Numerical Simulation of Poly-Dispersed Bubbly Flow Using a Multi-Fluid Model*

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Abstract
Numerical methods for predicting poly-dispersed bubbly flows in bubble columns are indispensable in the design of bubble column reactors. The objectives of the present study are (1) to experimentally investigate the effects of a bubble size distribution on poly-dispersed bubbly flows in an open vessel and (2) to examine the applicability of an (N+2)-field model (NP2) to poly-dispersed bubbly flows. Distributions of void fraction and liquid velocity in air-water bubble plumes in the vessel are measured using an experimental setup with a bubble injection device by which the ratio of the volume flow rate of large bubbles to that of small bubbles is controlled to a desired value. The main conclusions obtained are as follows: (1) experimental data on the effects of a bubble size distribution on air-water bubble plumes are obtained, and (2) NP2 gives good predictions for poly-dispersed bubble plumes.

Key words: Bubble, Multi-Phase Flow, Numerical Simulation, Bubble Column, Bubbly Flow, Multi-Fluid Model, Bubble Plume

1. Introduction

Bubble columns are widely used for gas–liquid contactors in many industrial fields such as chemical, petrochemical, bioengineering and so forth. Numerical methods for predicting bubbly flows are indispensable for the design of bubble column reactors. Various numerical methods for predicting bubbly flows such as a homogeneous model[1], a two-fluid model[2],[3], a bubble tracking method[4],[5] and an interface tracking method[6],[7],[8] have been proposed. Among these methods, the two-fluid model possesses the highest applicability to bubbly flows in large-scale systems[9]. However, it cannot take into account a bubble size distribution. A poly-dispersed bubbly flow, which consists of bubbles of various diameters, is to be formed in a Fischer-Tropsch reactor[10] for a GTL (Gas To Liquid) plant[11], which has been being developed by JOGMEC (Japan Oil, Gas and Metals National Corporation). Since drag and lift forces acting on a bubble depend on the bubble diameter[12], we have to take into account the bubble size distribution and its effects on the interfacial forces to accurately predict poly-dispersed bubbly flows.

An (N+2)-field model (NP2), a hybrid combination of a multi-fluid model and an interface tracking method, has been proposed by Tomiyama & Shimada[13]. When we simulate poly-dispersed bubbly flows using NP2, we can classify bubbles into N groups in terms of their diameters, and ensemble-averaged behavior of each bubble group is calculated.
by the multi-fluid model. However, its applicability to poly-dispersed bubbly flows has not been verified yet. In addition, few experimental studies have been carried out on the effects of a bubble size distribution on poly-dispersed bubbly flows.

The objectives of the present study are (1) to obtain experimental data on the effects of a bubble size distribution on poly-dispersed bubbly flows and (2) to examine the applicability of NP2 to poly-dispersed bubbly flows. Distributions of void fraction and liquid velocity in air-water bubble plumes in a small vessel are, therefore, measured using an experimental setup equipped with a bubble injection device by which the ratio of the volume flow rate of large bubbles to that of small bubbles is controlled to a desired value. Numerical simulations using NP2 are also carried out to examine its applicability to poly-dispersed bubbly flows.

2. Outline of NP2

2.1 Basic and constitutive equations

As shown in Fig. 1, NP2 classifies the phase fields into \( N+2 \) groups: a continuous gas field \( (G) \), a continuous liquid field \( (L) \) and \( N \) dispersed bubble fields \( (B_m, m = 1, 2, ..., N) \). The volume fractions \( \alpha \) satisfy

\[
\alpha_G + \alpha_L + \sum_{m=1}^{N} \alpha_{Bm} = 1
\]  

\( \text{(1)} \)

For simplicity, the gas and liquid phases are assumed to be incompressible Newtonian fluids without phase change. The cell-volume averaged conservation equations for the number density \( n_{Bm} \) of the \( m \)-th bubble field and the volume fractions of the two continuous phase fields are given by

\[
\frac{\partial n_{Bm}}{\partial t} + \nabla \cdot (n_{Bm} \mathbf{V}_{Bm}) = -\gamma_{Gb} + R_m
\]  

\( \text{(2)} \)

\[
\frac{\partial \alpha_G}{\partial t} + \nabla \cdot (\alpha_G \mathbf{V}_c) = \sum_{m=1}^{N} \Gamma_{Gb}
\]  

