Numerical investigation of small fish accelerating impulsively to terminal speed

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Abstract
The present paper discusses acceleration in the swimming of a small three-dimensional fish with two motions, carangiform and anguilliform. Flow fields generated by fish deformations are investigated numerically by the constrained interpolation profile method in combination with an immersed boundary method. The three-dimensional vortical structure visualized using a second invariant and the pressure field around the fish body show that a fish with anguilliform motion accelerates more rapidly than one with carangiform motion because of a larger thrust due to the strong transverse vortex in the wake of the fish and a large pressure variation around the fish body. It is also found that the time variations of inline swimming speed of a small fish and the fluid force acting on it can be estimated using a free-fall model, and the fluid force can be expressed by a linear function of the fish speed. This function consists of a thrust part that is independent of fish speed and a viscous drag part that is proportional to fish speed. Thus, time histories of swimming speed, swimming distance, and fluid force can be predicted by simple functions from rest to terminal speed.

Key words: Small fish, Swimming, Acceleration, Fluid force, Fluid–structure interaction, CIP method, Numerical simulation, Vortical structure

1. Introduction

Fish morphology and motion are being applied increasingly to robotics as a new form of underwater propulsion that does not use screws. For example, biomimetic two-joint robots have been developed that capitalize on the high maneuverability and speed of swimming dolphins (Nakashima et al., 2006). Differences in thrust and lift between robots modeled on dolphins and sea turtles have been discussed to establish an efficient propulsion system; these robots have great potential for applications such as water quality surveys in particularly shallow areas (Hosotani et al., 2010). In addition, small aquatic robots (e.g., those the size of a killifish; about 5 cm long) powered by compact fuel cells are expected to become available as lightweight and long-running propulsors (Takada et al., 2010).

A great deal of knowledge gleaned from fish swimming has been used to design propulsion systems and develop their swimming capabilities. Lighthill (1960, 1969) established the fundamental theoretical aspects of fish swimming, and Newman (Newman, 1973; Newman and Wu, 1973) proposed generalized slender-body and fluid-force models for a fish-like body. Since then, a general equation has been suggested to represent the Froude efficiency of a slender body placed in a uniform potential flow in relation to the motion of the body (Cheng and Blickhan, 1994). From an experimental point of view, cumulative observations and measurements have led to bending rules for natural propulsors (including fish) during steady motion—a mean flexion ratio of 0.65 and a mean flexion angle of 26.5° (Kelsey et al., 2014)—and the fact that the Strouhal number $St$ of most swimming animals is in the range of 0.25–0.35 (Eloy, 2012). The flow velocity fields generated by swimming fish have been gradually revealed by measurements such as those made using particle image velocimetry (PIV) (Lauder et al., 2007). Visualizations using tail dye tubes have shown three-dimensional chain-linked vortex-ring structures in the wake of robotic fish, the hydrodynamic forces on which have also been obtained (Tan et al., 2007).

Moreover, numerical studies have helped us to understand the mechanisms of fish swimming by showing that the
fluid forces generated by a moving fish are correlated with the vortical structures in its wake. For example, two-dimensional numerical simulations of the flow fields induced by a deforming fish-like airfoil placed in a uniform flow established that a negative drag coefficient (i.e., against the flow direction) is possible; the foil achieved large net thrust when a strong jet-like flow as a result of a reversed Kármán vortex street appeared in its wake (Akimoto and Miyata, 1993; Nakaoka and Toda, 1994). Recently, pioneering three-dimensional simulations (Liu et al., 1997; Liu and Kawachi, 1999) have succeeded in reproducing the complicated chain-linked vortex-ring structures that are seen behind swimming live fish. There is now much discussion about fish swimming efficiency and net thrust in relation to vortical wake structures and surface pressure distributions of aquatic bodies. By considering a reversed two-dimensional Kármán vortex street, some numerical studies have shown a correlation between fish acceleration and vortical wake structures (Borazjani et al., 2008, 2009, 2010; Brücker and Bleckmann, 2007).

One of the present authors has previously investigated the self-propulsion of a two-dimensional deforming NACA airfoil as an analog of a small fish with moderate Reynolds numbers of the order of $10^2$–$10^4$, and has proposed a simple model for predicting the speed of its center of mass (Ogata and Ogasawara, 2012). The present paper extends that work to a small three-dimensional fish that is accelerated by fluid forces that it generates by swimming. Whereas Li et al. (2012) established the speed of the center of mass of a zebrafish by three-dimensional computation and Borazjani et al. (2010) discussed the speed of some different types of fish, the present paper shows that the speed in three dimensions can still be predicted by our previous two-dimensional model.

