \sigma \) in the streamwise direction, \( -10 \leq y \leq 10 \) in the vertical direction, and \( 0 \leq z \leq \pi \) in the spanwise direction, respectively. The corresponding numbers of grid points are \( (N_x, N_y, N_z) = (256, 224, 128) \). The grid spacings in the streamwise and the spanwise directions are \( \Delta x = 7.36 \times 10^{-2} \) and \( \Delta z = 2.45 \times 10^{-2} \).
respectively. The minimum grid spacing in the transverse direction is $\Delta y_{\text{min}} = 0.3 \times 10^{-2}$ and the maximum spacing is $\Delta y_{\text{max}} = 0.41$. The dimensions of each driver part, i.e., a turbulent boundary layer, are $0 \leq x^D \leq 3\pi$ in the streamwise direction, $0 \leq y^D \leq 3$ in the vertical direction, and $0 \leq z^D \leq \pi$ in the spanwise direction, respectively, where the superscript $D$ denotes the driver part. The corresponding numbers of grid points are $(N^D_x, N^D_y, N^D_z) = (128, 96, 128)$ for each boundary layer. The time step is set to be $\Delta t = 10^{-3}$, which results in the local maximum Courant number of 0.5 in the main part. In the present simulation, the dimension in the main part is set larger than the summation of those in the driver parts in order to minimize the computational cost for the driver parts. Therefore, the outer regions of the main part are filled with the freestream velocities, as shown by the red lines in Fig. 1.

The inflow conditions are summarized in Table 1 and compared with those in the experiment by Naka et al. (2010). Due to the restrictions both in the experiment and the simulation, it was unable to match all the conditions. In the present study, we chose to match the ratio of the freestream velocities, i.e., $U_H/U_L = 2$, and the momentum thickness Reynolds number on the low-speed side, i.e., $Re_\theta = 330$.

3. Periodic forcing

In the present study, the periodic forcing using the piezo-film actuator (PFA) of Naka et al. (2010) is numerically mimicked by transversely oscillating the inlet velocity field. To be concrete, the inflow field obtained by combining two cross-sections of turbulent boundary layers, $u^D_i(y, z, t)$, is periodically oscillated and fed at the inlet of the mixing layer as

$$u_i(0, y, z, t) = u^D_i(y - A \sin(\Omega_c t), z, t),$$

where $A$ and $\Omega_c$ denote the forcing amplitude and the angular velocity.

Following Naka et al. (2010), the amplitude is fixed to be $A = 0.14$. The angular frequency is set to be $\Omega_c = 0.83$ (Case A) and $\Omega_c = 3.87$ (Case B), which correspond to the second sub-harmonic and the first sub-harmonic of the natural frequency of the mixing layer, respectively. These values are chosen because Naka et al. (2010) report that the development of vorticity thickness is promoted by the second sub-harmonic forcing, while it is delayed by the first subharmonic forcing. The present forcing conditions are listed in Table 2. Figure 2 (a) shows the time trace of the origin of the mixing layer, i.e., the splitter plate. The mean velocity and the streamwise turbulent intensity profiles at the inlet are plotted in Figs. 2 (b) and (c).
4. Results and discussion

4.1. Vortical structures

Figure 3 shows the instantaneous vortical structures identified by the local pressure, \( p' = p - p_{\infty} \), and the second invariant of the velocity gradient tensor, \( Q = \left( \frac{\partial u_i}{\partial x_j} \right) \left( \frac{\partial u_j}{\partial x_i} \right) \), which roughly correspond to large-scale vortices and fine-scale vortices, respectively. Snapshots of the spanwise vorticity in an \( x-y \) cross-section are depicted in Fig. 4. In the base flow (i.e., the uncontrolled case), two-dimensional spanwise vortical structures, viz., roller structures, develop gradually and collapse as they flow downstream. In the controlled cases, the large roller structures are directly induced by the periodic forcing, and they appear with equal intervals roughly corresponding to the forcing wavelengths shown in Table 2. As compared to the uncontrolled case, the trajectory of the large vortical structure in Case A significantly meanders in the transverse direction, while that in Case B stays mostly on the centerline. These roller structures are clearly observed up to around \( x = 10 \) before they break down. In addition, quasi-streamwise vortices in the upstream region look to be attenuated by the roller structures.

4.2. Statistics

In order to discuss the control effects more quantitatively, modifications in the statistics are examined in this section. The statistics are accumulated in a non-dimensional time period of \( \Delta t_{st} = 280 \) after the flow has reached its statistically steady state.

Figure 5 shows the mean streamwise velocity. The contour in the uncontrolled case (Fig. 5) confirms the spatial development of the mixing layer. The profiles indicate velocity deficits in all cases due to the two turbulent boundary layers, i.e., the effect of the splitter plate, which remain up to around \( x = 14 \) in the low-speed side (\( y < 0 \)). In Case B (at the higher forcing frequency), the velocity deficit disappears faster than those in the uncontrolled case and Case A (at the lower forcing frequency). The gradient of \( U \) is kept larger than the other cases; namely, the mixing is suppressed. It can also be found that the center of the mixing layer is shifted toward the positive transverse direction, which is opposite to the observation in the experiment by Naka et al. (2010). This discrepancy may be due to the difference in the boundary layer thicknesses at the inlet. While the boundary layer thicknesses on the upper and the lower sides are set the same in...
the present simulation, that on the upper side is 62% of that on the lower side in the experiment, as shown in Table 1.