\( \text{(3)} \)

\[
\frac{\partial \alpha_L}{\partial t} + \nabla \cdot (\alpha_L \mathbf{V}_c) = 0
\]  

\( \text{(4)} \)

where \( t \) is the time, \( \mathbf{V} \) the velocity, \( \gamma_{Gb} \) and \( \Gamma_{Gb} \) are the transfer rates of the bubble number density and the gas volume fraction between the \( m \)-th bubble group and the continuous gas field, respectively, and \( R_m \) is the transfer rate of the bubble number density into the \( m \)-th bubble group due to bubble coalescence and breakup. Since NP2 adopts the so-called one-field formulation for the two continuous phases, we use the velocity \( \mathbf{V}_c \) (\( = \mathbf{V}_G = \mathbf{V}_L \)) for the mixture \( c \) of the continuous gas and liquid phases in Eqs. (3) and (4). Though most of two-fluid models employ the conservation equation of bubble volume fraction \( \alpha_b \), NP2 uses the conservation equation of \( n_{Bm} \). This is because we cannot obtain accurate predictions with the \( \alpha_b \) conservation equation when a bubbly flow simulation is conducted using a cell size less than a bubble diameter. 

The cell-volume averaged momentum equations for the $m$-th bubble field and the mixture are given by
\[
\frac{\partial V_{bm}}{\partial t} + V_{bm} \cdot \nabla V_{bm} = -\frac{1}{\rho_{bm}} \nabla P + g - \frac{1}{\rho_{bm} \alpha_{bm}} \left( M_{Lbm} + M_{\Gamma m} + M_{Rm} \right)
\]  
(5)
\[
\frac{\partial V_{c}}{\partial t} + V_{c} \cdot \nabla V_{c} = -\frac{1}{\rho_{c}} \nabla P + F_{\mu} + F_{s} + g + \frac{1}{\rho_{c} \alpha_{c}} \sum_{n=1}^{N} \left( M_{Lbn} + M_{\Gamma n} \right)
\]  
(6)
where $\rho$ is the density, $P$ the pressure, $g$ the acceleration of gravity, $M_{Lbm}$ the interfacial momentum transfer between the $m$-th bubble group and the liquid phase, $M_{\Gamma m}$ the momentum transfer associated with $\Gamma_{GBm}$, $M_{Rm}$ the momentum transfer due to $R_m$, $F_{\mu}$ the viscous and turbulent diffusion and $F_{s}$ the force due to surface tension. The $\alpha_{c}$ is the volume fraction of the mixture ($\alpha_{c} = \alpha_{G} + \alpha_{L}$), and the mixture density $\rho_{c}$ is defined by
\[
\rho_{c} = \frac{\rho_{L} \alpha_{L} + \rho_{G} \alpha_{G}}{\alpha_{L} + \alpha_{G}}
\]  
(7)
The $F_{\mu}$ is given by
\[
F_{\mu} = \frac{1}{\rho_{c} \alpha_{c}} \nabla \cdot \left( \alpha_{c} \mu_{c} \left[ \nabla V_{c} + (\nabla V_{c})^T \right] \right)
\]  
(8)
where the superscript $T$ denotes the transpose, and $\mu_{c}$ is the mixture viscosity defined by
\[
\mu_{c} = \frac{\mu_{L} \alpha_{L} + \mu_{G} \alpha_{G}}{\alpha_{L} + \alpha_{G}}
\]  
(9)
where $\mu_{L}$ is the sum of molecular and eddy viscosities of the continuous liquid phase and $\mu_{G}$ the viscosity of the continuous gas phase. The surface tension force $F_{s}$ is given by
\[
F_{s} = \frac{\sigma \kappa \delta_{s} n_{s}}{\rho_{c}}
\]  
(10)
where $\sigma$ is the surface tension, $\kappa$ the sum of the two principal curvatures of the interface, $\delta_{s}$ the delta function which is zero except at the interface and $n_{s}$ the unit normal to the interface. The $M_{Lbm}$ is evaluated using the following standard constitutive equation:
\[
M_{Lbm} = M_{Dm} + M_{VMm} + M_{LFm} + M_{TDm}
= \frac{3}{4} \frac{\alpha_{bm}}{d_{bm}} C_{Dm} P \left[ V_{bm} - V_c \right] \left[ V_{bm} - V_c \right] \\
+ \alpha_{bm} C_{VMm} P \left[ \frac{DV_{bm}}{Dt} - \frac{DV_{c}}{Dt} \right] \\
+ \alpha_{bm} C_{LFm} P \left[ V_{bm} - V_c \right] \times \text{rot} V_{c} \\
+ C_{TDm} P k_{L} \nabla \alpha_{bm}
\]  
(11)
where $M_{D}$ is the drag force, $M_{VM}$ the virtual mass force, $M_{LF}$ the lift force, $M_{TD}$ the turbulent dispersion force, $d$ the bubble diameter, $C_{D}$ the drag coefficient, $C_{VM}$ the virtual mass coefficient, $D/Dt$ the substantial derivative, $C_{LF}$ the lift coefficient, $C_{TD}$ the turbulent dispersion coefficient and $k_{L}$ the turbulent kinetic energy of the continuous liquid field.