In this paper, we investigate numerically the acceleration of a small three-dimensional fish (e.g., a killifish) that commences swimming via an impulsive start. Although most fish have dorsal and pectoral fins, those of a killifish are relatively small; it is thought to swim using mainly its caudal (tail) fin. Therefore, studies of killifish swimming are considered to be directly and readily applicable to fish-like robots that have no fins other that a caudal fin. Specifically, we use the constrained interpolation profile (CIP) method (Yabe and Aoki, 1991; Xiao et al., 1996) with an immersed boundary method (Saiki and Biringen, 1996; Nishida and Tajiri, 2009) to simulate the interactions between a laminar flow and a deforming body in a Cartesian grid system. In Section 2, we introduce the basic equations and numerical methods for the killifish model and the flow interactions. In Section 3, we present flow fields of the vortical structure and pressure fields for different Reynolds numbers and deformation models. In Section 4, we discuss fish acceleration in relation to the previous two-dimensional study, and we present concluding remarks in Section 5.

2. Basic equations and numerical methods

The fish shape considered here is that of a small killifish. Figure 1(a) and (b) shows vertical views of two types of fish motion defined later by Eq. (4), and Fig. 1(c) and (d) shows side and rear views, respectively. The fish is covered with triangular meshes for the purposes of the immersed boundary method.

![Fig. 1. Geometry of killifish covered with triangular meshes: (a) and (b) vertical views of model (A) and (B), respectively; (c) side view; (d) rear view.](image)

When a fish of density $\rho_f$ and volume $V$ begins to swim in stationary fluid of density $\rho_f$, the fish is accelerated by fluid forces. We assume here that the fish swims only forward in the $x$ direction by means of cosine-wave deformation, and that any initial rotation or buoyancy of the fish can be neglected. Takizawa et al. (2005) simulated the sudden turn of a killifish at the beginning of swimming (known as a C-start) by imposing a simple
folding and unfolding motion around its center of gravity and by varying the position of the center of gravity by changing the density distribution in the body of the fish. Such translational and rotational accelerations may be effective for a fast start by a live fish (Tytell and Lauder, 2002; Borazjani et al., 2012). However, here we deal with the fundamental aspects of fish acceleration; we do not take the center of gravity into account, and a time average of the lateral force in one period is zero during swimming in the present paper. Likewise, we neglect any unbalanced moments or forces on the grounds that neither fish nor small robots tend to bend their own bodies, as this would induce rotation that would interfere with purely horizontal travel. Thus, the fish velocity $u_0(t)$ obeys Newtonian mechanics as follows:

$$M \frac{du_0(t)}{dt} = F_x = P + D,$$

$$P = \int_{\text{surface}} (-p n_x) dA,$$

$$D = \int_{\text{surface}} (\tau_{xj} n_j) dA,$$

where $M = \rho_b V$ is the mass of the fish and $F_x$ is the $x$ component of the fluid force obtained from the surface integrals $P$ and $D$ of pressure $p$ and viscous stress $\tau_{xj}$, respectively. Equation (1) is normalized using reference length $L$ and reference speed $U$ as follows:

$$a \dot{V} \frac{d\dot{u}_0}{dt} = \dot{F}_x = \frac{F_x}{\rho_f U^2 L^2} \equiv \dot{P} + \dot{D},$$

where $\dot{i} = U/t/L$, $\dot{u}_0 = u_0/U$, and $\dot{V} = V/L^3$ are dimensionless time, fish velocity, and fish volume, respectively. Here, the reference length $L$ is taken as the fish body length. The values of body-to-fluid density ratio $\alpha = \rho_b/\rho_f$ explored here are $\alpha = 1$ and $4$. For a live fish, a value of $\alpha \approx 1$ allows it to swim smoothly in water (Lighthill, 1960). However, we expect $\alpha > 1$ for a robot because of its solid body parts (e.g., surfaces, motors).

We employ Eq. (3) for the lateral traveling-wave oscillations of the fish midline, and we use two forms of amplitude $H(x)$ as in Eq. (4):

$$h(x,t) = H(x) \cos \left( \frac{2\pi}{\lambda} (X - f \delta t) \right), \quad \frac{dh(x,t)}{dt} = 2\pi \dot{f} H(x) \sin \left( \frac{2\pi}{\lambda} (X - f \delta t) \right),$$

$$H(x) = \begin{cases} A_{1\alpha} \left( \frac{13}{8} X^2 - \frac{33}{40} X + 0.2 \right), & \text{(A)} \vspace{0.5em} \\
A_{2\alpha} \left( -(X-1)^2 + 1 \right), & \text{(B)} \end{cases}$$

where $\lambda$, $f$, $A_{1\alpha} = (0.1)$, and $X = \hat{x} - \hat{x}_0 (0 \leq X \leq 1)$ are, respectively, the dimensionless wavelength, frequency, tail amplitude, and distance from the fish head located at $\hat{x}_0$; the head and tail correspond to $X = 0$ and $X = 1$, respectively. Model (A) in Fig. 1(a) is intended to simulate carangiform locomotion, in which essentially only the rear of the fish body moves (Dong and Lu, 2005; Morgansen et al., 2002; Borazjani and Sotiropoulos, 2008). Model (B) in Fig. 1(b) is intended to simulate anguilliform locomotion, in which the entire body moves like an eel swimming (Akimoto and Miyata, 1993; Borazjani et al., 2010). Models (A) and (B) in Eq. (4) are concave and convex functions, respectively. Thus, fish swimming characteristics under any kinds of function $H(x)$ between (A) and (B) (Ogata and Ogasawara, 2012) can be also presumed by results shown later.

