Direct numerical simulation for modification of sinusoidal riblets

Kie OKABAYASHI*

*Japan Aerospace Exploration Agency (JAXA)
6-13-1 Osawa, Mitaka, Tokyo 181-0015, Japan
E-mail: okabayashi.kie@jaxa.jp

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Abstract
This paper proposes a new riblet configuration. A traditional sinusoidal riblet was modified with the aim of reducing pressure drag. The height of the side wall in the newly configured riblet is lowered toward the node of the sinusoidal curve to reduce the large pressure drag, while the riblet height is maintained at the anti-node position as it has been reported to be the most effective for straight or traditional sinusoidal riblets. In this paper, effective configuration parameters and the drag-reducing performance of the modified sinusoidal riblet are investigated through parametrically conducting direct numerical simulation (DNS). First, the optimal combination of the wavelength and amplitude of the spanwise sinusoidal curve configuration is determined with a fixed riblet spacing $s$ and height $h$, which are located at around the design point of straight riblets. The effective combination obtained is similar to that reported in previous studies on traditional sinusoidal riblets. Second, the optimal vertical amplitude $a$ of the modified sinusoidal riblet is determined with the effective settings mentioned above. Unfortunately, with $a/h < 0.2$, the drag-reducing performance is similar to that of traditional sinusoidal riblets. Furthermore, the performance deteriorates with $a/h \geq 0.2$. However, the drag-reducing performance of the modified sinusoidal riblet is improved when $s^*$ deviates from the design point compared to that of comparable straight or traditional sinusoidal riblets. This indicates that flow-condition robustness is improved for the modified sinusoidal riblet. Finally, the mechanism of robustness is investigated. The contributions of the bottom and side walls to drag are calculated, and the difference of each contribution between the traditional and modified sinusoidal riblets is shown to discuss the influence caused by the lowered side wall. In addition, a quadrant analysis is conducted to examine the change in momentum transfer above the modified sinusoidal riblet.

Keywords: Riblet, Drag reduction, Turbulent flows, Flow control, Direct numerical simulation, Computational fluid dynamics

1. Introduction

Attention must be paid to sustainable aviation with an aim toward improving fuel efficiency and reducing CO$_2$ emission. Reduction of skin friction drag will contribute significantly toward improving aerodynamic performance as friction drag accounts for approximately half of the total drag of an aircraft under cruising condition. As is well known, several reduction techniques for skin friction drag have been proposed. Unfortunately, most of them cannot be used for practical applications because of technical problems or cost restrictions. Against this background, recently, several research groups have paid renewed attention to riblets as realizable flow control devices. For example, a research group developed an innovative shape-forming method for riblets (Stenzel et al., 2011). The structure of the riblet can be formed simultaneously during the painting process by using this method. This method will overcome the disadvantages of riblets, such as additional weight that offsets fuel saving, installation and maintenance costs, environment resistance, and difficulties in maintenance. In an attempt to analyze the application of riblets to commercial aircrafts, a flight test was conducted on an Airbus A320 aircraft, and a 2% reduction in total air drag was observed (Szodruch, 1991). The reduction in drag corresponds to an annual saving of more than 50000 liters of fuel per aircraft in normal regular service (MBB Transport
Aircraft Group, 1988). Generally, an approximately 8% reduction in turbulent skin friction drag can be achieved with a well-designed riblet (Viswanath, 2002).

Bechert et al. (1997) concluded from their experiments that ‘blade riblets,’ which are arranged as a row of thin fences along the streamwise direction, is the optimal 2D configuration. In contrast, the effectiveness of 3D riblets has been discussed in a few studies (Pollard, 1997; Viswanath, 2002). Several variations of streamwise configurations have been analyzed as 3D riblets such as zigzag riblets (Akinori et al., 2002; Sha et al., 2005; Grötheberger et al., 2012), sinusoidal riblets, (Peet et al., 2008; Peet et al., 2009; Grötheberger et al., 2012) and other riblets (Bruse et al., 1993; Wilkinson et al., 1988; Bechert et al., 1997a; Miki et al., 2011). While several streamwise configurations have been analyzed as 3D riblets, sinusoidal riblets, in particular, are reported to be more effective than straight riblets (Peet et al., 2008; Peet et al., 2009).

A few examples of previous studies on sinusoidal riblets are as follows: Grötheberger et al. (2012) reported that the drag reduction rate improved under appropriate combinations of wavelength and amplitude for a sinusoidal riblet with a height spacing (h/s) ratio of 0.3 and 0.9. However, they also reported that a straight riblet having an h/s ratio of 0.5 showed the maximum drag reduction. Sinusoidal riblets having an h/s ratio of 0.3, however, showed drag reduction rates that are comparable to that of straight riblets having an h/s ratio of 0.5. They suggested that shallowed grooves had a higher resistance to mechanical wear, an alleviation of manufacturing issues, and reduced weight. Peet et al. (2008) and Peet et al. (2009) analyzed flow field and turbulence statistics obtained by large eddy simulation (LES). Peet et al. (2009) organized the resulting drag reduction rate by using an excitation parameter $\epsilon = a^2T^3$ (a: amplitude, $T$: period), which was a relevant parameter with respect to skin friction modification by spanwise oscillatory motion. Peet et al. (2009a) performed a theoretical analysis on the prediction of turbulent skin friction on a geometrically complex surface such as riblets. The derived expression indicated several contributions to the skin friction. Particularly, with regard to the contribution of turbulence, sinusoidal riblets with an adequate wavelength achieved a drag reduction of 2% compared to straight riblets. They also suggested that the mechanism for suppressing additional turbulence is similar to the effects observed in a spanwise-oscillating wall.