2.2 Correlations
The drag coefficient $C_{D}$ in Eq. (11) is given by
\[
C_{Dm} = C_{Dm} \phi_{bm}
\]  
(12)
where $C_{Dn}$ is the drag coefficient for a single bubble, $\phi$ the drag multiplier which accounts for the effects of bubble swarm on $C_{Dn}$. The $C_{Dn}$ is given by (16)

$$C_{Dn} = \max \left[ \min \left( \frac{16}{Re_m} \left( 1 + 0.15Re_m^{0.87} \right), \frac{48}{Re_m} \cdot \frac{8}{3} \cdot \frac{Eo_m}{Re_m} + 4 \right) \right]$$  \hspace{1cm} (13)

where $Re$ is the bubble Reynolds number and $Eo$ the Eötvös number:

$$Re_m = \frac{\rho_L |V_{bm} - V| d_m}{\mu_L}$$  \hspace{1cm} (14)

$$Eo_m = \frac{g(\rho_L - \rho_m) d_m^2}{\sigma}$$  \hspace{1cm} (15)

where $\mu_L$ is the liquid viscosity.

The drag multiplier $\phi$ in Eq. (12) is given by (17)

$$\phi_m = \left[ A - B \ln(L_m/d_{me}) \right]^2$$  \hspace{1cm} (16)

$$A = 0.745(d/d_0) + 0.703 \quad (d_0 = 5 \times 10^{-3} \text{ m})$$  \hspace{1cm} (17)

$$B = 0.256(d/d_0) - 0.160$$  \hspace{1cm} (18)

The way to evaluate $L_m$ and $d_{me}$ in Eq. (16) is shown in Appendix.

The lift coefficient $C_{LF}$ is given by (18)

$$C_{LF} = \begin{cases} \min[0.288 \tanh(0.121Re_m)], f(Eo_m) & (Eo_m < 4) \\ f(Eo_m) & (4 \leq Eo_m \leq 10) \\ -0.29 & (10 < Eo_m) \end{cases}$$  \hspace{1cm} (19)

$$f(Eo_m) = 0.00105Eo_m^3 - 0.0159Eo_m^2 - 0.0204Eo_m + 0.474$$

where $Eo_m$ is the modified Eötvös number in which the maximum horizontal dimension $d_H$ of a bubble is adopted as a characteristic length:

$$Eo_m = \frac{g(\rho_L - \rho_m) d_m^2}{\sigma}$$  \hspace{1cm} (20)

The $d_H$ is given using a correlation for the aspect ratio $E = (d_V/d_H)$ of an ellipsoidal bubble, where $d_{V_m}$ is the maximum vertical dimension of a bubble. An empirical correlation of $E$ proposed by Vakrushov & Efremov (19) is used in this study.