![Fig. 2. Thickness $Z_{th}(X)$ of the fish in Fig. 1(c) from head to tail.](image)

The $x - y$ cross section of the fish in Fig. 1(a) and (b) is that of a NACA0012 airfoil whose centerline bends as in Eq. (3). We define the thickness $Z_{th}(X)$ in the $x - z$ plane of Fig. 1(c) using the form shown in Fig. 2:
\[ Z_{th}(X) = 0.2 \cdot \left(12.349X^2 - 24.783X^4 + 18.459X^3 - 8.391X^2 + 2.736X + 0.031\right) \quad (0 < X \leq 1), \]

where the maximum and minimum thickness are taken at \( Z_{th}(X_{\text{max}} = 0.427) = 0.0917 \) and \( Z_{th}(X_{\text{min}} = 0.860) = 0.0343 \), respectively, and we set \( Z_{th}(X = 0) = 0 \). However, the fish midline, which corresponds to the \( x \) axis in Fig. 2, is slightly bent to make the non-symmetric shape of Fig. 1(c). The cross section of the fish in the \( y-z \) plane is an ellipse whose major and minor axes are determined using NACA0012 and \( Z_{th}(X) \), respectively, at \( 0 < X \leq 1 \). The normalized volume of the present fish, \( \dot{V} = 9.0 \times 10^{-3} \) in Fig. 1, can be obtained using a volume integral (a sum of thin elliptical disks) in the \( x \) direction.

The incompressible Navier–Stokes equations are also normalized, and we use a non-inertial coordinate system in which the fish with velocity \( \mathbf{U}_f(t) = (\dot{u}_f(t), 0, 0) \) is fixed in space. Therefore, the normalized fluid velocity \( \mathbf{u} \) in the inertial coordinate system is replaced with \( \mathbf{u}^* = \mathbf{u} - \mathbf{U}_f(t) \) in the non-inertial coordinate system, giving the dimensionless Navier–Stokes equations as

\[ \nabla \cdot \mathbf{u}^* = 0, \]

\[ \frac{\partial \mathbf{u}^*}{\partial t} + (\mathbf{u}^* \nabla) \mathbf{u}^* = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}^* + \mathbf{G}^* - \frac{d\mathbf{U}_c}{dt}, \]

where \( \hat{p} = p/p_0 \) with a reference pressure \( p_0 = \rho U^2 \). The initial pressure and velocity of the fluid are \( \hat{p}(x, t=0) = 1.0 \) and \( \mathbf{u}^*(x, t=0) = (0, 0, 0) \), respectively, and the fish is then accelerated according to Eq. (2), noting that \( \dot{u}_f(t) < 0 \) when \( F_s < 0 \) in the inertial coordinate system. Hence, the inflow boundary of the \( x \) component of velocity is \( \mathbf{u}^*(x = 0, t) = -\dot{u}_f(t) > 0 \) in the non-inertial coordinate system. The virtual boundary (VB) method (Nishida and Tajiri, 2009), which is one of the immersed boundary methods used for fluid–structure interaction (FSI), is employed to identify a deforming body in a fluid, and an external force \( \mathbf{G}^* \) is also added:

\[ \mathbf{G}^* = (\mathbf{u}^* \nabla) \mathbf{u}^* + \nabla \hat{p} - \frac{1}{Re} \nabla^2 \mathbf{u}^* + \frac{d\mathbf{U}_c}{dt} + \frac{\mathbf{U}_p^* - \mathbf{u}^*}{\Delta t}, \]

where \( \mathbf{U}_p^* \) is either the fluid velocity interpolated at immersed boundary (IB) points near the surface or the deformation speed inside the body. A quadratic interpolation is used for the IB points to precisely capture the velocity change in the boundary layer on the body surface; the velocity at these points becomes \( \mathbf{U}_p^* \) by \( \mathbf{G}^* \). The final term on the right-hand side of Eq. (7) is the fish acceleration, which can be obtained directly from Eq. (2). Equation (2) is solved using a first-order explicit scheme, and Eqs. (6) and (7) are solved using the CIP–CUP (combined unified procedure) method (Yabe et al., 2001) with a fractional step and a bi-conjugate gradient stabilization (Bi-CGSTAB) method to solve the Poisson-type equation for calculating pressure such as that used in the previous two-dimensional study (Ogata and Ogasawara, 2012).

It should be noted that there is no characteristic flow speed in this problem because the fish is at rest initially. Hence, we take the phase speed \( U = Lf_0 \equiv c_0 \) of the fish as the reference speed to define the Reynolds number \( Re = L^2 f_0 / V_f \), where \( V_f \) and \( f_0 \) are the kinematic viscosity of the fluid and a reference frequency of the fish deformation, respectively. For example, we have \( Re \approx 4500 \) in water (\( V_f = 1.0 \times 10^{-7} \text{ m}^2/\text{s} \)) for a typical killifish body length \( L = 3.0 \text{ cm} \) and an oscillation frequency \( f_0 = 5.0 \text{ Hz} \). Hereinafter, we omit the “\(^*\)” sign from dimensionless variables for ease of notation.