Drag-reducing performance is necessarily improved by sinusoidal riblets compared to comparable straight riblets (Peet et al. (2008); Peet et al. (2009); Peet et al. (2009a)). However, this benefit is easily suppressed by the increase in pressure drag. A larger amplitude not only decreases friction drag, but also significantly increases the pressure drag. However, it is not meaningful to set too low an amplitude because the drag-reducing performance becomes close to that of straight riblets. The trade-off point was found (Peet et al. (2009)), but the net drag reduction is not drastically changed compared to straight riblets.

The objective of this research is to modify a traditional sinusoidal riblet configuration to curb the increase in the pressure drag. The effective configuration parameters and drag-reducing performance of the modified sinusoidal riblet are investigated by parametrically conducting direct numerical simulation (DNS). The resulting DNS database are analyzed to clarify the mechanism of drag reduction of the new riblet.

2. Modification of Sinusoidal Riblets

Figure 1(a) represents the schematic of a traditional sinusoidal riblet. The streamwise direction is denoted as $+x$. For a traditional sinusoidal riblet, the greater the angle between the flow and tip, the larger the pressure drag. The pressure drag is the largest at the ‘node’ position (e.g. $x_2$), while it becomes almost zero at the ‘anti-node’ position (e.g. $x_1$). Figure
Fig. 2: Top and side views of the configuration of sinusoidal and modified sinusoidal riblets.

1 (b) represents the schematic of the modified sinusoidal riblet. This new configuration is expected to reduce the pressure drag; the riblet height is lowered toward the node position (e.g. \( x_2 \)), while at the anti-node position (e.g. \( x_1 \)), the riblet height is maintained as has been reported to be the most effective for straight or traditional sinusoidal riblets. Figure 2 represents the top and side views of the sinusoidal and modified sinusoidal riblets. The variation of the riblet height is patterned sinusoidally with an amplitude of \( a \) as shown in Fig. 2. At the node position, the riblet height is \( h = 2a \). How to decide \( a \) will be discussed later. The wavelength of the variation in the riblet height is necessarily half of \( \lambda \), which is the wavelength of the spanwise sinusoidal curve. Spanwise sinusoidal curves are specified by wavelength \( \lambda \) and amplitude \( A \), as shown in Fig. 2. Sinusoidal curves are also specified by wavelength \( \lambda \) and angle \( \beta \) (Peet et al., 2008; Peet et al., 2009), which is defined as

\[
\beta = \tan^{-1} \left( \frac{2a}{\lambda} \right),
\]

and shown in Fig. 2. For the convenience of organizing the computational results, each case sets a combination of wavelength \( \lambda \) and angle \( \beta \).

3. Outline of Computation

3.1. Computational Condition

The object of analysis is a channel flow that has a flat surface on the upper wall and a riblet surface on the lower wall. Figure 3 shows an example of the computational domain. The increase and decrease in drag caused by riblets are calculated by comparing the drag of the upper and lower walls (Choi et al., 1993). Periodicity is assumed in the streamwise \((x)\) and spanwise \((z)\) directions. A triangular and a trapezoidal cross-section are adopted as the \( y-z \) cross-section. Figure 4 represents the computational mesh of each cross-section. The triangular cross-section with a tip angle of \( \alpha = 90^\circ \) is used for validation. The result is compared with DNS (Choi et al., 1993) and experimental results (Walsh, 1982; Bechert et al., 1997). The trapezoidal cross-section is the target of this study. The trapezoidal riblet was proposed as a suboptimal shape considering its application to aircraft (Bechert et al., 1997). This shape is durable for long flights and simultaneously supports the drag-reducing performance of optimal blade riblets. The riblet height \( h \) of the trapezoidal riblet is set as half the spanwise spacing of grooves \( s \), which is confirmed to be the effective size (Bechert et al., 1997; Gritlneiderger et al., 2012). All the computational cases will be shown later together with the calculated drag reduction rates. The geometrical parameters of the computational domain for each case, except for validation cases, can be found in the Appendix.

The channel width is set as \( 2\delta \). The averaged wall-shear velocity of the two-flat-plate channel, \( u_{e, flat} \), is defined by

\[
u_{e, flat} = \sqrt{\tau_{w}/\rho},
\]

where \( \tau_{w} \) is the averaged friction stress. In this study, the pressure gradient \( \partial P/\partial x = -\tau_{w}/\delta \) is applied as the driving force. The Reynolds number \( Re = \nu_{e, flat}\delta/\nu \) is set as 180 in order to compare with previous DNS results (Choi et al., 1993). The wall unit \( f^+ \) is defined by

\[
u_{e, flat}/\nu.
\]
3.2. Definition and Error of Drag Reduction Rate

The friction drag reduction rate $DR_f$ is defined as

$$DR_f = \frac{D_f - D_{f\text{flat}}}{D_{f\text{flat}}}$$  \hspace{1cm} (2)$$

where $D_f$ is the friction drag, which is obtained from the area integration of the streamwise component of the wall shear stress $\tau_{w1}$:

$$D_f = \int_A \tau_{w1} dA.$$  \hspace{1cm} (3)$$

In eq. (2), the subscripts rib and flat represent the ‘riblet surface (lower side of domain)’ and ‘flat surface (upper side of domain)’, respectively.

The streamwise component of the wall shear stress $\tau_{w1}$ is calculated as

$$\tau_{w1} = -\mu \left[ (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \mathbf{n} \right] \cdot \mathbf{e}_x$$  \hspace{1cm} (4)$$

Here, $\mathbf{n}$ is the normal vector on the wetted area, and $\mathbf{e}_x$ is the unit vector in the streamwise direction.