The virtual mass coefficient $C_{VM}$ in Eq. (11) is given by (12)

$$C_{VM} = \frac{E_m \cos^{-1} E_m - \sqrt{1 - E_m^2}}{E_m \sqrt{1 - E_m^2} - E_m \cos^{-1} E_m}$$  \hspace{1cm} (21)

The turbulent kinetic energy $k_L$ is evaluated using the following correlation that accounts for the bubble-induced turbulence (3):

$$k_L = \sum_{n=1}^{N} \alpha_{bn} |V_{bn} - V|^2$$  \hspace{1cm} (22)

3. Experiment

3.1 Experimental setup and conditions

Void and liquid velocity distributions in air-water bubble plumes in a small rectangular
vessel were measured. A schematic of the experimental setup is shown in Fig. 2 (a). The dimensions of the small vessel were 250, 30 and 485 mm in width, depth and height, respectively. Air and tap water at room temperature and atmospheric pressure were used for the gas and liquid, respectively. Initial water level was 150 mm. The air injection device was installed in the bottom of the vessel. The top and cross-sectional front views of the device are shown in Fig. 2 (b). It consisted of a stainless steel pipe of 2.6 mm in inner diameter and 24 stainless steel pipes of 0.2 mm in inner diameter. The former was used to release large bubbles (Bubble group $m = \ell$), and the latter to generate small bubbles ($m = s$). Two independent air supply lines were connected to the former and the latter pipes, by which the ratio of the air flow rate of the large bubbles $Q_\ell$ to that of the small bubbles $Q_s$ was controlled to a desired value. In the experiments, the ratio $Q_\ell : Q_s$ was varied from 0:6, 1:5, 2:4, ... to 6:0, while the total flow rate ($Q_\ell + Q_s$) was kept constant at $1.68 \times 10^{-6}$ m$^3$/s.

Mean diameters $d$ of the large and small bubbles were measured by the following two methods.

One is based on the number of bubbles passing through a horizontal plane per unit time (count method). By using high-speed video images of the bubble plumes we measured the times $t_\ell$ and $t_s$ required for 100 large bubbles and 400 small bubbles passing through the plane of $z = 125$ mm, respectively. Then the mean diameter $d_m (m = \ell, s)$ was evaluated by

$$d_m = \left( \frac{6}{\pi} \frac{N_m}{N_m} \right)^{1/3} \quad \text{(23)}$$

where $\theta$ is the mean bubble volume, $N_m$ the number of bubbles passing through the plane ($N_\ell = 100$ and $N_s = 400$). Though this method is simple, it gives accurate mean diameters since no assumption is required for bubble shapes. However, this method is not applicable to bubbly flows of high bubble number density due to the overlaps of bubble images. Hence, the validity of the following method was examined.

The other method is based on the bubble image (bubble image method). If the shape of a bubble is assumed to be oblate spheroidal, the sphere-volume equivalent diameter $d$ of the
bubble can be calculated using the following equations:

\[ S = \pi d_H d_V \]  
\[ C = 2d_H \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \phi} \, d\phi \]  
\[ k^2 = 1 - E^2 \]  
\[ \theta = \frac{\pi}{6} d_H^2 d_V \]  
\[ d = \left( \frac{60}{\pi} \right)^{1/3} \]

where \( d_H \) and \( d_V \) are the major and minor axes of a bubble, respectively, and \( S \) and \( C \) are the surface area and peripheral length of a bubble, respectively. By using a planimeter (TAMAYA digitizing Area-Line Meter, Super PLANIX β), we measured the values of \( S \) and \( C \) for 100 small bubbles and 50 large bubbles, which were randomly selected from images for \( 100 < z < 125 \) mm.

Horizontal distributions of time-averaged void fraction \( \alpha_B \) were measured at the elevation \( z \) of 125 mm by using an electric conductivity probe\(^{(20)}\). The sampling period and frequency for each measurement points were 15 minutes and 2000 Hz, respectively. Horizontal distributions of time-averaged liquid upward velocity \( V_L \) were also measured at the same elevation using a Laser Doppler velocimetry (LDV) system (DANTEC 60X83, Processor 58N10). The sampling number was 50000. The measurement uncertainties in \( \alpha_B \) and \( V_L \) are 2.5% and 1.0%, respectively.

### 3.2 Results and discussion

Images of flow patterns are shown in Fig. 3. Large bubbles rise up rectilinearly and small bubbles spread in the horizontal direction. The rising velocities of small bubbles entrained into the wake of a preceding large bubble increase, and small bubbles rapidly approach to the large bubble. The flow pattern strongly depends on the ratio \( Q_i : Q_s \), even though the total volume flow rate \( (Q_i + Q_s) \) is constant.

