Application of the Weis-Fogh mechanism to a ship propulsion system (Analysis based on the two-dimensional potential flow on the opening and closing stages)

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Abstract

A two-dimensional ship propulsion model using the Weis-Fogh mechanism is analyzed by the potential flow theory and the conformal mapping. The purpose of this study is to clarify the most effective operating condition in which the loss is minimum. The opening and the closing stages are carefully studied. In both stages, a fixed point of the wing moves in the direction perpendicular to the flow direction. The condition to prevent the vortex shedding when the wing separates from the wall is to equalize the both sides pressure at the trailing edge of the wing, and the acceleration of the wing motion satisfying the condition is clarified. On the other hand, in the closing stage the condition to satisfy the so-called Kutta condition is applied at the wing edge. This condition means that the wing prevents the separation nor generation of the jet which produces high thrust but more loss. The unsteady forces and the moment on the wing are also shown.

Keywords : Two-dimensional potential flow, Ship propulsion system, Weis-Fogh mechanism, Fluid dynamic forces, Unsteady flow

1. Introduction

The Weis-Fogh mechanism (Weis-Fogh, 1973 and Lighthill, 1973) has been known as a novel mechanism of lift generation in which the wings generate the circulations and the lift without shedding the vortices. After these first two papers were published, theoretical studies (Edwards and Cheng, 1982, Tsutahara and Kimura, 1984), and experimental studies for three-dimensional models were also given by Maxworthy (1979). Some numerical simulations by Haussling (1979) and Tsutahara et al. (1989) for two-dimensional model, and Ro and Tsutahara (1997), and Tsutahara and Murakami (1997) for three-dimensional model have been published. Even recently, Crowdy (2009) presented an exact solution to the translating stage of two-dimensional theoretical model. Kolomenskiy et al., (2011) published a long paper, and analyzed the two-dimensional flows in details using the potential theory and numerical simulations. In all studies, wing is assumed to be plane and non-deformable.

On the other hand, engineering applications of this mechanism has been few. Furber and Ffowcs Williams (1979) presented a result in which the performance of an axial compressor increases by narrowing the clearance between the stators and the rotors, and they explained this effect is due to the Weis-Fogh effect.

Tsutahara and Kimura (1985, 1986, 1987) presented two-dimensional model for a ship propulsion device applying

the two-dimensional Weis-Fogh effect model. They showed that this model actually worked by sailing test of a ship model (Tsutahara and Kimura, 1987) and Tsutahara et al. (1990) also showed that the efficiency is improved by controlling the wing motion. On the other hand, they also developed the two-stage type model, in which the two sets of wings are placed in a square duct (Tsutahara, et al. 1987, 1988, Ro, et al., 1990). They also applied the same model to a pump (Tsutahara, et al., 1989) and to a blower (Tsutahara and Kimura, 1994). Takahashi et al. (1992) and Tsutahara et al. (1992) presented a rotating-type Weis-Fogh pump, and they concluded the performance of this type pump depends on the shape of the duct as ordinary cross flow pumps.

In this paper, the authors will revisit the propulsion mechanism using the Weis-Fogh mechanism proposed by Tsutahara and Kimura (1985), and clarify the condition to make the flow to be a potential one.

2. Two-dimensional model of the Weis-Fogh mechanism and its application to ship propulsion

The authors believe that the application of the potential theory to this mechanism is not just an approximation but the potential flow can be a real one, by controlling the wing motion carefully. In the original Weis-Fogh mechanism, the flow separation inevitably occurs and vortices are shed in the opening (fling) stage as shown by Maxworthy (1979) and Haussling (1979). However, in the propulsion model proposed by Tsutahara and Kimura (1985) and Ro et al. (1990), which is shown in Fig.1(a) and (d), the separation is not significant and flow reattaches due to the uniform flow to the wings. This means that the potential flow can be realized except the vicinity of the leading edge of the each wing. In the opening stage, however, the vortex shedding will occur. In the closing (clap) stage as shown in Fig.1(c) and (f), a jet is produced and this violates the potential flow analysis, but this jet also can be eliminated by controlling the closing velocity and the efficiency will be improved avoiding generation of the jet. It should be noted here that the jet itself produces large thrust, but it needs much larger input and the efficiency becomes smaller as described by Tsutahara and Kimura (1986). These facts are supported by a simulation of the vortex method by Ro, et al.(1990).