In both sinusoidal and modified sinusoidal riblets, the friction drag as well as the pressure drag has to be considered. The total drag $D$ is defined as the sum of the friction drag $D_f$ and the pressure drag $D_p$:

$$D = D_f + D_p$$
$$= \int_A (\tau_{w1} + P_w n_x) \ dA$$  \hspace{1cm} (5)$$

Here, $P_w$ is the pressure above the wetted surface, and $n_x$ is the streamwise component of the normal vector. It should be noted that $D = D_f$ in straight cases and on the upper flat wall. Similar to $DR_f$ defined in eq. (2), the total drag reduction rate is defined as

$$DR_{\text{total}} = \frac{D_{\text{rib}} - D_{\text{flat}}}{D_{\text{flat}}}.$$  \hspace{1cm} (6)$$
Table 1: Setup parameters for validation cases.

<table>
<thead>
<tr>
<th>case</th>
<th>cross-section</th>
<th>$s^*$</th>
<th>$h^*$</th>
<th>$h/s$</th>
<th>$s/\delta$</th>
<th>$N_{up}$</th>
<th>$L_o/\delta$</th>
<th>$L_a/\delta$</th>
<th>$N_v \times N_b \times N_f$</th>
<th>$\Delta \xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>flat</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4\pi</td>
<td>2</td>
<td>2</td>
<td>$2 \times \Phi\xi$</td>
<td>$256 \times 128 \times 256$</td>
</tr>
<tr>
<td>2</td>
<td>triangle ($\alpha = 90^\circ$)</td>
<td>40</td>
<td>20</td>
<td>0.5</td>
<td>0.227</td>
<td>8</td>
<td>4\pi</td>
<td>2</td>
<td>$2 \times \Phi\xi$</td>
<td>$256 \times 128 \times 256$</td>
</tr>
<tr>
<td>3</td>
<td>triangle ($\alpha = 90^\circ$)</td>
<td>20</td>
<td>10</td>
<td>0.5</td>
<td>0.1135</td>
<td>16</td>
<td>4\pi</td>
<td>2</td>
<td>$2 \times \Phi\xi$</td>
<td>$256 \times 128 \times 512$</td>
</tr>
<tr>
<td>4</td>
<td>trapezoidal</td>
<td>20</td>
<td>10</td>
<td>0.5</td>
<td>0.1135</td>
<td>16</td>
<td>4\pi</td>
<td>2</td>
<td>$2 \times \Phi\xi$</td>
<td>$256 \times 128 \times 512$</td>
</tr>
</tbody>
</table>

1-K (Kim et al. (1987))
2-C (Choi et al. (1993))
3-C (Choi et al. (1993))
\(\tilde{\alpha}=0\) (Choi et al. (1993))
\(\tilde{\alpha}=60\) (Choi et al. (1993))

Table 2: Drag reduction rates of validation cases (*: experiment by Walsh (1982), **: DNS by Choi et al. (1993).

| case | cross-section | $s^*$ | $h^*$ | $h/s$ | $s/\delta$ | $N_{up}$ | $L_o/\delta$ | $L_a/\delta$ | $DR_{crit}$ ($\Delta DR$) |
|------|---------------|------|------|-------|----------|--------|-----------|-----------|----------------|-----------|
| 2    | triangle ($\alpha = 90^\circ$) | 40   | 20   | 0.5   | 0.227    | 8       | 4\pi      | 2         | $3.2\%$ (+3\%, +2\%) | $-3.0\%$ (-3\%, -5\%,-3\%) |
| 3    | triangle ($\alpha = 90^\circ$) | 20   | 10   | 0.5   | 0.1135   | 8       | 4\pi      | 2         | $3.0\%$ (-3\%) | $-6.2\%$ (-7.6\%) |
| 4    | trapezoidal   | 20   | 10   | 0.5   | 0.1135   | 16      | 4\pi      | 2         | $2 \times \Phi\xi$ | $256 \times 128 \times 256$ | 8.8      |

Choi et al. (1993) obtained error bounds for statistical sampling errors of drag measurements by comparing the wall shear rates of two flat walls in a plane channel simulation. They reported a remaining fluctuation of approximately ±2% after averaging over 4000 viscous time units (\(tn^+ \beta \xi /\nu\)). In the two-flat-wall channel DNS conducted in this study, the remaining fluctuation was approximately ±1% after averaging over 4000 viscous time units. Therefore, the sampling data of more than 4000 viscous time units are used for statistical processing of all the cases.

3.3. Numerical Method

The unsteady simulation method is based on the fractional step method for incompressible flow. The convective and viscous terms are discretized by the central finite difference of 2nd order accuracy. The Adams-Bashforth method of 2nd order accuracy is applied for time marching. These calculations are performed on JAXA Supercomputer System (JSS and JSS2).

3.4. Validation

The code and computational setup are validated by comparing them with previous DNS and experimental results of a two-flat-plate channel and standard straight riblets. Table 1 lists the setup parameters for validation cases. Reference data for each case are as follows: A two-flat-plate channel (DNS by Kim et al. (1987)), triangle-grooved straight riblets (experiment by Walsh (1982); DNS by Choi et al. (1993); experiment by Bechert et al. (1997)), and a trapezoidal-grooved straight riblet (experiment by Bechert et al. (1997)). Information regarding reference DNS is also listed in Table 1.

3.4.1. Drag Reduction Rates

The calculated drag reduction rates for cases 2-4 are shown in Table 2. The experimental (Walsh, 1982; Bechert et al., 1997) and DNS results (Choi et al., 1993) with the respective error bounds are also shown for comparison. The results are reasonable considering that error bounds of only ±1% were observed.