![Fig. 3 Images of bubble plumes](image-url)
Mean diameters of large and small bubbles $d_l, d_s$ evaluated by the two methods are shown in Fig. 4. The open and closed circles show the mean diameter evaluated by the count method and that by the bubble image method, respectively. The difference between two values of $d_l$ (= 2.2 - 3.4 mm) is less than 6 %, and that of $d_s$ (= 6.1 - 11.9 mm) is less than 14 %. This result confirms that the bubble image method well evaluates mean bubble diameter.

Measured void distributions are shown in Fig. 5, where $x$ denotes the horizontal position and the center of the vessel corresponds to $x = 0$ mm. The void profile is strongly affected by the ratio $Q_l : Q_s$. The $\alpha_l$ at $x = 0$ mm increases with $Q_s$, and the void profile becomes flatter as $Q_l$ increases. This increase is caused by the rectilinear motion of large bubbles, and the flat $\alpha_l$ profile is by bubble dispersion.

Measured distributions of $V_L$ are shown in Fig. 6. Circulating flows are formed. The value of $x$ where the sign of $V_L$ changes from positive to negative decreases with increasing $Q_s$, and $V_L$ in the center region ($x \leq 8$ mm) increases with $Q_l$. These tendencies are mainly due to the momentum transfer induced by large bubbles.

The fact that void and liquid velocity profiles strongly depend on a bubble size distribution suggests that a bubble size distribution should be accurately taken into account in numerical simulations of poly-dispersed bubbly flows. The present experimental data can be utilized for the validation of various multi-fluid models.
4. Numerical simulation

Numerical simulations of the bubble plumes were carried out using NP2. Bubbles were classified into two groups, large and small bubbles. Measured mean diameters \(d_A\) and \(d_S\) shown in Fig. 4 were used. Inlet gas velocities of large and small bubbles were calculated from the total cross-sectional areas of the pipes and the gas volume flow rates. The turbulent dispersion of large bubbles was neglected \((C_{TD} = 0)\) because large bubbles rose rectilinearly. The turbulent dispersion coefficient \(C_{TD_s}\) for small bubbles was 2.0, following the recommendation by Shimada et al.\(^{(21)}\) The number density and momentum transfers between the two bubble groups were neglected \((R_m = M_{Rm} = 0, m = t, s)\), since few bubble coalescence and breakup were observed in the experiments. A bubble which reached a computational cell containing a free surface was regrouped into the continuous gas phase. Uniform cubic cells of 2.72 mm in size were used. Simulations were conducted for \(t = 0 \sim 50\) s with a constant time step, \(\Delta t = 0.1\) ms. Since predicted \(\alpha_B\) and \(V_L\) at \(z = 125\) mm became quasi-steady for \(t \geq 10\) s, mean values of them were evaluated for \(t = 10 \sim 50\) s. Figures 7 and 8 shows the mean predicted \(\alpha_B\) and \(V_L\), respectively. Good agreement implies that NP2 can give good predictions for the effects of a bubble size distribution on bubbly flows, provided that an accurate bubble size distribution is given.
5. Conclusions

Distributions of void fraction and liquid velocity in air-water bubble plumes in a small vessel are measured by making use of a bubble injection device by which the ratio of the volume flow rate of large bubbles to that of small bubbles is controlled to a desired value. Numerical simulations of poly-dispersed bubble plumes are carried out using an \((N+2)\)-field model \((NP2)\). As a result, the following conclusions are obtained.

(1) Experimental data on the effects of a bubble size distribution on air-water bubble plumes are obtained.

(2) Mean bubble diameters in bubble plumes can be accurately measured using a method utilizing bubble images.
(3) NP2 gives good predictions for poly-dispersed bubble plumes.

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References


Appendix

The \( m \) \((m = 1, 2, \ldots, N)\), where \( N \) is the number of bubble groups) of a bubble group \( B_m \) was numbered in order of increasing a bubble diameter. The multiplier \( \phi \) for the drag coefficient in bubble swarm was given by Eq. (16). The \( L_{me} \) and \( d_{me} \) in Eq. (16) were evaluated assuming that a drag force acting on bubbles of a group \( B_m \) was reduced by the bubbles in the groups of a larger or equal diameter. The \( d_{me} \) of the group \( B_m \) was evaluated as the mean diameter of all these bubbles. The \( L_{me} \) was calculated by

\[
L_{me} = \left( \frac{1}{N} \sum_{k=1}^{N} n_{B_k} \right)^{1/3}
\]