Derivation of Predicted Pressure Drop Equation on Granular Particle Plug Transportation in Horizontal Pipe*

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Abstract

In a plug transportation that is one of gas-solid two-phase flows, the prediction equations on particle velocity within a plug and pressure drop in a horizontal pipe have been formulated. The agreement between values calculated by these equations and experiments in which solid-air mass flow rate, pipe diameter, kinds of particle were changed was obtained. The difference between them is almost within 10%. In order to confirm the validity of supposition to derive these equations, particle velocity distribution within a plug in the directions of flow and radius, and particle velocity transformation from a stationary bed to a plug have been analyzed by high speed camera and PIV. The results show that there is no particle velocity distribution in a plug, and particles are accelerated uniformly in extra part of a plug. Namely, particles in a plug are fixed relative to each other and so they all move with the same velocity.

Key words: Pressure Drop, Particle Velocity, Pipe Line, Gas-Solid Two-Phase Flow, PIV

1. Introduction

Pneumatic transportation, which is one form of particulate transportation, has many advantages over container transportation, belt conveyors, and other types of mechanical transportation. These merits include a high-speed, sanitary, and inexpensive transportation process. Pneumatic transportation can be broadly characterized as dilute-phase high-speed transportation and dense-phase low-speed transportation. Dense-phase low-speed transportation, called plug transportation, is often adopted from the viewpoint of pipe wall wear and particle breakage prevention, as well as energy savings.

In the design of a plug transportation line, it is necessary to estimate the pressure drop accurately from the viewpoint of cost and energy reduction and reliability. These factors have been studied by many researchers, and many equations to predict pressure drop have been developed^{(1)–(5)}. However, these equations cannot be assumed to be practical because the difference between the theoretical calculation and experimental results becomes large when the transportation conditions, such as the types of particles and pipe diameter, are changed. Additionally, there is the problem of changing the coefficient in the Ergun's equation according to the particle shape^{(1)–(3)}. For the pressure drop through a static packed bed, the author has derived a modified Ergun's equation that considers pipe diameter and particle shape⁽⁶⁾. The predicted results agree with experimental results within 6%, and the applicability of this equation is high for many types of particles, air velocity, and pipe diameter. However, this equation requires the particle velocity and a preliminary experiment

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in order to apply it to plug transportation.

The purpose of this study is to derive a prediction equation for pressure drop that has a high application for various pipe diameters, types of particles and transportation conditions, and does not require a preliminary experiment. In addition, the flow conditions of the particles are measured, and the validity of the assumptions in the derivation of the prediction equation is verified.

Nomenclature

A	: Pipe cross-sectional area	[m ²]
D	: Pipe diameter	[m]
d_p	: Particle diameter	[m]
F	: Stress at the front and back end of a plug	[Pa]
g	: Acceleration of gravity	[m/s ²]
l_{ex}	: Single extra plug length	[m]
l_p	: Single plug length	[m]
L_p	: Total plug length	[m]
L_T	: Pipe length	[m]
M_s	: Mass flow rate of particles	[kg/s]
P	: Air pressure	[Pa]
ΔP	: Pressure drop	[Pa]
U_a	: Superficial air velocity	[m/s]
U_{SP}	: Slip velocity	[m/s]
u_s	: Particle velocity in the plug	[m/s]
W_p	: Plug velocity	[m/s]
ε	: Porosity	[—]
μ_w	: Coefficient of friction	[—]
ρ_a	: Air density	[kg/m ³]
ρ_B	: Bulk density	[kg/m ³]
ρ_S	: Particle density	[kg/m ³]
σ_r	: Principal radial stress	[Pa]
σ_x	: Principal stress in flow direction	[Pa]
τ_w	: Shear stress at the wall	[Pa]
ϕ	: Angle of internal friction	[deg]
ϕ_s	: Ratio of straight length on periphery	[—]

2. Derivation of equation

Figure 1 shows a stable transportation plug in a horizontal pipe. In a stable state, each plug sweeps up the stationary particles in front and leaves them behind a stationary layer of the same thickness⁽¹⁾⁽⁷⁾. The plug length over which particles are filled in the cross-section is represented by l_p . The part in l_p is called a plug, and the part which exists between a plug and a stationary bed is called the external plug. l_{ex} is the length that is equal to the particle volume of the extra part divided by the cross-sectional area of the pipe. It is assumed that particles in l_p are transported in a fixed state with each other, that is, they are transported as a lump, and u_s is assumed to be the same regardless of the position in the plug. In addition, it is assumed that particles in an external plug are accelerated uniformly from 0 to u_s , and the mean particle velocity in an external plug is regarded as $0.5u_s$.