The above discussion leads to the following conclusions as far as the flow is assumed two-dimensional. The potential flow can be realized by controlling the wing motion, and if so the loss will not occur and the propulsion efficiency will be the maximum. Of course, the loss-zero operation is impossible because of the viscosity and three-dimensionality, that is, the tip vortex generation will occur at both upper and lower tips of the wing. As to the viscosity effect, the boundary layer is thin when the Reynolds number is high and the viscosity can be neglected for most part of the flow field as far as the flow does not separate. The efficiency will be high enough when the wing motion is kept to fit the two-dimensional potential flow assumption. So far, in the potential flow analysis, only the kinetic aspects have been emphasized especially the circulation. However, in order to prevent the vortex shedding, the dynamics especially the pressure distribution must be considered.

The authors will present an exact solution of the flow for opening and the closing stages which are singly connected region flows, and clarify the condition of the wing motion to realize the potential flow. The authors will also revisit the mechanism of the two-dimensional propulsion model presented in Tsutahara and Kimura (1985) and shown in Fig.1, in which the point of $1/4$ chord length of the wing P moves perpendicularly to the walls (sailing direction).

Fig. 1 Two dimensional model of ship's propulsion mechanism using the Weis-Fogh mechanism. A wing opens with the point p , which is the $1/4$ chord length point in the case presented here, traversing the water channel as in (a) and (d). The ship is moving upward, and in this model the fluid flows downward. Then the wing translates in the channel as in (b) and (e) keeping the attack angle or changing it. Finally it closes with touching its leading edge as in (c) and (f).

3. Analysis for opening and closing stages

3.1 Wing motions and the conformal mapping

The flow fields for the opening and the closing stages are single connected region and that for the translating stage is the doubly connected region and both are different from each other. Then in this analysis, only the opening stage and the closing stage are considered and the translating stage will be studied in another paper. The effect of the width of the channel is neglected because the wing is very close to the one wall. In the opening (fling) stage, we shall adopt Fig. 1(a) neglecting the left wall. The analysis is performed by the conformal mapping and the flow direction is taken in the real axis of z -plane ($z = x + iy$) as shown in Fig. 2(a), which is considered to be physical plane. The point of $1/4$ chord length from the leading edge is assumed to move in the direction (y -direction) perpendicular to the sailing direction (x -direction) with the velocity V (not constant) touching the trailing edge with the lower wall. This $1/4$ chord length point corresponds to the action point or the axis to move the wing.

Actually the flow is analyzed in ζ -plane by a conformal mapping through the function

$$z = K \cdot (\zeta + 1)^{1-\alpha/\pi} \cdot (\zeta - 1)^{\alpha/\pi} \quad (1)$$

where

$$K = c/2 \cdot (1 - \alpha/\pi)^{\alpha/\pi-1} \cdot (\alpha/\pi)^{-\alpha/\pi} \quad (2)$$

and c is the chord length of the wing and α is the opening angle as shown in Fig. 2(a). By this condition the both sides of the trailing edge A_1 and A_2 are translated to $\zeta = -1$ and $\zeta = 1$ respectively, and the leading edge B is to $\zeta = 1 - 2\alpha/\pi$ as shown in Fig. 2(b). This mapping is the same as Lighthill's. In this stage, the trailing edge moves in x -direction with the velocity $V \tan \alpha$, and in order to keep the trailing edge at the origin of the z coordinates the whole z -plane moves at the same velocity. The flow velocity U is assumed constant but the V shall be time dependent.

On the other hand, in the closing (clap) stage, we adopt the stage in Fig. 1(c) but the figure should be rotated 180 degrees and the left wall is made correspond with the real axis of the z -plane as in the opening stage. Then the $3/4$ chord length point moves with velocity $-V$ touching the trailing edge with the wall this time, and the incoming flow is reversed. Then the analysis is done by the same conformal mapping as shown in Fig. 2 without velocities of the wing, flows, and moving point of the wing. Exact definition of forces will be described later, but configuration is the same.

3.2 The complex velocity potential

In the opening stage, the wing opens with the angular velocity $\Omega = 4V/(3c \cos \alpha)$ and translates in the x -direction with the velocity $V \tan \alpha$ as a whole. But it is more convenient to keep the trailing edge on the origin for conformal mapping. Then the coordinates in the z -plane is moved with the trailing edge and the incoming flow U is changed to $U + V \tan \alpha$.

Fig. 2 Conformal mapping from z -plane (physical plane) to ζ -plane (mapped plane).