3.4.2. Turbulence Statistics

Figure 5 represents the turbulence statistic profiles at various spanwise locations of triangle-grooved straight riblets with a tip angle \(\alpha = 90^\circ\) (case 2 and 3). Variations among the locations are observed only near the riblets when \(s^+ = 20\), while the influence of riblet penetrates farther into the channel when \(s^+ = 40\). These tendencies are similar to the results of Choi et al. (1993). Table 3 represents the increase and decrease in the maximum values of turbulence intensities and Reynolds stress between the flat and riblet surfaces. Choi et al. (1993) reported that the maximum values decreased in the drag-reducing configuration (\(s^+ = 20\)), and increased at the tip in the drag-increasing configuration (\(s^+ = 40\)). The increase and decrease of present results show similar tendencies as Choi et al. (1993). There are differences between the present results and those reported by Choi et al. (1993). This may be due to the difference in tip angle: the tip angle of available data is 60° while they also computed the case of 90°. Nonetheless, the signs of most values are same as Choi et al. (1993).

Finally, as for case 1, the mean velocity, root-mean-square velocity fluctuation, and energy budget show agreement.
Fig. 5: Turbulence statistics of channel flow with triangle-grooved straight riblets. Black: bottom, red: middle point, green: tip.
with DNS data of Kim et al. (1987) (figure omitted).

4. Determination of Effective Parameters

4.1. Spanwise Amplitude $A$ and Wavelength $\lambda$

The effective combination of wavelength $\lambda$ and amplitude $A$ of traditional sinusoidal riblets is determined by parametrically conducting DNS. The configuration of the most effective combination will be the baseline for modification. Hereafter, the trapezoidal cross-section is employed. In this section, the influence of $\lambda$ and $A$ on the drag-reducing performance is assessed. The spacing and height of the riblet are fixed at $s^+ = 20$ and $h/s = 0.5$, respectively, which are located around the design point of straight riblets. Table 4 lists the computational cases and the resulting drag reduction rates. Cases 5, 8, 9, and 10 are set to examine the influence of $\lambda$ with a fixed angle $\beta$ (corresponding to $A/\lambda$). On the other hand, cases 6, 7, 8, and 11 are set to examine the influence of angle $\beta$ with a fixed $\lambda$.

The friction drag reduction rate is improved for each sinusoidal riblet compared to that of straight riblets. The total drag reduction rates are also improved in some cases with a maximum reduction rate of 7.8% (case 8), while the reduction rate of the comparable straight riblet (case 4) is 6.1%.

Grüneisenberger et al. (2012) conducted experiments on sinusoidal riblets with trapezoidal cross-sections having $h/s$ ratios of 0.5, which are similar to the ones used in the present study. They reported that $A/\lambda > 0.04$ deteriorated the drag-reducing performance, while the drag-reducing performance was similar to that of straight riblets when $A/\lambda$ was 0.04 or less. In contrast, Peet et al. (2009) reported from their LES results that the drag-reducing performance was improved with sinusoidal riblets having rectangular cross-sections, i.e., ‘blade riblets,’ compared to that of straight blade riblets: a total drag reduction rate of $-14.6\%$ was obtained using a similar configuration as case 8, i.e., $s^+ = 16$, $h/s = 0.5$, $A/\lambda = 0.031$, $\lambda^+ = 1080$, while a total drag reduction rate of $-11.2\%$ was obtained with comparable straight riblets. Despite the differences in cross-sectional shape, this tendency is consistent with present results.

Figure 6 represents the drag reduction rates for different values of $\tan \beta$, with a fixed wavelength $\lambda^+ = 1131$ (cases 4, 6, 7, 8, and 11). The larger the value of $\tan \beta$, the better the friction drag reduction rate. However, the total drag reduction rate deteriorates, except for two cases, when the value of $\tan \beta$ is less than 0.2 (cases 8 and 11). Particularly, the drag reduction is not observed when $\tan \beta$ is 0.628 (case 6). The solid lines in Fig. 6 represent the approximated curves obtained from the calculation point. The friction drag reduction rate is proportional to $\tan \beta$, and the pressure drag rate ($D_P/D_{RL}$) is proportional to $\tan^2 \beta$. Peet et al. (2009) reported that the pressure coefficient, $C_p$, is proportional to the $\tan \beta$. This corresponds to the present results. They suggested that this correlation between $C_P$ and $\tan^2 \beta$ was due to the conversion of transverse kinetic energy $k \sim (U \tan \beta)^2$. From the approximated curves of the total drag reduction rate (green line + black line), $\tan \beta = 0.1$ seems to be the most effective parameter. However, when $\tan \beta = 0.1$ and $\lambda^+ = 1131$ (case 11), the calculated total drag reduction rate is not the best among all the cases.

Cases with different wavelength $\lambda$ are compared. Figure 7 represents the drag reduction rate for various $\lambda^+$ with fixed
\[ \tan \beta \] (cases 5, 8, 9, and 10). It should be noted that the scale of ordinate is different from that of Fig. 6. The value of \( \lambda \) has little influence on the pressure drag. Thus, the pressure drag is determined mostly by the value of \( \tan \beta \). In addition, the difference in the friction drag for different values of \( \lambda \) is less than that for different values of \( \tan \beta \). The total drag reduction rate is the largest at \( \lambda^+ = 1131 \) among the four cases computed. Peet et al. (2009) predicted that the most effective wavelength was \( \lambda^+ \sim 1000 \). They concluded that a value of \( \lambda^+ \) that was larger or smaller than 1000 disrupted the self-maintenance cycle of turbulence in wall flows, which increased drag. The characteristic advection distance of streamwise vortex before breakup or the length of low-speed streak is approximately 1000 in the wall unit. The present results show that the total drag reduction rate is the largest at approximately \( \lambda^+ = 1000 \), as predicted by Peet et al. (2009). However, considering the error bounds, these four data points cannot infer that the local maximum is at approximately \( \lambda^+ = 1000 \). Thus, from this analysis, riblets having an angle \( \beta \) of 5–10° are found to be more effective than straight riblets, while the effective value of \( \lambda \) cannot be determined. Considering the above-mentioned discussion, the configuration of case 8 (\( \lambda^+ = 1131 \), \( \tan \beta = 0.188 \)) is employed as the baseline for modification, as it seems to be one of the most effective combinations.