The minute part length dx within l_p is taken, and the force balance is derived under the assumption that all stresses are function of x only.

$$\frac{dP}{dx} + \frac{d\sigma_x}{dx} + \frac{4\tau_w}{D} = 0 \quad (1)$$

Here, σ_x is the principal internal particle stress in the flow direction, P is the air pressure, τ_w

Fig.1 Pressure within a plug in horizontal pipe

Fig.2 Particle velocity (Plug velocity axis)

Fig.3 Ratio of straight length

is the shear stress at the wall, and D is the pipe diameter. Konrad⁽¹⁾ derived the equation for the principal internal particle stress in the radial direction σ_r by deriving the Mohr's stress circle. For passive failure, i.e., $\sigma_r > \sigma_x$

$$\sigma_r = K_w \sigma_x + (K_w + 1)c \cos \phi \cos(\omega + \phi_w) \quad (2)$$

$$K_w = \frac{1 + \sin \phi \cos(\omega + \phi_w)}{1 - \sin \phi \cos(\omega + \phi_w)} \quad (3)$$

$$\sin \omega = \frac{\sin \phi_w}{\sin \phi} \quad (4)$$

$$\tan \phi_w = \mu_w \quad (5)$$

where ϕ is the angle of internal friction, and c is the internal particle cohesion.

Next, the stress due to gravity on the pipe wall P_g is considered. Konrad⁽¹⁾, Pan and Wypych⁽²⁾, and Tsuji⁽³⁾ obtained this stress from hydrostatic pressure theory. However, the particles are solid, so this method may not be applied. Instead, this paper considers P_g as a unit force by plug weight.

$$P_g = \frac{\frac{1}{4} \rho_B \pi D^2 g dx}{\pi D dx} = \frac{1}{4} \rho_B g D \quad (6)$$

Particles obey passive failure when they are transported near a rigid body⁽¹⁾. Then, Eq.(7) can be derived from Eqs.(2) and (6).

$$\sigma_r = K_w \sigma_x + (K_w + 1)c \cos \phi \cos(\omega + \phi_w) + \frac{1}{4} \rho_B g D \quad (7)$$

The pressure drop within l_p is assumed constant in a stable plug transportation, and Eq.(8) is obtained by integrating for σ_x from Eqs.(1), (7) and Coulomb's law.

$$\sigma_x = k \exp \left[-\frac{4\mu_w K_w}{D} x \right] - \frac{\Delta P}{l_p} \frac{D}{4\mu_w K_w} - \frac{(K_w + 1)c \cos \phi \cos(\omega + \phi_w)}{K_w} - \frac{\rho_B g D}{4K_w} - \frac{C_w}{\mu_w K_w} \quad (8)$$

In this equation, k is a constant of integration, ΔP is the pressure drop through the plug, μ_w is the coefficient of friction and C_w is the particle-wall cohesion. From Janssen's analysis⁽¹⁾⁽⁸⁾, stresses at the front and back end of a plug can be assumed to be the same value. Therefore, $\sigma_x = F$ at $x=0, l_p$.

$$\frac{\Delta P}{l_p} = \frac{4\mu_w K_w F}{D} + \frac{4\mu_w (K_w + 1)c \cos \phi \cos(\phi_w + \omega)}{D} + \rho_B g \mu_w + \frac{4C_w}{D} \quad (9)$$

Konrad et al. determined F from the difference in the momentum between the stationary

layer and l_p . However, his approach might not be applicable since there is l_{ex} between them. Therefore, the axis of plug velocity W_p is taken as shown in Fig.2, and F is obtained by considering the momentum balance between l_{ex} and l_p .