The complex velocity potential is expressed as a function of ζ as

$$F(\zeta) = \frac{\Omega K^2}{2 \sin 2\alpha} \left[-(\zeta+1)^{2-2\alpha/\pi} (\zeta-1)^{2\alpha/\pi} + \zeta^2 + \zeta \left(2 - \frac{4\alpha}{\pi} \right) \right] + KV \tan \alpha (\zeta+1)^{1-\alpha/\pi} (\zeta-1)^{\alpha/\pi} - K\zeta(U + V \tan \alpha) \quad (3)$$

where the first term is the exact solution by Edwards and Cheng (1982) for the flow where the wing only rotates about the trailing edge which is at rest, the second term represents a uniform flow with velocity $V \tan \alpha$ which corresponds to the translation velocity of the wing. The last term represents the amended term by which the uniform flow is corrected to be $-U$ at infinity in the z -plane (KU in the ζ -plane), which is the relative flow velocity to the ship body.

3.3 Unsteady pressure forces and moment

Unsteady pressure is obtained (Imai, 1970) by

$$p_b = -\rho \left(\frac{\partial \phi}{\partial t} \Big|_z + \frac{1}{2} |u|_z^2 \right) + p_o \quad (4)$$

where

$$\begin{aligned} \frac{\partial \phi}{\partial t} \Big|_z &= \frac{\partial \phi}{\partial t} \Big|_\zeta + \operatorname{Re} \left\{ \frac{\partial F}{\partial \zeta} \cdot \frac{d\zeta}{dt} \right\} \Big|_z = \operatorname{Re} \left\{ \frac{\partial F}{\partial t} \Big|_\zeta \right\} + \operatorname{Re} \left\{ \frac{\partial F}{\partial \zeta} \cdot \frac{d\zeta}{dt} \right\} \Big|_z \\ &= \operatorname{Re} \left\{ \frac{\partial F}{\partial \alpha} \cdot \frac{d\alpha}{dt} + \frac{\partial F}{\partial V} \cdot \frac{dV}{dt} \right\} + \operatorname{Re} \left\{ \frac{\partial F}{\partial \zeta} \cdot \frac{d\zeta}{dt} \right\} = \operatorname{Re} \left\{ \frac{\partial F}{\partial \alpha} \cdot \Omega + \frac{\partial F}{\partial V} \cdot \dot{V} \right\} + \operatorname{Re} \left\{ \frac{\partial F}{\partial \zeta} \cdot \left(-\frac{\partial z / \partial \alpha}{\partial z / \partial \zeta} \cdot \Omega \right) \right\} \end{aligned} \quad (5)$$

$$|u|_z^2 = \left| \frac{\partial F}{\partial \zeta} \cdot \frac{d\zeta}{dz} \right|_z^2 \quad (6)$$

and

$$-\frac{\partial z / \partial \alpha}{\partial z / \partial \zeta} \cdot \Omega = \frac{1}{\pi} \left(\frac{1 - \zeta^2}{2\alpha/\pi + \zeta - 1} \right) \log \frac{(\pi/\alpha - 1)(\zeta - 1)}{(\zeta + 1)} \cdot \Omega \quad (7)$$

The first two terms on the right-hand side of Eq.(5) are obtained by simple partial differential and $\Omega = d\alpha/dt$. In the last term $-(\partial z / \partial \alpha) / (\partial z / \partial \zeta) \cdot \Omega$ is given by differentiating both sides of Eq.(1) by t and put $dz/dt = 0$ because this differentiation is defined at a fixed in the z -plane.

Then the forces and the moment about the 1/4 chord length point are given, respectively, by

$$F_x + i \cdot F_y = -i \cdot \int_{-1}^1 p_b \frac{dz}{d\zeta} d\zeta \quad (8)$$

and

$$M_{1/4} = -\operatorname{Re} \left(\int_{-1}^1 p_b \cdot (\bar{z} - 3c/4 \exp(-i\alpha)) \frac{dz}{d\zeta} d\zeta \right) \quad (9)$$

The integral are evaluated along the straight line from $z = -1 + 0.001i$ to $z = 1 + 0.001i$ in order to avoid the singularity at $z = a$, but the suction force acting at the leading edge is successfully estimated. It should be noted that the above forces and moment are those from fluid only and the input force to move the wing must include the reaction force from the wall because the wing touches the wall at the trailing edge(opening stage) and leading edge(closing stage). The reaction force is defined by

$$F_W = M_{1/4} / (3c/4 \cdot \cos \alpha) \quad (10)$$

when the moment is positive (anti-clockwise). Of course, when the moment is negative (clockwise) the trailing edge detaches from the wall surface and the reaction force does not exist.