4.2. Vertical Amplitude \( a \)

The influence of the vertical amplitude \( a \) on the drag-reducing performance is assessed to determine the effective value of \( a \). As discussed in the previous section, the configuration of case 8 (\( s^+ = 20 \), \( h/s = 0.5 \), \( \lambda^+ = 1131 \), \( \tan \beta = 0.188 \)) is employed as the baseline traditional sinusoidal riblet. Table 5 lists the computational cases and the resulting drag reduction rates. The vertical amplitudes of \( a/h = 0.05 \), 0.1, 0.2, and 0.3 are adopted. The data of the comparable straight and traditional sinusoidal riblets are also shown in Table 5. Figure 8 represents the cross-sectional computational meshes of each case at the node position.

Figure 9 represents the drag reduction rates for different values of \( a/h \). Both the friction drag reduction rate and the total drag reduction rate are certainly the smallest at \( a/h = 0.1 \). However, the drag reduction rates are almost unchanged considering the error bounds. No notable change is observed in the pressure drag rate \( D_f/D_{fl} \). On the contrary, the total drag reduction rate deteriorates at \( a/h \geq 0.2 \) along with the deterioration of the friction drag reduction rate. This may be because the downwash of high-speed fluid (sweep), which increases the Reynolds stress, can easily enter the riblet grooves owing to the lowered riblet height. Choi et al. (1993) proposed a well-known drag reduction mechanism of straight riblets: streamwise vortices followed by downwash cannot enter the riblet grooves with adequate spacing and height. The present result is consistent with this mechanism. This mechanism suggests that \( s^+ = 20 \), \( h/s = 0.5 \) are effective regardless of variations in streamwise configuration. Therefore, the design point of a riblet’s cross-section is usually set around \( s^+ = 20 \), \( h/s = 0.5 \). According to the data of cases 12-15, unfortunately, lowering the riblet height toward the node position of the sinusoidal riblet does not show the expected effect at around the design point. Nonetheless, the effect of the modified sinusoidal riblet when the flow condition deviates from the design point will be assessed in the following section. The vertical amplitude \( a/h \) is adopted as 0.1 because it has no bad effect on drag reduction at the design point.
Table 5: Drag reduction rates of modified sinusoidal riblets with trapezoidal cross-sections ($s^* = 20$, $h/s = 0.5$, $\lambda^* = 1131$ and $\tan \beta = 0.188$). Result of comparable straight (case 4) and traditional sinusoidal riblets (case 8) are also shown.

<table>
<thead>
<tr>
<th>case</th>
<th>$s^*$</th>
<th>$h^*$</th>
<th>$h/s$</th>
<th>$\lambda^*$</th>
<th>$A/A^*$</th>
<th>$\beta$/deg</th>
<th>$\tan \beta$</th>
<th>$a/h$</th>
<th>$D_{R_4}$</th>
<th>$D_{R_{total}}$</th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>20</td>
<td>10</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-6.2%</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
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<td>0.5</td>
<td>1131</td>
<td>0.03</td>
<td>0.188</td>
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<td>0</td>
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<td>-7.8%</td>
</tr>
<tr>
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<td>10</td>
<td>0.5</td>
<td>1131</td>
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<td>0.188</td>
<td>0.05</td>
<td>0</td>
<td>-9.7%</td>
<td>-7.1%</td>
</tr>
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<td>20</td>
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<td>0.5</td>
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<td>0.03</td>
<td>0.188</td>
<td>0.1</td>
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<td>-10.5%</td>
<td>-8.1%</td>
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<td>14</td>
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<td>10</td>
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<td>1131</td>
<td>0.03</td>
<td>0.188</td>
<td>0.2</td>
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<td>-8.9%</td>
<td>-6.5%</td>
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<td>0.3</td>
<td>0</td>
<td>-8.1%</td>
<td>-5.7%</td>
</tr>
</tbody>
</table>

(a) $a/h = 0$ (case 8) (b) $a/h = 0.05$ (case 12) (c) $a/h = 0.1$ (case 13) (d) $a/h = 0.2$ (case 14) (e) $a/h = 0.3$ (case 15)

Fig. 8: Cross-sectional computational meshes near the modified sinusoidal riblets at the node position.

Fig. 9: Drag reduction rate $s$ versus $a/h$ ($s^* = 20$, $h/s = 0.5$, $\lambda^* = 1131$, $\tan \beta = 0.188$).

5. Drag Reduction Rate versus $s^*$

The drag-reducing performance of the modified sinusoidal riblet is examined when the flow condition deviates from the design point. The target configuration is case 13, where $h/s = 0.5$, $\lambda/s = 55.36$, $A/A^* = 0.03$, and $a/h = 0.1$. Spacing $s^*$ is set as the parameter here. Table 6 lists the computational cases and the resulting drag reduction rates. Comparable straight and traditional sinusoidal cases are also computed. Here, case 30 (modified sinusoidal riblet, $s^* = 12$) was planned but not conducted: the computation was diverged at the small cell of the node position even if the time increment was quite small.

Figure 10 represents the drag reduction rates for various values of $s^*$. Experimental results of comparable straight riblets by Bechert et al. (1997) are also shown. Although there are differences of 1-2%, the present data qualitatively agree with their data; the value of $s^*$ for minimal drag reduction rate ($s^* = 17$) and the shape of the curve are same. These similarities indicate the validity of the present results on various values of $s^*$. The larger the value of $s^*$, the larger the pressure drag rate $D_p/D_{flat}$. However, $D_p/D_{flat}$ is almost constant compared to the friction drag reduction rate.