$$F = \frac{1}{2} \frac{G_s}{A} u_s \quad (10)$$

G_s is the mass flow rate of particles that are taken into a plug and discharged from a plug. u_s is different from W_p due to the particle flow. As for G_s , Eq.(11) is obtained from Fig.2.

$$G_s = A \rho_B (W_p - u_s) \quad (11)$$

The analogy to gas-liquid systems is well-established for gas-fluidized beds by Brooke Benjamin⁽⁹⁾. This is applied to the relation between W_p and u_s .

$$W_p = u_s + 0.542 \sqrt{gD} \quad (12)$$

Equation(13) is obtained from Eqs.(10)~(12).

$$F = 0.271 \rho_B u_s \sqrt{gD} \quad (13)$$

For non-cohesive particles,

$$C_w = c = 0 \quad (14)$$

Therefore, Eq.(15) is derived from Eqs.(9), (13) and (14).

$$\frac{\Delta P}{l_p} = \mu_w \rho_B g + 1.084 \mu_w K_W \rho_B u_s \sqrt{\frac{g}{D}} \quad (15)$$

On the other hand, Ergun derived the equation for a pressure drop in a plug⁽¹⁰⁾. However, it is well known that there is a difference between calculations by Ergun's equation and experimental results due to D and the particle shape⁽¹¹⁾⁽¹²⁾. Therefore, the author suggests a modified Ergun's equation into which these factors are incorporated⁽⁶⁾.

$$\frac{\Delta P}{l_p} = 150 \left(1 + \frac{2d_p(1-0.645\phi_s)}{3D(1-\varepsilon)} \right)^2 \frac{\eta(1-\varepsilon)^2}{\varepsilon^3 \{(1-0.645\phi_s)d_p\}^2} U_{SP} + 1.75 \frac{\rho_a(1-\varepsilon)}{\varepsilon^3(1-0.645\phi_s)d_p} U_{SP}^2 \quad (16)$$

Here, η is the air viscosity, d_p is the particle diameter, ρ_a is the air density, ε is the porosity ($=1-\rho_B/\rho_s$), U_{SP} is the slip velocity and ϕ_s is the particle straight ratio. ϕ_s is the ratio of straight length l_i to circumference l_s in the second dimensional projection of the particle, as shown in Fig.3. The number of the linear portion is given as N_s , and ϕ_s is defined as follows.

$$\phi_s = \sum_{i=1}^{N_s} l_i / l_s \quad (17)$$

In Eq.(17), ϕ_s is a real number from 0 to 1. For example, $\phi_s=0$ indicates a spherical particle and $\phi_s=1$ indicates a cubic particle. Then, U_{SP} is treated as Eq.(18) in this paper.

$$U_{SP} = U_a - u_s \quad (18)$$

Here, U_a is the air velocity. Equation(19) is obtained by solving for u_s from Eqs.(15), (16) and (18).

$$u_s^2 - \left(\frac{a}{b\rho_a} + 2U_a + \frac{1.084\mu_w K_W \rho_B}{b\rho_a} \sqrt{\frac{g}{D}} \right) u_s + \frac{aU_a}{b\rho_a} + U_a^2 - \frac{\rho_B g \mu_w}{b\rho_a} = 0 \quad (19)$$

$$a = 150 \left(1 + \frac{2d_p(1-0.645\phi_s)}{3D(1-\varepsilon)} \right)^2 \frac{\eta(1-\varepsilon)^2}{\varepsilon^3 \{(1-0.645\phi_s)d_p\}^2} \quad (20)$$

$$b = 1.75 \frac{(1-\varepsilon)}{\varepsilon^3(1-0.645\phi_s)d_p} \quad (21)$$

Two solutions are found for u_s from Eq.(19), but the larger value gives a negative slip velocity. Therefore the smaller value is the true value for u_s . In short, u_s is given by Eq.(22).

$$u_s = \frac{1}{2} \left(\frac{a}{b\rho_a} + 2U_a + \frac{1.084\mu_w K_W \rho_B}{b\rho_a} \sqrt{\frac{g}{D}} \right) - \frac{1}{2} \sqrt{\left(\frac{a}{b\rho_a} + 2U_a + \frac{1.084\mu_w K_W \rho_B}{b\rho_a} \sqrt{\frac{g}{D}} \right)^2 - 4 \times \left(\frac{aU_a}{b\rho_a} + U_a^2 - \frac{\rho_B g \mu_w}{b\rho_a} \right)} \quad (22)$$