Next, we consider an acceleration model of a three-dimensional deforming proto-killifish. The fish in Fig. 1 impulsively begins to deform at \( t = 0 \) in stationary fluid, following Eq. (3) with \( H(x) \) of Eq. (4). The Reynolds number is in the range 300–9,000, and both wavelength \( \lambda \) and frequency \( f \) are 1.0. The computational regions are \( (0.0 \leq x \leq 28.53, 0 \leq y \leq 5.89, 0 \leq z \leq 7.75) \) and the numbers of grid points are \( (N_x, N_y, N_z) = (190, 90, 100) \). Each space is divided into nonuniform sections with minimum widths \( (\Delta x_{\text{min}}, \Delta y_{\text{min}}, \Delta z_{\text{min}}) = (2.00 \times 10^{-2}, 8.69 \times 10^{-3}, 8.00 \times 10^{-3}) \) near the fish. The fish head is located at \( (x_0, y_0, z_0) = (11.193, 2.945, 3.875) \). The calculations are conducted until \( t = 20.0 \) and 30.0 for \( \alpha = 1.0 \) and 4.0, respectively.
3. Flow patterns of a small fish during acceleration

First, we focus on the three-dimensional vortical structures and pressure fields of the swimming fish. Various numerical simulations of the interactions between swimming fish and the associated flows have shown that fish shed three-dimensional ring vortices from the trailing edge of their tails by effectively “kicking” fluid in their wake. Flow is also induced by regions of positive or negative pressure around the body of the fish. Both these effects lead to the production of net thrust (Liu et al., 1999; Borazjani et al., 2010; Li et al., 2012).

The vortical structure can be visualized by means of the second invariant $Q = \left( \Omega_{ij}, \Omega_{ij} - S_{ij}, S_{ij} \right) / 2$, where $S_{ij}$ and $\Omega_{ij}$ are symmetric and antisymmetric tensors of the velocity gradient, respectively (Chong, 1990; Jeong and Hussain, 1995). Flow fields such as those of the second invariant can be depicted in inertial coordinates by a simple coordinate transformation to allow the fish acceleration to be seen clearly.

Figures 3 and 4 compare the pressure distributions at $t = 8.0$ near the fish body deforming according to model (A) or (B), respectively, for $\alpha = 4.0$ and $Re = 1,000$ or 9,000. The larger deformations associated with model (B) lead to larger pressure variations around the belly and tail fin because of the larger deformation velocity $dh/dt$ of the fish midline in Eq. (3). Note that the pressure distribution of each fish does appear to change appreciably with Reynolds number. This is because the pressure near the body of a fish is determined mainly by its shape and its deformation velocity $dh/dt$, which includes tail amplitude and swimming frequency in Eq. (3) and is proportional to the amplitude function $H(x)$. Therefore, the fluid around the body surface under model (B) is pushed strongly in the positive $x$ direction, the reaction to which causes the fish to accelerate more than it does under model (A).

Next, Fig. 5 displays time histories of the vortical structure under model (A) for $Re = 9,000$, and Fig. 6 shows snapshots of the vortical structures at $t = 8.0$ for increasing Reynolds number; Figs. 7 and 8 do the same for model (B). It is found in Figs. 6 and 8 that fish moving using either model (A) or (B) can accelerate rapidly with increasing Reynolds number, and that complicated 3-D chain-linked vortex-ring structures and strong double-row vortices (Borazjani et al., 2010; Tan, G.-K, 2007) remain in the wake of each fish after the same number of kicks. In contrast, at smaller Reynolds numbers such as $Re = 1,000$ in Figs. 6 and 8, vortices diffuse because of the greater influence of viscosity. As already mentioned in Section 1, the reversed Kármán vortex street generated by a deforming wing is a source of jet-like flow behind its tail in two dimensions. Here, we consider the strong vortices generated by a three-dimensional fish just behind its tail (Brücker and Bleckmann, 2007) also to be a source of aquatic propulsion with increasing Reynolds number. In addition, the small viscous shear stress on the body surface is regarded as another reason that fish can swim quickly at large Reynolds numbers. This aspect will be discussed in the next section.

Fig. 3. Pressure at $t = 8.0$ for $Re = 1,000$ and 9,000 for model (A) with $\alpha = 4.0 : p = 1.015$ (orange); $p = 0.985$ (green). Left and right columns are vertical and lateral views, respectively.

Fig. 4. Pressure at $t = 8.0$ for $Re = 1,000$ and 9,000 for model (B) with $\alpha = 4.0 : p = 1.015$ (orange); $p = 0.985$ (green). Left and right columns are vertical and lateral views, respectively.
Fig. 5. Isosurface of second invariant $Q = 0.1$ for model (A) with $\alpha = 4.0$ and $Re = 9,000$ at initial state and $t = 1.0$, 3.0, and 5.0. Left and right columns are vertical and lateral views, respectively.