The total drag reduction rate of the traditional sinusoidal riblet is worse than that of the straight riblet at $s^* > 25$, while it better at $s^* = 17 - 25$. Thus, the drag-reducing performance of the traditional sinusoidal riblet deteriorates easily when the flow condition deviates from the design point. In contrast, a better drag reduction rate is observed also at a higher value of $s^*$ with the modified sinusoidal riblet. This indicates that flow-condition robustness is improved with the modified sinusoidal riblet.
Table 6: Drag reduction rates of the modified sinusoidal riblet (h/s = 0.5, λ/s = 55.36, tan β = 0.188 and a/h = 0.1) for various values of $s^*$. Results of comparable straight and traditional sinusoidal riblets are also shown.

<table>
<thead>
<tr>
<th>case</th>
<th>streamwise</th>
<th>$s^*$</th>
<th>$h^*$</th>
<th>$h/s$</th>
<th>$A^*$</th>
<th>$A/A^*$</th>
<th>$β$ [deg]</th>
<th>$tan β$</th>
<th>$a/h$</th>
<th>$D_{Req}$</th>
<th>$D_{Req}$ (modified)</th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>straight</td>
<td>20</td>
<td>10</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-6.2%</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
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<td>0.5</td>
<td>1131</td>
<td>55.36</td>
<td>0.03</td>
<td>10.7</td>
<td>0.188</td>
<td>-10.3%</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>modified</td>
<td>20</td>
<td>10</td>
<td>0.5</td>
<td>1131</td>
<td>55.36</td>
<td>0.03</td>
<td>10.7</td>
<td>0.188</td>
<td>-10.5%</td>
<td>-10.0%</td>
</tr>
<tr>
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<td>0.5</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-6.9%</td>
<td>-</td>
</tr>
<tr>
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<td>961</td>
<td>55.36</td>
<td>0.03</td>
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<td>0.188</td>
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<td>-</td>
</tr>
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<td>0.5</td>
<td>961</td>
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<td>0.03</td>
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<td>-10.6%</td>
<td>-8.4%</td>
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<td>0</td>
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<td>-</td>
</tr>
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<td>0.188</td>
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<td>-2.3%</td>
</tr>
<tr>
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</tr>
<tr>
<td>22</td>
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<td>0.5</td>
<td>-</td>
<td>-</td>
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<td>0</td>
<td>0</td>
<td>+2.1%</td>
<td>-</td>
</tr>
<tr>
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<td>1696</td>
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<td>+3.9%</td>
</tr>
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<td>0.188</td>
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<td>-</td>
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<td>-</td>
</tr>
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<td>0.5</td>
<td>2002</td>
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<td>0.03</td>
<td>10.7</td>
<td>0.188</td>
<td>+10.9%</td>
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</tr>
<tr>
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<td>modified</td>
<td>37</td>
<td>18.5</td>
<td>0.5</td>
<td>2002</td>
<td>55.36</td>
<td>0.03</td>
<td>10.7</td>
<td>0.188</td>
<td>+4.3%</td>
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</tr>
<tr>
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<td>straight</td>
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<td>0.5</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-6.3%</td>
<td>-</td>
</tr>
<tr>
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<td>6</td>
<td>0.5</td>
<td>679</td>
<td>55.36</td>
<td>0.03</td>
<td>10.7</td>
<td>0.188</td>
<td>-7.6%</td>
<td>-5.4%</td>
</tr>
<tr>
<td>30</td>
<td>modified</td>
<td>12</td>
<td>6</td>
<td>0.5</td>
<td>679</td>
<td>55.36</td>
<td>0.03</td>
<td>10.7</td>
<td>0.188</td>
<td>diverged</td>
<td>diverged</td>
</tr>
</tbody>
</table>

Fig. 10: Drag reduction rates versus $s^*$ ($h/s = 0.5, λ/s = 55.36, tan β = 0.188, a/h = 0.1$).

6. Discussion

The mechanism of robustness of a modified sinusoidal riblet is examined. In the first place, a modified sinusoidal riblet was developed with the aim of reducing the pressure drag by decreasing the surface area of the riblet’s side wall. Including this effect, the cause of robustness is assessed from the following aspects:

- Reduction in pressure drag owing to decreased surface area
- Reduction in friction drag owing to decreased surface area
- Reduction in momentum transfer over the modified sinusoidal riblet

6.1. Decrease of Surface Area

Figure 11 represents the ratio between the surface area of the riblet surface (lower wall), $A_{rib}$, and that of the flat surface (upper wall), $A_{flat}$, $A_{rib}/A_{flat}$ is separated into bottom and side wall contributions. In addition, for comparison with Fig. 13(a), the contribution from side wall is separated into those from the surface facing the main stream (“front wall”) and those from reverse face of the front wall (“back wall”). In the case of straight riblets, the contribution from the side wall is split into the contribution from the ‘front wall’ and ‘back wall,’ for the convenience of comparison between sinusoidal and modified sinusoidal riblets, although there is no difference in ‘front’ and ‘back’ faces in the case of straight riblets.
Fig. 11: The ratio of the surface area of the riblet surface to the flat surface \( (h/s = 0.5, \lambda/s = 55.36, \tan \beta = 0.188, a/h = 0.1) \). \( A_{\text{rib}} = A_{\text{rib}_{\text{twist}}} + A_{\text{rib}_{\text{tuck}}} + A_{\text{rib}_{\text{cut}}} \).

Fig. 12: Difference in drag reduction rates between sinusoidal and modified sinusoidal riblets \( (h/s = 0.5, \lambda/s = 55.36, \tan \beta = 0.188, a/h = 0.1) \).