The pressure drop in a plug is obtained by substituting u_s calculated from Eq.(22) into Eq.(15). However, Eq.(15) must be expanded for the total pipe length because there are not

only plugs that are a particle lump, but also air lumps in the horizontal pipe. The relationship between the pipe length L_T and L_P is derived as

$$L_P = \frac{M_s L_T}{A \rho_B u_s} \quad (23)$$

The pressure drop through an air lump can be neglected because it is much smaller than that in the plug. So, substituting Eq.(23) for Eq.(15) can be regarded as an expansion for the pipe length.

$$\frac{\Delta P}{L_T} = \frac{M_s}{A \rho_B u_s} \left[\rho_B g \mu_w + 1.084 \mu_w K_w \rho_B u_s \sqrt{\frac{g}{D}} \right] \quad (24)$$

Therefore, particle velocity and pressure drop in a horizontal pipe can be calculated from Eqs.(22) and (24).

3. Experimental apparatus and method

Figure 4 shows the experimental apparatus used for this study. Three pipe diameters were used, i.e., $D=25.6, 38.0$ and 50.0 mm. The total pipeline length is about 13 m, and the pipe material is polyacrylic. An air chamber is used to collect the compressed air in an air source with an air compressor and a pressure-reducing valve is installed after the air chamber to regulate the compressed air. A flow meter is installed at the rear of the pressure reducing valve to measure the air flow rate Q_a and a load cell in a blow tank is installed to measure the particle mass flow rate M_s . In addition, two pressure sensors are installed in the horizontal pipe to measure the pressure in the pipe, as shown in Fig.4. Pressure in the pipe is obtained as the mean value for 30 sec captured every 0.05 sec in stability plug transportation. The transportation conditions are set to become stable by a pressure reducing valve and a particle feeder valve. The transportation conditions are $U_a=1.1 \sim 5.0$ m/s and solid-air mass flow rate ratio $m_T=21 \sim 105$.

A high-speed camera and PIV are used to measure the particle velocity alteration from the stationary bed to L_P , and the distribution of the particle velocity in a plug in the horizontal and vertical directions. The validity of supposition in the derivation of the proposed equation is then verified. The frame rate of the high-speed camera is 2000 fps and the resolution is 1024×1024 pixels. The interrogation window of the PIV is 45×45 pixels, and the correlation coefficient is more than 0.7 and the resolution of the PIV is 0.056 mm/pixel. A metal halide light is irradiated at the pipe wall and luminance distribution by

Fig.4 Experimental apparatus

Fig.5 Measurement of particle velocity by digital video camera

Table1 Particle properties

Solids	Rapseed	Polystyrol pellet	Polyethy -lene pellet	Nipolon hard	Plastic pellet
Mean particle diameter d_p (mm)	1.98	2.80	3.27	3.54	6.00
Straight length ratio ϕ_s	0.020	0.369	0.184	0.132	0.000
Voidage ε	0.420	0.383	0.378	0.416	0.461
Bulk density ρ_B (kg/m ³)	435	678	591	531	572
Particle density ρ_s (kg/m ³)	1050	1100	950	910	1061
Angle of internal friction ϕ (deg)	25.3	35.4	27.0	30.7	30.0
Coefficient of friction μ_w^*	0.365	0.439	0.360	0.288	0.250
Shape	Sphere	Column	Disk	Oval	Sphere

* Against polyacrylic plate

particles in the plug is obtained. Therefore particle velocity is obtained by PIV. It is believed that the rotation does not affect the results of the PIV because there are no rolling particles in the plug. Though there are many researches which measured W_p , the result which measured particle velocity distribution in the plug has never been reported. In addition, a digital video camera is used to confirm the validity of the PIV results and to measure u_s . This camera's frame rate is 30 fps. Some particles are colored to facilitate the measurement of u_s . To measure u_s , the colored particles in the plug are first found in the image recorded by the digital video camera. Then, some frames are advanced and u_s is calculated from the number of advanced frames and the particle transfer length, as shown in Fig.5. The high-speed camera and digital video camera are positioned within the dashed box in Fig.4. The high-speed camera is used for three different test conditions, i.e., $D=38$ and 50 mm polyethylene pellet, and $D=38$ mm plastic pellet.