3.4 Definition of forces

The definition of forces for the opening stage is as follows.

$$F_T = F_x \quad (11)$$

$$F_I = -F_y - F_W \quad \text{when the moment is positive} \quad (12)$$

$$F_I = -F_y \quad \text{when the moment is negative} \quad (13)$$

where F_T is the thrust (output) and F_I is the input force (wing driving force) which drive the wing and they are shown in Fig. 3. The force F_W is the reaction force from the wall but the friction is neglected, so that only force perpendicular to the wall is considered.

3.5 Definition of non-dimensional parameters

The non-dimensional parameter for the circulation Γ about the wing is expressed by

$$\bar{\Gamma} = \Gamma / (Uc) \quad (14)$$

On the other hand, the non-dimensional parameters for the wing driving velocity V and the acceleration $\dot{V}$ are given as follows, respectively.

$$\bar{V} = V/U \quad (15)$$

$$\bar{\dot{V}} = \dot{V} / (U^2/c) \quad (16)$$

The non-dimensional parameters for the moment $M_{1/4}$, the thrust F_T , the reaction force F_W , and the wing driving force F_I acting on the wing are defined as follows, respectively.

$$C_M = M_{1/4} / (\rho U^2 / 2 \cdot c^3) \quad (17)$$

$$C_T = F_T / (\rho U^2 / 2 \cdot c^2) \quad (18)$$

$$C_W = F_W / (\rho U^2 / 2 \cdot c^2) \quad (19)$$

$$C_I = F_I / (\rho U^2 / 2 \cdot c^2) \quad (20)$$

Fig. 3 The forces and moment acting on the wing for opening and closing stages. In the analysis, the figure for the closing stage is rotated 180 degrees and the configuration becomes the same as the opening stage, but the all vectors should be reversed.

where C_M denotes the moment coefficient; C_T the thrust coefficient; C_W the reaction force coefficient; C_I the wing driving force coefficient, respectively.

4. Characteristics of two-dimensional ship propulsion model

All the calculation presented in this paper are performed by Mathematica V.10.1.0.0.

4.1 Opening stage

In the opening stage the separation occurs at the leading edge, and mostly the shedded vortex moves towards the lower-side of the wing (see Fig. 3(a)). When there is no incoming flow ($U = 0$) the separation region becomes wide (Maxworthy 1979) and the potential flow should be modified by introducing the discrete vortex as shown in Edwards and Cheng (1982). But the incoming flow makes the separation region much smaller, sometimes the separation is negligibly small (Tsutahara 1988), then in this case the potential theory can be applied as a good approximation.

A sample of the wing motion is shown in Fig.4, in which the wing has an acceleration but the position of the wing changes rapidly and then the time interval is varied.

We will show the characteristics of the propulsion model and find the condition in which the wing detaches without shedding vortex. It should be achieved by adding an acceleration to the velocity V to equalize the pressure at both sides of the trailing edge of the wing when the wing separates from the wall.

Fig. 4 The wing motion when the 1/4 chord length point moves in the direction perpendicular to the wall.

(a) Variation of the opening angle for two cases with zero-acceleration and non-zero acceleration

(b) Variation of the angular velocity for two cases with zero-acceleration and non-zero acceleration

Fig. 5 Time histories of the opening angle and the angular velocity of the wing.

Fig. 6 Streamlines for $\bar{V} = 1.0$, $\dot{V} = 0$, and $\alpha = \pi/9$ in (a) in which the flow velocity at the leading edge is infinity and the flow will separate there but in actual operations the separated flow will reattach, and the flow except the vicinity of the leading edge recovers the potential flow. Pressure distribution is shown in (b) for $\bar{V} = 1.0$, $\dot{V} = 20.0$, and $\alpha = \pi/9$.

Fig. 7 The dimensionless circulation about the wing which should be positive considering the succeeding translating.

4.1.1 Streamlines and pressure distribution

Figure 6 (a) shows the streamlines for $\bar{V} = 1.0$, $\dot{V} = 0$, and $\alpha = \pi/9$. Fig. 6 (b) shows the pressure field for $\bar{V} = 1.0$, $\dot{V} = 20.0$, and $\alpha = \pi/9$. It should be noted that the streamlines (the kinematic part of the flow) does not depend on the acceleration of the wing motion but the pressure (the dynamic part of the flow) depends on it. The former is completely determined by the velocity distribution on the boundary, but the latter depends on the acceleration too.

4.1.2 Circulation

During the opening stage, the circulation around the wing is not so important, but when the wing separates from the wall this circulation should be transferred to the translating (fling) stage, and in this stage the circulation is essential parameter for thrust evaluation.

Figure 7 shows the relationship between the dimensionless circulation $\bar{\Gamma}$ about the wing and the angles α of the wing for variable wing driving velocity $\bar{V}$. The high wing driving velocity is necessary to obtain the large circulation but the wing should detach from the wall at the opening angle before about $\pi/6$.