Fig. 6. Isosurface of second invariant $Q = 0.1$ for model (A) with $\alpha = 4.0$ for $Re = 1000, 3000, 5000,$ and $9000$ at $t = 8.0$. Left and right columns are vertical and lateral views, respectively.
Fig. 7. Isosurface of second invariant $Q = 0.1$ for model (B) with $\alpha = 4.0$ and $Re = 9,000$ at initial state and $t = 1.0$, 3.0, and 5.0. Left and right columns are vertical and lateral views, respectively.

Fig. 8. Isosurface of second invariant $Q = 0.1$ for model (B) with $\alpha = 4.0$ for $Re = 1000, 3000, 5000$, and 9000 at $t = 8.0$. Left and right columns are vertical and lateral views, respectively.
4. Acceleration models of a small three-dimensional fish

4.1 Comparison and evaluation of fish speed

Next, we investigate the acceleration characteristics of swimming fish. Figure 9(a) shows examples of the time histories of fish speed \( U_F(t) \) and fluid force \( F(t) \) in Eq. (2) under model (A) for \( \alpha = 1.0 \) and \( Re = 5,000 \). It is found that the fish speed \( U_F \) increases (Borazjani, 2010) whereas fluid force \( F \) decreases with temporal oscillations because of the wavy deformation.

A time average in each period, defined as follows, is introduced to clearly evaluate fish swimming:

\[
\bar{a}(n) = \frac{1}{T} \int_{t=nT}^{(n+1)T} a(t) dt, \quad a = (U_F, F),
\]

where \( n \) is the number of motion periods of the fish. Equation (9) gives discrete data with respect to time, as shown in Fig. 9(b). At the beginning of swimming, the average fish speed \( \bar{U}_F(n) \) increases with time because of the large fluid force \( \bar{F} \). However, this average force gradually approaches zero as the fluid force \( F \) becomes oscillatory around zero, as shown in Fig. 9(a). Eventually, the average fish speed \( \bar{U}_F(n) \) approaches an asymptotic value known as the terminal speed. In other words, the fish is able to swim at a constant speed because \( P \) (thrust) and \( D \) (drag) are in equilibrium. The maximum numbers of motion periods are \( n = 20 \) and \( 30 \) for \( \alpha = 1.0 \) and \( 4.0 \), respectively, because it takes longer for a fish with a larger value of \( \alpha \) to reach terminal speed. From now on, we use time histories of mean horizontal speed \( \bar{U}_F(n) \) and fluid force \( \bar{F} \) to evaluate fish acceleration, and we omit the “~” sign representing time-averaged variables for ease of notation.

![Fig. 9.](image)

Fig. 9. (a) Time histories of fish speed \( U_F \) (black line) and fluid force \( F \) (red line) under model (A) for \( \alpha = 1.0 \) and \( Re = 5,000 \). (b) Time averages (Eq. (9)) of \( \bar{U}_F(n) \) (black circles) and \( \bar{F} \) (red squares) in each period of Fig. 9(a).

![Fig. 10.](image)

Fig. 10. Comparisons of time-averaged histories at \( Re = 5,000 \): (a) fish speed \( U_F \); (b) fluid force \( F \). Red: (A), blue: (B). Full circles: \( \alpha = 1.0 \), open squares: \( \alpha = 4.0 \).

Figure 10 compares average fish speeds and fluid forces under models (A) with (B) for \( \alpha = 1.0 \) and \( 4.0 \). Again, it can be seen that fish swim faster under model (B) than they do under model (A), as was shown also in Figs. 3–8. When designing an aquatic robot, it is particularly important to be able to predict time histories of speed and fluid force.
using relatively simple functions of time; these will clearly depend on the Reynolds number, as shown in Figs. 6 and 8.

Time histories of $P$, $D$, and $F = |P + D|$ in Eq. (2) for $\alpha = 4.0$ are plotted in Fig. 11 to establish their functional dependence on Reynolds number. At the beginning of swimming under both models, a large fluid force $F$ is generated mainly by a large pressure force $P$ that pushes stationary fluid around the fish body, whereas $D$ is still relatively small because a boundary layer has not yet developed. Thus, the fish will be accelerated by $F$ or mainly $P$. It is found that the value of $F$ under model (B) in Fig. 11(b) is approximately three times larger than that under model (A) in Fig. 11(a).

The larger absolute values of $P$ under model (B) can also be understood by comparing Figs. 3 and 4. However, with time, both $P$ and $D$ converge on certain values and $F$ approaches zero for all Reynolds numbers. This is because $|P|$ gradually decreases because pressure around a fish head increases by Bernoulli’s principle with increasing fish speed, whereas $D$ increases because of the viscous shear stress generated by the boundary layer on the body surface as it accelerates; eventually, the fish swims at a constant speed. Under large Reynolds number, $D$ is small such as friction coefficient of a flat plate for laminar boundary layer, and the absolute value of $P$ is also small because pressure around a head also becomes large by large fish speed owing to acceleration, but pressure around the fish belly and tail is almost determined by lateral deformation $H(x)$, thus, the pressure is not so changed and difference of pressure field mainly appears around a fish head under different Reynolds numbers. The characteristic can be clearly seen in Fig. 4.