Each ratio of sinusoidal riblets is similar to that of straight riblets. All three riblets have the same \( A_{\text{rib}_{\text{tuck}}} / A_{\text{flat}} \), \( A_{\text{rib}_{\text{twist}}} / A_{\text{flat}} \) and \( A_{\text{rib}_{\text{cut}}} / A_{\text{flat}} \) of the modified sinusoidal riblet is decreased by 9.3% compared to straight or sinusoidal riblets. The total \( A_{\text{rib}} / A_{\text{flat}} \) is decreased by 5.5%.

6.2. Effect of Decrease of Surface Area

Figure 12 represents the difference in drag reduction rates between the traditional sinusoidal riblet and modified sinusoidal riblets. The difference indicates the effect of the lowered side wall. The effect on both the pressure and friction drag is small at around the design point \( (s^+ = 17 - 20) \). In contrast, the friction drag is highly influenced by the decrease in the riblet’s side wall with a larger value of \( s^+ \), while the pressure drag is nearly unchanged.

The contribution to the friction drag from bottom, front, and back walls are examined. The cases of \( s^+ = 17 \) and \( s^+ = 30 \) are compared. The case of \( s^+ = 17 \) is adopted as the design point. The case of \( s^+ = 30 \) is adopted as the typical size with which the modified sinusoidal riblet shows its robustness. Figure 13(a) represents the ratio between each drag on the riblet surface (lower wall) and the total drag on the flat surface (upper wall). The friction drag on the riblet surface \( D_{\text{rib}} \) is separated into contributions from bottom, front, and back walls. It should be noted that the pressure drag on the riblet surface \( D_{\text{prb}} \) is not separated into contributions from front and back walls. For straight riblets, the contribution from the side wall is split into the contribution from ‘front’ and ‘back’ walls, as shown in Fig. 11. Figure 13(b) represents the difference of each contribution in Fig. 13(a) between the traditional sinusoidal and modified sinusoidal riblets, which indicates the effect of the lowered side wall.

As for the pressure drag, both \( D_{\text{prb}} / D_{\text{flat}} \) itself and the difference are small compared to those of the friction drag, as shown in Figs. 10 and 12. Nonetheless, the reduction in the pressure drag by the modified sinusoidal riblet is confirmed in Fig. 13(b). Furthermore, the effect of the decrease in the side-wall area at \( s^+ = 30 \) is larger than that at \( s^+ = 17 \), which contributes to robustness.

Here, the contribution to the surface area and the contribution to the friction drag from each wall are compared. As
shown in Fig. 11, $A_{hub, prev}/A_{flat}$ is larger than $A_{rib, prev}/A_{flat}$ and $A_{hub, back}/A_{flat}$. However, $D_{frich, prev}/D_{flat}$ is smaller than $D_{frich, back}/D_{flat}$ and $D_{frich, back}/D_{flat}$. This corresponds to the larger wall shear stress on the side wall; the side wall is exposed to turbulence vortices, while the flow stagnates inside the grooves. $D_{frich, back}/D_{flat}$ is smaller than $D_{frich, prev}/D_{flat}$ while $A_{rib, prev}/A_{flat}$ and $A_{rib, back}/A_{flat}$ are same. This suggests separation from the riblet tip. The flow may tend to stagnate behind the front wall.

Figure 13(b) shows that $D_{frich, prev}/D_{flat}$ of the modified sinusoidal riblet is larger than that of the traditional sinusoidal riblet. This is obvious because downwash can easily enter the grooves through the lowered side wall. It should be noted that the increase at $s^+ = 30$ is smaller than that at $s^+ = 17$. It can also be one of the reason for robustness. For a larger $s^+$, the diameter of the streamwise vortex is relatively small; thus, downwash can easily enter the grooves without the need to lower the side wall. Therefore, the difference between the traditional and modified sinusoidal riblets becomes smaller at a larger $s^+$.

In contrast, $\Delta D_{frich, prev}/D_{flat}$ and $\Delta D_{frich, back}/D_{flat}$ of the modified sinusoidal riblet are smaller than that of the traditional sinusoidal riblet. Moreover, the reduction at $s^+ = 30$ is larger than that at $s^+ = 17$, which also contributes to robustness. This may be attributed to not only the decrease in surface area but also the decrease in momentum transfer (discussed later).

6.3. Change of Momentum Transfer

Figure 14 shows the Reynolds shear stress for each quadrant of the traditional sinusoidal riblet and the modified sinusoidal riblet. Choi et al. (1993) reported that the second and fourth quadrants were reduced with a drag-reducing configuration, while they were increased at the riblet tips with a drag-increasing configuration. In contrast, the first and third quadrants were almost constant above the riblet surface.
Fig. 14: Reynolds shear stress $\overline{|\tau|^2}$ from each quadrant. At the riblet side, bottom of groove is set to be $y/\delta = 0$. Magenta: first, red: second, green: third, blue: fourth, black: total. Solid line: flat-surface side, circle: riblet side. Circle and x represent data at node and at anti-node respectively.
As shown in Fig. 14(a), the second and fourth quadrants are reduced with a drag-reducing configuration. Accordingly, the Reynolds stress is reduced. Same results are obtained with the traditional sinusoidal riblet at \( s^* = 17 \) (figure omitted). This tendency agrees with Choi et al. (1993). Riblets changed only the second and fourth quadrants, while the first and third quadrants were nearly unchanged. This implies that riblets modify only the organized motion of streamwise vortices, which are accompanied by sweep and ejection (Choi et al., 1993).

When \( s^* = 30 \), the second quadrant of the traditional sinusoidal riblet is increased, as shown in Fig. 14(b). Not only the peak value of the second quadrant, but also the value at \( y^*/\delta > 0.3 \) is increased compared to those above the flat surface side. Accordingly, the Reynolds stress is also increased especially at the anti-node position. In contrast, as shown in Fig. 14(c), the increase in the second quadrant of the modified sinusoidal riblet is not as much as that of the traditional sinusoidal riblet. Although the value at \( y^*/\delta > 0.3 \) is increased, the peak value is reduced compared to flat surface side. The increase in the second quadrant may be caused by spanwise sinusoidal configuration, as it is not observed with the comparable straight riblet (figure omitted). Therefore, the modified sinusoidal riblet suppresses the increase in the Reynolds stress at a larger \( s^* \) by the spanwise sinusoidal configuration.