Rapeseed, polystyrol pellets, polyethylene pellets, nipolon hard and plastic pellets are used as the particles in the experiments. Table 1 shows their properties.

4. Experimental result

Figure 6 shows a result of the particle velocity vectors in a plug analyzed by PIV, which is the result of a plastic pellet and $m_T=62.8$. The vectors are the mean value for 50 frames. Figure 6 shows that all of the speed vectors point toward the flow direction, and the particles do not move in vertical direction.

Then, u_s in flow direction measured by PIV is compared with that measured by the digital video camera in order to confirm the validity of the PIV result. u_s by the digital video camera is the mean value of a hundred particles. Particle velocity by PIV is the mean value for 50 frames and three plugs in the area of $D \times D$. Figure 7 shows that each value almost agrees. Additionally, there were little error vectors in the PIV analysis and therefore results of the PIV are correct.

To clarify the particle velocity distribution in the plug, the relationship between the particle velocity by PIV and the position in the vertical direction y/D is presented in Fig.8. Figure 8 shows that the velocity vectors of all positions are almost the same value regardless of the cross-sectional position. These results are the values at the wall. Tsuji⁽¹³⁾ and Chen⁽¹⁴⁾ suspected that there may be non-uniform velocity distribution in a plug. But in

Fig.6 Velocity vector in a plug
(Plastic pellet, $m_T=62.8$)

Fig.7 Confirmation on validity of analysis by PIV

Fig.8 Particle velocity distribution in a plug

Fig.9 Particle velocity at each position
(Polyethylene pellet, $D=50\text{mm}$, $m_T=40.2$)

the present study particle velocity is the same value in a cross-sectional area in the plug because there is uniform velocity distribution in Fig.8 and in Tanaka's simulation research result⁽¹⁵⁾, and there are no rolling particles.

Next, the particle velocity variation in the mainstream direction from the stationary layer to l_p is examined. The area at the same position for the mainstream direction is taken, and the velocities in each region of this area are measured by PIV and are shown in Fig.9. This particle velocity is the mean value over this area where particles had been satisfied to the pipe wall in order to remove float particles. Figure 9 shows there is no change in the particle velocity in the flow direction in the plug. From the result of Figs.6, 8 and 9, it is said that particles in l_p are transported in a fixed state relative to each other, that is, they are transported in a lump. Therefore u_s can be assumed to be a constant value regardless of the position in a plug. Additionally, Fig.9 shows particles in an external plug are accelerated uniformly, hence the mean particle velocity in an external plug can be regarded as $0.5u_s$.

5. Analysis of equation

The validity of the equation for predicting particle velocity in a plug and pressure drop derived by Konrad⁽¹⁾, Pan and Wypych⁽²⁾, Tsuji⁽³⁾, Mi and Wypych⁽⁴⁾ and Aziz⁽⁵⁾ is examined. Figures 10 to 15 show U_a along the horizontal axis and the ratio of the predicted value to the measured one along the vertical axis. The ratio with respect to u_s is shown as α , and the ratio with respect to $\Delta P/L_T$ is shown as β . u_s is taken for the mean value measured by the digital video camera since u_s does not change in a plug. u_s calculated from each researcher's equation is used when calculating $\Delta P/L_T$. Experiments on u_s are not compared with the calculated value of Aziz at this time since he did not derive an equation on it. Pan and Wypych, and Mi and Wypych derived the equation about W_p , not u_s . Therefore, u_s is calculated from Eq.(12) and compared with the experiments. Still, ρ_a , which is required to calculate these values, is obtained from the measured internal pipe pressure.