It is also found in Fig. 11 that although the absolute values of the pressure and viscous forces $|P| + |D|$ diminish, $F = |P + D|$ increases with Reynolds number because $|D| = D$ becomes appreciably smaller than $|P|$. However, the time histories of both $P$ and $D$ do not change very much for $Re > 3,000$. Next, we discuss the correlation between speed $U_F$ and fluid force $F$ in Eq. (2) to consider the acceleration performance of a fish.

![Fig. 11. Dependence of $P$, $D$, and $F = |P + D|$ in Eq. (3) on the Reynolds number for $\alpha = 4.0$. Columns (A) and (B) correspond to models (A) and (B), respectively.](image-url)
4.2 Estimation of fish speed

When time histories of both fish speed and fluid force are obtained as \( U_f(t) \) and \( F(t) \), the fluid force can also be expressed as a function of fish speed as \( F(U_f) \). Figures 12 and 13 show \( F(U_f) \) under both models (A) and (B) for \( \alpha = 1.0 \) and 4.0, respectively. The discrete data of fish speed after the first two periods are interpolated by dashed lines. Figures 12 and 13 indicate that the dimensionless fluid force \( F \) is a linear function of dimensionless fish speed \( U_f \) after the first two periods, regardless of the deformation mode or density ratio \( \alpha \). Therefore, if \( F \) is composed of a constant thrust \( a \) and a drag \( bU_f \) as \( F = a - bU_f \), then we can obtain the time history of the fish speed \( U_f(t) \) as

\[
a \dot{V} \frac{d\dot{U}_{\text{est}}}{dt} = F(\dot{U}_{\text{est}}) = a - b \dot{U}_{\text{est}},
\]

where \( \dot{U}_{\text{est}}(t) \) is an estimated solution of fish speed \( U_f(t) \) and \((a,b)\) are dimensionless positive coefficients; the “\(^\wedge\)“ sign is only shown again here to distinguish between dimensional and dimensionless variables. In addition, the original dimensional form of Eq. (10) for the dimensional fish speed \( U_{\text{est}} = U_{\text{est}} \) is

\[
m \frac{dU_{\text{est}}}{dt} = a \cdot \rho \cdot L^2 \dot{U}^2 - b \cdot \rho \cdot L^2 U \cdot U_{\text{est}} = a \cdot \rho \cdot L^4 f_0^2 - b \cdot \rho \cdot L^3 f_0 \cdot U_{\text{est}}, \quad m = \rho_b V
\]

The right-hand side of Eq. (11) consists of a constant first term and a second term that is proportional to \( U_{\text{est}} \). In the previous two-dimensional simulations (Ogata and Ogasawara, 2012), it was found that the speed of a two-dimensional deforming airfoil could also be estimated using Eq. (10). Indeed, Eqs. (10) and (11) are similar to that for a freely falling sphere of diameter \( d \):
Fig. 14. Time histories of fish speed for various Reynolds numbers for $\alpha = 1.0$. Columns (A) and (B) correspond to models (A) and (B), respectively. Upper: whole calculation; lower: enlarged figure around beginning of deformation. Points are simulation results, and solid lines are fitted curves of Eq. (13).

Fig. 15. Time histories of fish speed for various Reynolds numbers for $\alpha = 4.0$. Columns (A) and (B) correspond to models (A) and (B), respectively, as in Fig. 14.
\[
m \frac{dU_{sp}}{dt} = F(U_{sp}) = mg - 3\pi \mu du \cdot U_{sp}, \quad m = \rho V \frac{1}{6} \pi d^3. \tag{12}
\]

This is Stokes' law for small (viscosity dominated) Reynolds numbers, where \( \mu \) is viscosity and \( U_{sp}(t) \) is the free-fall speed of the sphere, which is accelerated by the constant acceleration of gravity and decelerated by viscous drag.

The first and second terms on the right-hand side of Eq. (10) can be regarded as thrust and drag, respectively; they correspond to the acceleration due to gravity and the viscous drag in Eq. (12) by comparison with Eq. (11). The exact solution \( U_{est} \) of Eq. (10), the swimming distance \( L_{est}(t) \), and \( F_{est} \) can be easily obtained as follows:

\[
U(t)_{est} = \frac{a}{b} \left[ 1 - \exp \left( -\frac{b}{V} t \right) \right], \tag{13}
\]

\[
L(t)_{est} = \int U(t)_{est} dt = \frac{a}{b} \left[ t + \frac{\alpha V}{b} \exp \left( -\frac{b}{\alpha V} t \right) \right] - \frac{\alpha a V}{b^2} = \frac{a}{b} \left[ t + \frac{\alpha V}{b} \left( \exp \left( -\frac{b}{\alpha V} t \right) - 1 \right) \right]. \tag{14}
\]

\[
F(t)_{est} = \alpha V \frac{dU}{dt} = a - b U_{est} = a \exp \left( -\frac{b}{\alpha V} t \right). \tag{15}
\]

Figures 14 and 15 show time histories of fish speed \( U_f \) interpolated by \( U_{est} \) of Eq. (13) for \( \alpha = 1.0 \) and \( 4.0 \), respectively, for various Reynolds numbers. The coefficients \( (a, b) \) are obtained from \( F(U_f) \) in Eq. (10); we used the least-squares method for the dashed lines in Figs. 12 and 13.