4.1.3 Moment around the 1/4 chord length point

It should be noted that for the wing to open with the trailing edge touching the wall, the moment defined above must be positive, otherwise the edge detaches from the wall. Figure 8 shows the relationship between the moment coefficient C_M and the angles for variable wing driving velocity $\bar{V}$ and the acceleration $\dot{V}$. As is expected, the results show that an acceleration is necessary to obtain the positive moment. In this analysis, the mass of the wing is neglected. The actual operations, however, the mass of the wing is larger than the water and the moment becomes positive when the acceleration is smaller than that expected from this analysis.

(a) Moment coefficient for variable $\bar{V}$ without acceleration (b) Moment coefficient for variable acceleration $\dot{\bar{V}}$ when $\bar{V} = 1.0$

Fig. 8 The moment about the 1/4 chord length point which should be positive for the trailing edge of the wing to touch the wall during the opening stage.

(a) Thrust coefficient for variable $\bar{V}$ without acceleration (b) Thrust coefficient for variable acceleration $\dot{\bar{V}}$ when $\bar{V} = 1.0$

Fig. 9 The thrust coefficient for variable dimensionless wing driving velocity $\bar{V}$ and the acceleration $\dot{\bar{V}}$.

4.1.4 The thrust and the input force (wing driving force)

Figure 9(a) shows the relationship between the thrust coefficient C_T and the angles for variable wing driving velocity $\bar{V}$, and Fig. 9(b) shows the relationship between the thrust coefficient C_T and the angles for variable wing driving acceleration $\dot{\bar{V}}$.

The thrust acting on the wing increases with increasing wing driving velocity and the acceleration, but the large wing driving force is necessary to obtain the large thrust.

The wing driving force coefficient C_I is given in Fig. 10, and larger driving force is necessary when the acceleration is large and the opening angle is small. Of course this force includes the reaction force from the wall and if there is no acceleration the trailing edge detaches from the wall.

Fig. 10 The wing driving force coefficient for variable dimensionless wing driving acceleration $\dot{V}$ when $\bar{V} = 1.0$.

Fig. 11 The dimensionless wing driving acceleration $\dot{V}$ which make the pressure difference zero for variable dimensionless wing driving velocity $\bar{V}$.

4.1.5 Pressure difference between the both sides of the trailing edge when the wing separates

The difference between the pressures at the front and the rear sides is due to the unsteady term $\rho \partial \phi / \partial t$ and the kinetic energy term does not matter because the velocities are identical on the both sides. The pressure difference is expressed simply by

$$\operatorname{Re} \left(\frac{\partial F}{\partial \alpha} \bigg|_{\zeta=1} - \frac{\partial F}{\partial \alpha} \bigg|_{\zeta=-1} \right) \Omega + \operatorname{Re} \left(\frac{\partial F}{\partial V} \bigg|_{\zeta=1} - \frac{\partial F}{\partial V} \bigg|_{\zeta=-1} \right) \dot{V} = P_{A_2} - P_{A_1} \quad (21)$$

because the last term on the right side of Eq.(5) becomes 0 and some other terms disappear. In order to prevent the vortex shedding, the pressure difference should be disappear when the wing detaches from the wall. But this condition can not be achieved by a constant $\bar{V}$ and we will find the acceleration in $\bar{V}$ to equalize the pressures. The acceleration $\dot{V}$ is obtained by substituting 0 to the right-hand side, and expressed as

$$\dot{V} = \left[\frac{4K^2V}{3c \cdot \sin 2\alpha \cdot \cos \alpha} \left[2 \left(1 - \frac{2\alpha}{\pi} \right) \left\{ -\frac{2}{\tan 2\alpha} + \frac{2}{\pi} \log \left(\frac{1-\alpha/\pi}{\alpha/\pi} \right) + \tan \alpha \right\} - \frac{4}{\pi} \right] \right. \\ \left. + \frac{2KV}{\pi} \left\{ -\frac{\pi}{\cos \alpha} + \tan \alpha \cdot \log \left(\frac{\alpha/\pi}{1-\alpha/\pi} \right) \right\} + \frac{2KU}{\pi} \log \left(\frac{\alpha/\pi}{1-\alpha/\pi} \right) \right] \cdot \Omega \bigg/ \left\{ -\frac{8K^2V}{3c \cdot \sin 2\alpha \cdot \cos \alpha} \left(1 - \frac{2\alpha}{\pi} \right) + 2K \tan \alpha \right\} \quad (22)$$

Figure 11 shows the relationship between $\bar{V}$ which make the pressure difference zero and the opening angle. It should be noted that $\bar{V}$ is positive when the opening angle is small and $\bar{V}$ becomes infinity at the opening angle near $\pi/6$. Beyond that, $\bar{V}$ becomes negative. The fact that $\bar{V}$ becomes infinity means that the pressure difference can not be made zero by controlling the translation acceleration $\bar{V}$, because the second term in Eq.(21) or the denominator in Eq.(22) becomes 0. At this angle even the pressure at the trailing edge can not be controlled.