Fig.10 Comparison of experiment with calculation (Konrad)

Fig.11 Comparison of experiment with calculation (Pan and Wypych)

Fig.12 Comparison of experiment with calculation (Tsuji)

Fig.13 Comparison of experiment with calculation (Mi and Wypych)

Fig.14 Comparison of experiment with calculation (Aziz)

Fig.15 Comparison of experiment with calculation (This study)

Table2 Difference of pressure drop equation

		This study	Konrad	Pan and Wypych	Tsuji	Mi and Wypych	Aziz
Pressure drop	Gravity action	Plug weight	Hydraulic pressure				Plug weight
	Particle flow	l_p and l_{ex}	l_p and stationary bed				U_a and W_p
Slip velocity		$U_a - u_s$	$U_a + U_s - u_s$	$U_a - W_p$	$U_a - u_s$	Not consider	
Particle failure		Passive	Active	Passive	Active	Not consider

Fig.16 Pressure drop versus particle velocity

Table2 shows the difference of the pressure drop equation. The equations of Konrad, Pan and Wypych, Tsuji, Mi and Wypych, and Aziz are different from this study in terms of what gravity action is applied by the hydrostatic pressure. Additionally, when F is introduced, Aziz obtained it from the difference between U_a and W_p , while others obtained it from a momentum balance between the plug and stationary bed. Also, Konrad applied superficial particle velocity U_s to slip velocity, and Pan and Wypych considered particles as active failure and the slip velocity as the difference between U_a and W_p .

Figure 10 shows that the experimental results are different from the values predicted by Konrad because α ranges from 0.6 to 1.5. Note that calculation conditions are different from Ergun's equation because the superficial particle velocity was applied to U_{sp} . Subsequent calculations agree with the experiments when the particles are polyethylene pellets at $D=38$ mm, plastic pellets and rapeseed, and the U_a is from 2.0 to 3.0 m/s. However, the calculations differ from the experimental results under other conditions. β is larger than 1.0. This means that the pressure drop equation by Konrad estimates a value larger than actual value, and is applicable under limited conditions.

Figure 11 shows that the experimental results are different from the value predicted by Pan and Wypych. When U_a is less than 3.0 m/s, α and β are much different from 1.0, although the calculations have a good agreement with the experimental results when U_a is larger than 3.0 m/s, except for rapeseed. This difference is possibly due to taking U_{sp} from the difference between U_a and W_p . U_a becomes large, u_s and W_p also become larger. Therefore the effect of $0.542\sqrt{gD}$ in Eq.(12) on W_p or U_a becomes smaller. But as U_a becomes smaller, the effect of $0.542\sqrt{gD}$ becomes larger. This could be an origin of the difference. Therefore, U_{sp} cannot be taken for $U_a - W_p$.

α is smaller than 1.0 and β is about 2.0 according to Tsuji's equation, as shown in Fig.12. α and β are not shown in accordance to the experiments. The estimation of the effect of gravity action and particle flow on the pressure drop are far higher than the actual value.

α and β calculated from the equation of Mi and Wypych completely deviate from the experimental values, as shown in Fig.13. Mi and Wypych deduced a relational expression on u_s with U_a from the experimental value under a specific condition. Therefore, the difference occurs because the applicability of the specific case is lacking.

Regarding the value of β calculated from the Aziz equation in Fig.14, it is proven that

there is completely no coincidence between the calculated value and the experimental value. From this result, the pressure drop by the particle flow cannot be considered at the difference between U_a and W_p .

The above results indicate that all of the previously-derived equations on u_s and $\Delta P/L_T$ have a difference relative to the experimental results. Figure 15, on the other hand, shows that α and β are mostly in the range 0.90 to 1.10 regardless of pipe diameter, type of particle and transportation conditions. Therefore, the difference of the equations proposed in this study for u_s and $\Delta P/L_T$ is within $\pm 10\%$ for all test conditions. This means that the proposed equations have high adaptability for types of particles, pipe diameter and transport conditions. In this study, the definition of slip velocity, the determination of F and the estimation of the effect of gravity action and particle flux on pressure drop are all made appropriately to construct the correct prediction.