![Fig. 16. Dependence of coefficient \( a \) on \( Re \) under models (A) and (B) for \( \alpha = 1.0 \) (full circles) and \( 4.0 \) (open squares). Points are simulation results, and both dashed and solid lines are fitted curves of Eq. (16) with Table 1.](image)

![Fig. 17. Dependence of coefficient \( b \) on \( Re \) under models (A) and (B) for \( \alpha = 1.0 \) (full circles) and \( 4.0 \) (open squares). Points are simulation results, and both dashed and solid lines are fitted curves of Eq. (16) with Table 1.](image)

Equation (13) deviates from the simulation results \( U_f \) in Figs. 14 and 15 in only the early stages of swimming. It
is also found from Figs. 12 and 13 that the fluid force \( F(U_f) \) deviates from the dashed straight lines \( F(U_{est}) \) at the beginning of swimming for all Reynolds numbers, and especially for \( Re = 1,000 \). A possible reason for this is the initial condition of the lateral oscillation of the fish midline. The initial fish deformation speed, which is the time derivative of \( h(x,t) \), is thought also to be zero at \( t=0 \) and then to increase to the correct value of \( \frac{dh}{dt}(x,t) \) in Eq. (3) within a short time. However, \( \frac{dh(x,t=0)}{dt} \) is not zero in the present simulations. Therefore, the initial fish deformation speed \( \frac{dh(x,t=0)}{dt} \) here is considered to impart an appreciable amount of momentum to the fluid surrounding the fish at the beginning of deformation. The actual acceleration will be suppressed more at smaller Reynolds numbers because of the increasing effect of viscosity on the body surface as a result of the impulsive deformation. In contrast, Figs. 12 and 13 show that this effect diminishes as the Reynolds number is increased. Nevertheless, Eq. (13) can be used to describe the simulation results in general under both swimming models and density ratios for any Reynolds number. The numerical solutions for \( Re = 1,000 \) would also become identical to Eq. (13) if the initial lateral oscillation speed could be adequately modified.

Figures 16 and 17 show the variations of coefficients \((a,b)\) with Reynolds number. From these figures, we suggest the following approximate functions:

\[
a \cong \frac{Re}{C_1 + C_2 \cdot Re} = \frac{1}{C_1 + \frac{Re}{C_2}}, \quad b \cong \frac{\sqrt{Re}}{C_3 + \frac{C_4}{\sqrt{Re}}} = \frac{1}{\sqrt{Re} + \frac{C_4}{\sqrt{Re}}},
\]

(16)

where the pairs \((C_1,C_2)\) and \((C_3,C_4)\) are positive parameters that depend on fish properties such as shape and deformation. The solid and dashed curves in Figs. 16 and 17 are from Eq. (16) with coefficients \((C_1,C_2)\) and \((C_3,C_4)\) that were also obtained using the least-squares method and that are listed in Table 1. For all conditions (models (A) and (B), \( \alpha = 1.0 \) and 4.0) in the present paper, the fitted functions agree well with the simulation results, although coefficient \( b \) in Fig. 17 experiences some deviation for \( \alpha = 4.0 \).

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0(A)</td>
<td>2,235,402.7</td>
<td>690,605.0</td>
<td>484,064.7</td>
<td>134,781.3</td>
</tr>
<tr>
<td>4.0(A)</td>
<td>776.10</td>
<td>794.95</td>
<td>289.38</td>
<td>281.88</td>
</tr>
<tr>
<td>1.0(B)</td>
<td>10,985.8</td>
<td>2,906.5</td>
<td>4,927.4</td>
<td>1,394.1</td>
</tr>
<tr>
<td>4.0(B)</td>
<td>2,264</td>
<td>2,931</td>
<td>1,296</td>
<td>1,592</td>
</tr>
</tbody>
</table>

It is presumed by Eq. (16) that the coefficients are \((a \cong Re, b \cong \sqrt{Re})\) and \((a \cong \text{constant}, b \cong 1/\sqrt{Re})\) in the limit of \( Re \to 0 \) and \( Re \to \infty \), respectively. The previous two-dimensional study concluded that the coefficient \( a \) was independent of Reynolds number, but here we find that it gradually decreases with decreasing Reynolds numbers. The numerical solutions for the coefficient \( b \) also decrease, and they have a maximum value with respect to the Reynolds number.