As an indicator of the mixing enhancement or suppression, the momentum thickness, \( \theta \), is defined as

\[
\theta(x) = \frac{1}{\Delta U^2} \int_{-\infty}^{\infty} (U_H - U(x, y))(U(x, y) - U_L) \, dy,
\]

where \( \Delta U \) denotes the gap between the freestream velocities on the high-speed and low-speed sides, i.e., \( \Delta U = U_H - U_L \).

Figure 6(a) shows the streamwise development of the momentum thickness. As compared to the uncontrolled case, the momentum thickness is found to be thickened in both controlled cases. With the low frequency forcing (Case A), the momentum thickness seems to be constantly thickened as compared to that in the uncontrolled case. On the other hand, with the high frequency forcing (Case B), the development of the momentum thickness becomes milder after it is drastically increased in the range of 0 < \( x < 2 \).

The momentum thickness is not always adequate to evaluate the degree mixing because the inlet velocity profile itself oscillates in the transverse direction, by which the momentum thickness is automatically increased. Therefore, the vorticity thickness is examined as another indicator, which is defined as

\[
\delta_{\omega}(x) = \frac{\Delta U}{\partial U/\partial y}_{\text{max}},
\]

The vorticity thickness represents the highest mean shear in the mixing layer as a function of the streamwise coordinate. In order to exclude the effect of velocity deficits, the vorticity thickness is captured in 0 ≤ \( y < 1 \).

In Fig. 6(b), the computed vorticity thickness is plotted together with the experimental data of Naka et al. (2010). Here, in order to plot both data in the same figure, the 99% boundary layer thickness of the inflow on the low-speed side of Naka et al. (2010) is assumed to be \( \theta = (7/2)\delta \). In the present simulation, \( \delta_{\omega} \) is found to be thickened already at the inlet by the periodic forcing. While \( \delta_{\omega} \) monotonically increases in Case A, it is nearly constant in Case B in the region of 2 < \( x < 6 \) before it recovers to the uncontrolled level. This indicates that the mixing is temporarily suspended in the region of 2 < \( x < 6 \) with a high frequency forcing (Case B). The region where the vorticity thickness in Case B decreases agrees with the region where the turbulence suppression is found in Fig. 3. Namely, the stable large vortical structure created by the high frequency forcing suppresses the development of mixing layers.

As for the comparison with the experimental data, a good agreement can be found between the present results and Naka et al. (2010) in the uncontrolled case and the case of low frequency forcing (Case A). However, a quantitative difference can be found in the high frequency forcing case (Case B). The plateau in the vorticity thickness is observed in 7 < \( x < 10 \) in the experiment, while it is observed in 2 < \( x < 6 \) in the present simulation. Moreover, \( \delta_{\omega} \) is much larger in the experiment. This discrepancy is supposed to come from the assumption of the inlet velocity. Although the oscillation amplitude in the experiment is determined by the measurement under a no-wind condition because the laser displacement meter disturbs the flow, the actual oscillation amplitude in Naka et al. (2010) might have been smaller in the presence of the wind. The difference in the momentum thickness at the inlet, as shown in Table 1, may also have caused the difference.

Figures 7–9 show the Reynolds normal stresses. As shown in Fig. 7, the streamwise normal stress, \( u'\bar{u}' \), is increased by the control in the range of 0 < \( x < 1.5 \) in both cases. This is a direct consequence of the oscillation of the mean velocity profile at the inlet. In the subsequent region (2 < \( x < 5 \), \( u'\bar{u}' \) is increased in Case A and decreased in Case B. In the further downstream region, the change in \( u'\bar{u}' \) milder, but the values in the controlled cases are kept higher than that in
and lasts up to \( x = 2 \) and increases again in the downstream region \( (x > 5) \). The peak location shifts downstream compared to the other cases.

This suggests that the mixing suppression due to the stable roller structures are generated, as shown in Figs. 3 and 4. The enhancement of \( \overline{\nu'v'} \) remains further downstream due to the stable large spanwise vortices. As observed in Fig. 9, the spanwise normal stress, \( \overline{u'w'} \), is decreased in the upstream region \( (0 < x < 3) \) in the controlled cases, confirming the attenuation of quasi-streamwise vortices. In Case B, however, \( \overline{u'w'} \) increases again in the downstream region \( (x > 5) \) and the peak location shifts downstream compared to the other cases.