The followings are noted here in the connection between the opening stage and the translating stage. In the actual operation, the wing might become to have negative moment and detach at small opening angle, but the flow runs through the narrow space between the wing trailing edge and the wall and then the pressure drops there. The wing might be open again with keeping the narrow space. This scenario must be clarified in the study for the case in which the wing translates detaching the wall, and this stage will be studied in the future.

4.2 Closing stage

In the closing stage, the leading edge touches the wall and the trailing edge moves in the flow. As mentioned before, the condition to prevent the vortex shedding is to keep the flow velocity finite, that is, the point a must be a stagnation point in the ζ -plane using the mapping shown in Fig. 2. In this case, the relationship between $\bar{V}$ and opening angle α can be obtained from

$$\left. \frac{dF}{d\zeta} \right|_{\zeta=a} = 0 \quad (23)$$

and is shown in Fig. 12.

4.2.1 Streamlines and pressure distribution

Figure 13 shows the streamlines and the pressure field for the closing stage at $\alpha = \pi/6$ with $\bar{V} = 0.19992$ and $\bar{V} = 0$ in which the flow velocity at the trailing edge is finite and the pressure is also finite which corresponds to the so-called Kutta condition.

Fig. 12 The relationship between the dimensionless wing driving velocity $\bar{V}$ and the opening angle α for the condition to keep the flow velocity finite at the trailing edge.

Fig. 13 Streamlines and pressure distribution for the closing stage at $\alpha = \pi/6$ with $\bar{V} = 0.19992$ and $\bar{V} = 0$.

4.2.2 The acceleration for fitting the Kutta condition

The acceleration $\bar{V}$ is necessary and this acceleration is equal to that which keeps the pressures on both sides of the trailing edge. The unsteady pressure is given in Eq.(4) and the velocities on the both sides of the trailing edge is the same, so that this condition is simplified as $\partial\phi/\partial t$ being the same on both sides. Then the acceleration will be obtained by the equation

Fig. 14 The dimensionless wing driving acceleration $\bar{V}$ which satisfies Eq.(24).

(a) Moment coefficient

(b) Reaction force coefficient

(c) Wing driving force coefficient

(d) Thrust coefficient

Fig. 15 Moment and force acting on wing for the closing stage without generating vortex shedding. The scale of the horizontal axis is taken according to time process, that is, large angle to 0.

$$\left. \frac{d}{d\zeta} \left(\frac{\partial \phi}{\partial t} \right) \right|_{\zeta=a} = 0 \quad (24)$$

because there is no singularity at the trailing edge in the ζ -plane and $\partial\phi/\partial t$ is differentiable there. The exact formula for Eq.(24) is very lengthy and complicated, and the only result of the wing driving acceleration which satisfies Eq. (24) is shown in Fig. 14.

It is noted that the condition for the fluid velocity being finite at the trailing edge in Eq.(23) and that for the pressures on the both sides of the edge being the same in Eq.(24) are the same even in unsteady flow. The acceleration shown in Fig. 14 corresponds to the velocity shown in Fig. 12 where the gradient of the velocity curve denotes $dV/d\alpha$ but not dV/dt , so that the latter must be obtained through the relation $dV/dt = (dV/d\alpha) \cdot (d\alpha/dt)$.

A much simpler case is shown in Appendix B, where a flat plate in a uniform flow accelerates and rotates. The two conditions to keep the circulation constant and to make the same pressure on the both sides of the trailing edge are analytically shown to be the same.

4.2.3 Moment and Forces

The moment is positive when the opening angle is less than about $11\pi/36$, and this means that the reaction force from the wall is not negligible. As a result, the wing driving force increases as the opening angle becomes small. On the other hand, the thrust becomes small as the angle smaller.

It is noted here that in the closing stage a large thrust is expected because a strong jet is produced if the vortex shedding is allowed. But in this case the driving force to close the wing is much larger and the efficiency becomes small. So, in the closing stage, the wing should be slowly closed especially when the opening angle is small.