The definition of Reynolds number \( Re = \frac{L_f U_f}{v_f} \) implies that both coefficients \((a,b)\) will become zero in the limit of \( Re \to 0 \). This is because a fish either with \( f_0 = 0 \) or in fluid with \( v_f = \infty \) would be unable to swim, in which case neither thrust \( a \) nor drag \( bU \) would appear in Eq. (10). In contrast, as the Reynolds number is increased, the coefficient \( b \) still decreases, whereas the coefficient \( a \) increases; both approach finite values because the effects of viscosity become small (although the flow is never truly inviscid). Equation (16) agrees with the above properties, but its essential meaning and that of coefficients \( C_1 - C_4 \) (that include \( \alpha = \rho_f/\rho \), amplitude \( A_u \), and wavelength \( \lambda \)) remain to be clarified in future work. Although Eq. (16) suggests that coefficient \( b \) tends to zero for both \( Re \to \infty \) and \( Re \to 0 \), turbulent flow and boundary layers must be taken into consideration at very large Reynolds numbers. For example, a different Reynolds number \( \text{Re}_f = \frac{L_f U_f}{v_f} \) can be considered, where \( U_f \) is the terminal fish speed \( U_f(t \to \infty) \) at which the fish is swimming. In the case of \( Re = 9,000 \) under model (B), the value \( U_f(t \to \infty) \approx 0.5 \) from Figs. 14 and 15 (b) corresponds to \( \text{Re}_f \approx 4,500 \). The range of \( \text{Re}_f \) for small fish is roughly \( O(10^3) \) to \( O(10^5) \) (Liu et al., 1997; Wolfgang et al., 1999; Borazjiani et al., 2010). Our evaluations suggest that Eq. (16) can be applied up to...
Various important points are yet to be discussed, not least the efficiency determined by the swimming power, the thrust, and the drag. There have been previous studies of fish in uniform flow (Borazjani and Sotiropoulos, 2009; Liu and Kawachi, 1999) and two-dimensional fish undergoing acceleration (Ogata and Ogasawara, 2012). A challenge remains to optimize the design of a three-dimensional fish shape that includes fins, because a number of parameters would have to be considered for optimization. However, more efficient swimming would be one of the most important viewpoints of fish robotics; the rules for bending in natural propulsors could be solutions to this form of optimized motion.

Another interesting aspect is schools of fish. Although two side-by-side two-dimensional fish (Dong and Lu, 2007) or three fish in a diamond-shaped school (Deng, 2006) have been studied in uniform flows, advantages of fish schools in robotics applications have yet to be clearly determined. In future publications, we will address such issues and present further simulation results and those from experiments with a basic fish robot.

5. Conclusions

The speed of a small three-dimensional fish was investigated numerically from an impulsive start to terminal speed for two types of deformation and values of density ratio at various Reynolds numbers. It was found that such a fish accelerates much like a freely falling sphere. Thus, we could predict the speed of the fish using a simple analytical model, as in a previous study in two dimensions. Two pairs of unknown parameters in the model could be determined by only two conditions through experimental measurements of fish speed that could be made using a high-speed camera or a force meter. The model deviated slightly from the simulation results only at the beginning of swimming for small Reynolds numbers. This was because the fluid force $F_{\text{ext}}$ deviated from a linear function of fish speed just after the start of deformation. One possible solution to this would have been to modify the initial condition of the fish deformation speed. Nevertheless, the present model could estimate not only acceleration but also terminal speed for any Reynolds number.

In trying to design an aquatic fish-like robot, aspects such as the shape of its caudal fin (Kikuchi et al., 2014) and its deformation have to be considered. Flexible-fin propulsion devices aimed at creating artificial fish have also been investigated to assess the effects of elastic modulus and shape on the fluid forces (Terada and Yamamoto, 1999). However, by using the present model for a small fish, it would be possible to estimate performance values such as the terminal speed for various conditions of the fluid (Reynolds number) and the fish (density ratio, frequency, and body length). For example, in water, one fish with $(L, f_0) = (3.0\,\text{cm}, 4.0\,\text{Hz})$ and another with $(L, f_0) = (6.0\,\text{cm}, 1.0\,\text{Hz})$ have the same dimensionless solution because of similarity through $Re = L^2 f_0 / \nu$. Hence, the properties of any fish robots that have the same shape and deformation could be evaluated from a single robot. However, other interpolations similar to Eq. (16) may also be possible if aspects of the fish, such as its shape or the amplitude functions of Eq. (4), are changed. More parametric studies are required that would involve further changes to the parameters of both the fish and the fluid.

Various important points are yet to be discussed, not least the efficiency determined by the swimming power, the thrust, and the drag. There have been previous studies of fish in uniform flow (Borazjani and Sotiropoulos, 2009; Liu and Kawachi, 1999) and two-dimensional fish undergoing acceleration (Ogata and Ogasawara, 2012). A challenge remains to optimize the design of a three-dimensional fish shape that includes fins, because a number of parameters would have to be considered for optimization. However, more efficient swimming would be one of the most important viewpoints of fish robotics; the rules for bending in natural propulsors could be solutions to this form of optimized motion.

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