Additionally, the relationship between non-dimensional particle velocity and non-dimensional pressure drop in a plug is shown in Fig.16 in order to confirm the validity of the estimation by the proposed equation for pressure drop due to gravity action and particle flow. The plotting point in the figure is the value that is measured ΔP and divided by L_p calculated from Eq.(23) and $\rho_B g \mu_w$. Then, the experimental value of u_s was used in Eq.(23). The continuous line in the figure is the prediction line from Eq.(25), which is derived from Eq.(24), divided by Eq.(23) and $\rho_B g \mu_w$.

$$\frac{\Delta P}{L_p} \cdot \frac{1}{\rho_B g \mu_w} = 1 + 1.084 \frac{K_W u_s}{\sqrt{gD}} \quad (25)$$

The first term on the right side of Eq.(25) represents the pressure drop due to gravity, and the second term represents the pressure drop due to the particle flow. Therefore, the vertical axis at $u_s=0$ in Fig.16 indicates the pressure drop due to gravity and the slope of the line indicates the pressure drop due to the particle flow. Fig.16 shows that the experimental results agree with the prediction equation of this study. Therefore, it is said that the prediction equation of this study precisely estimates pressure drop by gravity and particle flow in all test conditions.

6. Conclusion

In this paper, particle flow in the plug and transportation characteristic are analyzed in granular particle plug transportation through the horizontal pipe. As particles, rapeseed, polystyrol pellets, polyethylene pellets, nipolon hard and plastic pellets are used. Pipe diameters are 25.6, 38.0 and 50.0mm and length is about 13m. Transportation conditions are $U_a=1.1 \sim 5.0\text{m/s}$ and $m_f=21 \sim 105$. The results are as follows:

- 1) Equations on particle velocity in the plug and pressure drop applied modified Ergun's equation have been formulated. Calculations by these equations agree with experiments within 10% in all test condition. Additionally, the pressure drop estimations due to gravity action and particle flow are appropriate.
- 2) Particle velocity at the wall vicinity is constant regardless of the position from the result of high speed camera and PIV. Then it is said particles in the plug are transported in a fixed state with each other from this study's and Tanaka's research result.
- 3) Particles in an external plug are accelerated uniformly and they all move with the same velocity in the plug from the result of high speed camera and PIV.

Appendix

Each researcher's equation is shown in the following. Eq.(14) is applied during calculation in this paper.

(1) Konrad

$$\frac{\Delta P}{L_T} = \frac{M_s}{A \rho_B u_{sm}} \left[2g\mu_w \rho_B + 2.168\mu_w K_W \rho_B u_{sm} \sqrt{\frac{g}{D}} \cdot \frac{1 - 2M_s / \rho_B A u_{sm}}{1 - M_s / \rho_B A u_{sm}} \right] \quad (26)$$

$$u_{sm}^3 + u_{sm}^2 \left(-\frac{(a'+f)}{b'\rho_a} - 2(U_a + U_s) - \frac{M_s}{\rho_B A} \right) + u_{sm} \left(\frac{a'(U_a + U_s)}{b'\rho_a} + (U_a + U_s)^2 - \frac{2g\rho_B \mu_w}{b'\rho_a} \right. \\ \left. + \frac{M_s(a'+2f)}{\rho_B A b'\rho_a} + \frac{2M_s}{\rho_B A} (U_a + U_s) \right) - \frac{M_s}{\rho_B A} \left(\frac{a'(U_a + U_s)}{b'\rho_a} + (U_a + U_s)^2 - \frac{2g\rho_B \mu_w}{b'\rho_a} \right) = 0 \quad (27)$$

$$a' = 150 \frac{\eta(1-\varepsilon)^2}{\varepsilon^3 d_p^2} \quad (28)$$

$$b' = 1.75 \frac{(1-\varepsilon)}{\varepsilon^3 d_p} \quad (29)$$

$$f = 2.168 \mu_w K_w \rho_B \sqrt{g/D} \quad (30)$$

$$U_s = M_s / \rho_s A \quad (31)$$

$$u_{sm} = \frac{l_p u_s + (l_{ex} u_s / 2)}{l_p + l_{ex}} \quad (32)$$

Usually, l_p is far larger than l_{ex} . Therefore u_{sm} is treated as u_s in this paper.