Figure 15 shows the moment and the forces acting on the wing. The wing motion is completely determined if the Kutta condition is applied, because the moving acceleration is determined if the opening angle and the velocity are given.

5. Conclusions

The dynamic properties of a two-dimensional propulsion mechanism using the Weis-Fogh mechanism is studied in which the wing motion is controlled by a linear translating motion of a point of 1/4 chord length. The opening and closing stages are analyzed by a potential theory and the conformal mapping. The condition to satisfy the potential flow, that is, the condition for preventing the vortex shedding is presented. Conclusions obtained are as follows.

- (1) In the opening stage, the positive acceleration is necessary for the wing to open touching the wall with its trailing edge and to produce the positive thrust.
- (2) When the wing detaches from the wall, the acceleration changes its sign from positive to negative to keep the pressures on the both sides of the trailing edge be the same and to prevent the vortex shedding. The point where the sign changes the acceleration has no effect.
- (3) In the closing stage, the Kutta condition is applied at the trailing edge, and the acceleration of the wing is shown.
- (4) Even in unsteady flows, the Kutta conditions that the flow velocity is finite at the trailing edge and the condition for the pressures on the both sides of the edge to be the same are shown identical.

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Appendix A

The pressure difference between the both sides of the trailing edge in Lighthill's model

Lighthill's original model of the Weis-Fogh mechanism is as follows. Two plates (wings) rotate around the trailing edge (touching with each other) at a constant angular velocity. The wings separate at about 120 degrees, but the reason is not clear. We shall calculate the pressure difference between the both sides of the trailing edge with the acceleration of angular velocity.

The mapping is as follows.

$$z = K \cdot (\zeta + 1)^{1-\alpha/\pi} \cdot (\zeta - 1)^{\alpha/\pi}, \quad K = c/2 \cdot (1 - \alpha/\pi)^{\alpha/\pi-1} \cdot (\alpha/\pi)^{-\alpha/\pi} \quad (\text{A.1,2})$$

The complex potential is

$$F(\zeta) = \frac{\Omega \cdot K^2}{2 \cdot \sin(2\alpha)} \left[-(\zeta + 1)^{2-2\alpha/\pi} \cdot (\zeta - 1)^{2\alpha/\pi} + \zeta^2 + \zeta \cdot \left(2 - \frac{4 \cdot \alpha}{\pi} \right) \right] \quad (\text{A.3})$$

The pressure difference is given by the following equation

$$\Delta p = p|_{\zeta=1} - p|_{\zeta=-1} = \left(\frac{\partial F}{\partial \alpha} \Big|_{\zeta=-1} - \frac{\partial F}{\partial \alpha} \Big|_{\zeta=1} \right) \cdot \Omega + \left(\frac{\partial F}{\partial \alpha} \Big|_{\zeta=-1} - \frac{\partial F}{\partial \alpha} \Big|_{\zeta=1} \right) \cdot \dot{\Omega} \quad (\text{A.4})$$

The pressure difference in the case of constant angular velocity is shown in Fig. 16 which show that the pressure difference decreases as the opening angle increases but it is still alive even at the opening angle $\pi/3$ but the acceleration of angular velocity can vanish the pressure difference and is shown in Fig. 17.

Considering the motion of the wing after detaching from the wall, the opening angle $\pi/3$ may be good choice as Lighthill mentioned, or impose some acceleration.

Appendix B

In this section, the so-called Kutta condition for a flat plane in a uniform flow in which the flow velocity at the trailing edge is finite and also the pressures on the both sides of the edge is the same is clearly identified for steady flows. However, for unsteady flows this identification is not so clear as far as the authors know. Therefore, this identification is shown here for a case in which the wing changes its attack angle and the flow has an acceleration.

The flow around a flat wing with chord length 4 in the z -plane is mapped in to the flow around a circular cylinder

Fig. 16 The pressure difference when the opening angular velocity is constant.

Fig. 17 The angular acceleration to vanish the pressure difference at the trailing edge.

whose diameter is 2 in the ζ -plane ($\zeta = \xi + i\eta$) by

$$z = \zeta + 1/\zeta \quad (\text{B.1})$$

(see for example Imai 1970), and the complex velocity potential is given by

$$f(\zeta) = U \cdot \left(e^{-i\alpha} \cdot \zeta + e^{i\alpha} / \zeta \right) + i\Gamma / (2\pi) \cdot \log \zeta \quad (\text{B.2})$$

where α is the attack angle and U is the flow velocity and both are functions of the time in this case, but the circulation Γ is a constant.