The Reynolds shear stress (RSS), \( -\overline{u'v'} \), is shown in Fig. 10. In Case B, the RSS is rapidly enhanced in \( 0 < x < 2 \), and damped after this region, while the distribution of the RSS is simply thickened in Case A. The decreased RSS in Case B indicates that the correlation between the streamwise and the transverse fluctuations are weakened. It is also found that the RSS starts to be enhanced again after \( x = 6 \). This suggests that the mixing suppression due to the stable roller structure takes effect from \( x = 2 \) and lasts up to \( x = 6 \).

These results, together with the three-dimensional visualization in Figs. 3 and 4, indicate that the mixing layer is significantly modified by the periodic forcing regardless of \( \Omega \), but in different manners. In particular, Case B has a region where the spatial growth of the Reynolds shear stress is suppressed by the stable large-scale roller structures.

Figure 11 (a) shows the turbulent energy thickness, \( \delta_k \), defined here as

\[
\delta_k = \int_{-\infty}^{\infty} \frac{u'^2 + v'^2 + w'^2}{2} \, dy,
\]

where the velocities are non-dimensionalized by the freestream velocity on the low-speed side. In the uncontrolled case,
\( \delta_k \) rapidly increases in the upstream region, \( 0 < x < 1 \), its growth is suspended in \( 1 < x < 2 \), and it mildly grows again in \( x > 2 \). In the controlled cases, \( \delta_k \) at the inlet is approximately three times higher than that in the uncontrolled case. In Case A, \( \delta_k \) decreases once in the region of \( 0 < x < 2 \) before it increases again. In Case B, in contrast, it is kept at a similar level, which is higher than that in the uncontrolled case. From these results, turbulence looks to be enhanced also in Case B; however, the increase of \( \delta_k \) in Case B should be attributed to the large-scale roller structure observed above.

Similarly to the turbulent energy thickness, the RSS thickness, \( \delta_{RSS} \), is defined as

\[
\delta_{RSS} = \int_{-\infty}^{\infty} \overline{u'v'} \, dy.
\]  

and plotted in Fig. 11 (b). In the uncontrolled case, \( \delta_{RSS} \) shows a trend similar to \( \delta_k \) shown in Fig. 11 (a). In Case A, \( \delta_{RSS} \) in the upstream region is slightly thickened by the control. In \( x > 7 \), however, the growth rate drastically increases. This is likely due to the meandering of the mixing layer as shown in Fig. 4(b). In Case B, in contrast, \( \delta_{RSS} \) in \( 3 < x < 5 \) takes values below those in the uncontrolled case before it increases again. The result confirms that the initial enhancement of turbulent kinetic energy in Case B is due to the large-scale rollers induced by the control and that the presence of those stable large-scale roller structures in the downstream region makes the streamwise and the transverse fluctuations more orthogonal to each other; thus the RSS is reduced.

Figures 12–15 shows the pre-multiplied spectra at \( x = 9.4 \). The left, the middle, and the right columns of Figs. 12–15 are the spectra in the uncontrolled case, Case A and Case B, respectively. Due to the spanwise-homogeneous forcing at the inlet, the powers of the streamwise and the transverse fluctuations are increased in the longer wavelength range, while the spanwise fluctuation remains nearly unchanged. The spectra in the higher frequency case (Case B) indicates a greater enhancement than that in the lower frequency case (Case A). Because the spanwise mean velocity at the inlet is zero, the periodic motion does not work to increase the spanwise velocity fluctuations. Figure 15 shows the co-spectrum of \(-u'v'\) at \( x = 9.4 \). While long wavelength components are enhanced in Case A, they are not in Case B. This streamwise location \((x = 9.4)\) is further downstream of the region where the RSS recovers in Case B. Namely, the recovery of the RSS in Case B is mainly due to a development of the shorter wavelength components.

5. Conclusions

Direct numerical simulation was performed for a turbulent mixing layer periodically forced at the inlet. The control parameters such as the amplitude and the frequency of forcing are set similar to those in the experiment of Naka et al. (2010): \( \Omega_c = 0.83 \) (low-frequency forcing, Case A) and \( \Omega_c = 3.85 \) (high-frequency forcing, Case B).

The visualized flow field and the Reynolds stresses show that the spanwise roller structures induced by the periodic forcing at different frequencies modify the flow structure in different manners. While the mixing look to be enhanced in Case A by the meandering motion, it is observed to be suppressed in Case B in the region right downstream of the inlet by the stable roller structures sitting on the centerline.

Since it is difficult to accurately evaluate the degree of mixing due to the velocity deficits at the inlet, two different measures are examined: the momentum thickness and the vorticity thickness. From the vorticity thickness, the mixing appears to be promoted with the low-frequency control (Case A), while it is temporarily suspended with the high-frequency control (Case B). Despite slightly different inlet conditions, the observed trends are in fair agreement with the experiment of Naka et al. (2010) in the uncontrolled case and Case A. In Case B, in contrast, a quantitative difference from the
experimental result is noticed.

The Reynolds stresses and the pre-multiplied power spectra indicate that a large two-dimensional roller structure is induced by the present spanwise-homogenous forcing. While unstable rollers in Case A lead to an increase in turbulent mixing, the stable roller structures in Case B are found to reduce the Reynolds shear stress and turbulent mixing.

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