(2) Pan and Wypych

$$\frac{\Delta P}{L_T} = \frac{M_s}{A(1-\alpha)\rho_B W_p} \left[2.168 \mu_w \rho_B K_w' \sqrt{\frac{g}{D}} W_p + \rho_B g \mu_w (2 - 1.175 K_w') \right] \quad (33)$$

$$\alpha = 0.542 \sqrt{gD} / W_p \quad (34)$$

$$K_w' = \frac{1 - \sin \phi \cos(\omega - \phi_w)}{1 + \sin \phi \cos(\omega - \phi_w)} \quad (35)$$

$$A_1 W_p^2 + A_2 W_p + A_3 = 0 \quad (36)$$

$$A_1 = b' \rho_a \quad (37)$$

$$A_2 = - \left[a' + 2.168 \mu_w K_w' \rho_B \sqrt{\frac{g}{D}} + 2b' \rho_a U_a \right] \quad (38)$$

$$A_3 = a' U_a + b' \rho_a U_a^2 - \rho_B g (2 - 1.175 \mu_w K_w') \quad (39)$$

(3) Tsuji

$$\frac{\Delta P}{L_T} = \frac{M_s}{\frac{\pi}{4} D^2 u_s (1-\varepsilon) \rho_s} \left[2.168 \mu_w K_w \rho_B u_s \sqrt{\frac{g}{D}} + 2 \mu_w \rho_B g + \frac{4}{D} \{ \mu_w (K_w + 1) c \cos \phi (\omega + \phi_w) + C_w \} \right] \quad (40)$$

$$C_1 (1 - u_s / U_a) + C_2 (1 - u_s / U_a)^2 = C_3 u_s / U_a + C_4 + C_5 \quad (41)$$

$$C_1 = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\rho_a}{(d_p / D)^2 U_a D \eta} \quad (42)$$

$$C_2 = 1.75 \frac{1-\varepsilon}{\varepsilon^3} \frac{1}{d_p / D} \quad (43)$$

$$C_3 = 2.168 \mu_w K_w (1-\varepsilon) \frac{\rho_s}{\rho_a F_{ra}} \quad (44)$$

$$C_4 = 2 \mu_w (1-\varepsilon) \frac{\rho_s}{\rho_a F_{ra}^2} \quad (45)$$

$$C_5 = \frac{4}{\rho_a U_a^2} \{ \mu_w (K_w + 1) c \cos \phi (\omega + \phi_w) + C_w \} \quad (46)$$

$$F_{ra} = U_a / \sqrt{gD} \quad (47)$$

(4) Mi and Wypych

$$\frac{\Delta P}{L_T} = \left(1 + 1.084 K_w' F_{rs}^{0.5} + 0.542 F_{rs}^{-0.5} \right) \frac{2g \mu_w M_s}{A W_p} \quad (48)$$

$$W_p = k (U_a - U_{a \min}) \quad (49)$$

$$U_{a \min} = \frac{\rho_s g \tan \phi_w \varepsilon^3 d_p^2}{180 (1-\varepsilon) \eta} \quad (50)$$

$$k = 105 \frac{\varepsilon d_p}{D} \left(\frac{\tan \phi_w}{\tan \phi} \right)^{1/3} \quad (51)$$

$$F_{rs} = W_p^2 / gD \quad (52)$$

(5) Aziz

$$\frac{\Delta P}{L_p} = E + \rho_B g K_w \left(\frac{U_a}{W_p} - 1 \right) \left(2 \tan \phi_w + \frac{4 \tan \phi}{\pi} \right) F_{rs} \quad (53)$$

$$E = \frac{\Gamma}{2} + \frac{4c}{\pi D} \pm \frac{4(K_w + 1)c \cos \phi \tan \phi \cos(\omega \pm \phi)}{\pi D} \quad (54)$$

$$\Gamma = \frac{4C_w}{D} \pm \frac{4}{D} (K_w + 1)c \cos \phi \tan \phi_w \cos(\omega + \phi_w) \quad (55)$$

In this study, Eq.(23) is applied for Eq.(53) to obtain $\Delta P/L_T$.

$$\frac{\Delta P}{L_T} = \frac{M_s}{A \rho_B u_s} \left[E + \rho_B g K_w \left(\frac{U_a}{W_p} - 1 \right) \left(2 \tan \phi_w + \frac{4 \tan \phi}{\pi} \right) F_{rs} \right] \quad (56)$$

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