The condition for the velocity at the trailing edge to be finite is well known as

$$\Gamma = 4\pi U \sin \alpha \quad (\text{B.3})$$

so that in order to keep the circulation constant

$$\dot{\alpha} = -\dot{U}/U \cdot \tan \alpha \quad (\text{B.4})$$

must hold.

On the other hand, the time derivative of Eq.(B.2) is obtained by

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \alpha} \cdot \dot{\alpha} + \frac{\partial f}{\partial U} \cdot \dot{U} + \frac{\partial f}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial t} \quad (\text{B.5})$$

In order to calculate the pressure at the trailing edge the last term is 0 because the mapping of the trailing edge on the ζ -plane does not change. So the following equation is obtained.

$$\begin{aligned} \frac{df}{dt} &= \frac{\partial f}{\partial \alpha} \cdot \dot{\alpha} + \frac{\partial f}{\partial U} \dot{U} \\ &= \dot{U} \xi \cos \alpha + \frac{\dot{U} \xi \cos \alpha}{\eta^2 + \xi^2} + U \eta \dot{\alpha} \cos \alpha + \frac{U \eta \dot{\alpha} \cos \alpha}{\eta^2 + \xi^2} - 2U \dot{\alpha} \text{Arg}(i\eta + \xi) \cos \alpha \\ &\quad + \dot{U} \eta \sin \alpha + \frac{\dot{U} \eta \sin \alpha}{\eta^2 + \xi^2} - U \xi \dot{\alpha} \sin \alpha - \frac{U \xi \dot{\alpha} \sin \alpha}{\eta^2 + \xi^2} - 2U \dot{\alpha} \text{Arg}(i\eta + \xi) \sin \alpha + \\ &\quad i \left(\dot{U} \eta \cos \alpha - \frac{\dot{U} \eta \cos \alpha}{\eta^2 + \xi^2} - U \xi \dot{\alpha} \cos \alpha + \frac{U \xi \dot{\alpha} \cos \alpha}{\eta^2 + \xi^2} + U \dot{\alpha} \cos \alpha \log(\eta^2 + \xi^2) - \dot{U} \xi \sin \alpha + \frac{\dot{U} \xi \sin \alpha}{\eta^2 + \xi^2} \right. \\ &\quad \left. - U \eta \dot{\alpha} \sin \alpha + \frac{U \eta \dot{\alpha} \sin \alpha}{\eta^2 + \xi^2} + \dot{U} \sin \alpha \log(\eta^2 + \xi^2) \right) \end{aligned} \quad (\text{B.6})$$

where Arg means the argument of complex number $\xi + i\eta$. The pressure difference on both side of the trailing edge in the wing in the z -plane is given by differentiating the real part of Eq.(B.6) with respect to η .

$$\begin{aligned} \frac{\partial}{\partial \eta} \left(\text{Re} \frac{\partial f}{\partial t} \right)_{\zeta} &= -\frac{2\dot{U} \eta \xi \cos \alpha}{(\eta^2 + \xi^2)^2} + U \dot{\alpha} \cos \alpha - \frac{2U \eta^2 \dot{\alpha} \cos \alpha}{(\eta^2 + \xi^2)^2} + \frac{U \dot{\alpha} \cos \alpha}{\eta^2 + \xi^2} + \dot{U} \sin \alpha - \frac{2\dot{U} \eta^2 \sin \alpha}{(\eta^2 + \xi^2)^2} \\ &\quad + \frac{\dot{U} \sin \alpha}{\eta^2 + \xi^2} + \frac{2U \eta \xi \dot{\alpha} \sin \alpha}{(\eta^2 + \xi^2)^2} - 2iU \dot{\alpha} \cos \alpha \text{Arg}'(i\eta + \xi) - 2i\dot{U} \sin \alpha \text{Arg}'(i\eta + \xi) \end{aligned} \quad (\text{B.7})$$

By substituting Eq.(B.4) to Eq.(B.7) for the trailing edge of the wing , the following value can be obtained.

$$\frac{\partial}{\partial \eta} \left(\operatorname{Re} \frac{\partial f}{\partial t} \right)_{\zeta=1+0i} = 2U \left(-\frac{\dot{U}}{U} \cdot \tan \alpha \right) \cos \alpha + 2\dot{U} \sin \alpha - 2iU \left(-\frac{\dot{U}}{U} \cdot \tan \alpha \right) \cos \alpha \operatorname{Arg}'(1+0i) - 2i\dot{U} \sin \alpha \operatorname{Arg}'(1+0i) = 0 \quad (\text{B.8})$$

The result means that the condition for the pressure on both sides of the trailing edge to be the same can be satisfied when the Kutta condition is extended to the unsteady flows. Therefore, the both conditions are identical.