7. Conclusion

A traditional sinusoidal riblet was modified with the aim of reducing the pressure drag. The riblet height is lowered toward the node of the sinusoidal curve, while the riblet height is maintained at the anti-node position as it has been reported to be the most effective for straight or traditional sinusoidal riblets. Effective configuration parameters and drag-reducing performance of the modified sinusoidal riblet are investigated by parametrically conducting DNS. The results are summarized as follow:

(1) Parametric analysis is conducted to determine the optimal wavelength \( \lambda \) and amplitude \( A \) of the spanwise sinusoidal curve configuration with a fixed riblet spacing and height, which are located at around the design point of straight riblets. The most effective wavelength and angle \( \beta = \tan^{-1}(2\pi A/\lambda) \) were \( \lambda^* \sim 1000 \) and \( \beta = 5 - 10^\circ \), respectively. These agree with the results of previous research.

(2) Then a parametric analysis is again conducted to determine the optimal vertical amplitude \( a \) of the modified sinusoidal riblet. Configuration parameters except \( a \), are fixed at the effective settings mentioned above. Unfortunately, with \( a/h < 0.2 \), the drag-reducing performance is nearly unchanged for the traditional sinusoidal riblet. Furthermore, the drag-reducing performance deteriorates with \( a/h \geq 0.2 \).

(3) However, the drag-reducing performance of the modified sinusoidal riblet is improved when \( s^* \) deviates from the design point compared to that of comparable straight and traditional sinusoidal riblets. This indicates that flow-condition robustness is improved with the modified sinusoidal riblet.

(4) This flow-condition robustness is caused by the following reasons:

- The pressure drag is suppressed.
- The friction drag at the bottom wall of the modified sinusoidal riblet is increased compared to that of the traditional sinusoidal riblet, because downward can easily enter the grooves. For a larger \( s^* \), the diameter of the streamwise vortex is relatively small; thus, downward can easily enter the grooves without the need to lower the wall. Therefore, when \( s^* \) is larger, the effect of the increase of the friction drag at the bottom wall is small.
- The friction drag at the side wall of the modified sinusoidal riblet is reduced compared to that of the traditional sinusoidal riblet. For a larger \( s^* \), the effect of the reduction is large. This may attributed to not only the decrease in surface area but also the decrease in momentum transfer: the modified sinusoidal riblet suppresses the increase in the Reynolds stress at a larger \( s^* \) by the spanwise sinusoidal configuration.

8. Appendix

Tables 7, 8, and 9 list the geometrical parameters of the computational domain. The cases correspond to those in Tables 4, 5, and 6, respectively. The grid points on each spacing is similar to those of Choi et al. (1993). In contrast, the streamwise length of the computational domain is \( 3 \sim 5.3 \) times longer than that used by Choi et al. (1993), as streaks were observed to be longer than the original domain length in the test calculations. The spanwise length is set to be \( 1.2 \sim 2.5 \) times longer than that used by Choi et al. (1993) such that the interaction of the turbulence structure can be fully observed. A uniform mesh with a spacing \( \Delta x^* = 7.5 \sim 11.0 \) is adopted in the streamwise direction. These \( \Delta x^* \) are much finer than 35.3, which was used by Choi et al. (1993).
Table 7: Geometrical parameters of computational domain for the cases listed in Table 4.

<table>
<thead>
<tr>
<th>case</th>
<th>$x^*$</th>
<th>$y^*$</th>
<th>$L_{0}/y^*$</th>
<th>$L_{1}/y^*$</th>
<th>$L_{2}/y^*$</th>
<th>$N_{x} \times N_{y} \times N_{z}$</th>
<th>$\Delta x^*$</th>
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<tr>
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<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>2 \times 0.28\pi</td>
<td>256 \times 128 \times 512</td>
</tr>
<tr>
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<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>2 \times 0.28\pi</td>
<td>256 \times 128 \times 512</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>2 \times 0.28\pi</td>
<td>256 \times 128 \times 512</td>
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<tr>
<td>9</td>
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<td>0.1135</td>
<td>$L_{0}/y^*$</td>
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<td>4</td>
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<td>256 \times 128 \times 512</td>
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<tr>
<td>10</td>
<td>20</td>
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<td>$L_{0}/y^*$</td>
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<tr>
<td>11</td>
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<td>$L_{0}/y^*$</td>
<td>16</td>
<td>2</td>
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Table 8: Geometrical parameters of computational domain for the cases listed in Table 5.

<table>
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<th>$y^*$</th>
<th>$L_{0}/y^*$</th>
<th>$L_{1}/y^*$</th>
<th>$L_{2}/y^*$</th>
<th>$N_{x} \times N_{y} \times N_{z}$</th>
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<td>-</td>
<td>16</td>
<td>-</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
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<tr>
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<td>0.1135</td>
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<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
<tr>
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<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
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</table>

Table 9: Geometrical parameters of computational domain for the cases listed in Table 6.

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<th>$y^*$</th>
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<th>$L_{2}/y^*$</th>
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</tr>
</thead>
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<td>-</td>
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<td>4\pi</td>
<td>2 \times 0.28\pi</td>
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<tr>
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<td>0.1135</td>
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<td>2 \times 0.28\pi</td>
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<tr>
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<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>0.1135</td>
<td>2\pi</td>
<td>16</td>
<td>2</td>
<td>4\pi</td>
<td>2 \times 0.28\pi</td>
</tr>
</tbody>
</table>

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