Numerical Simulation of Sand Erosion in a Square-Section 90-Degree Bend

Masaya SUZUKI**, Kazuaki INABA*** and Makoto YAMAMOTO***

**Graduate School of Mechanical Engineering, Tokyo University of Science,
1-14-6 Kudankita, Chiyoda-ku, Tokyo 102-0073, Japan
***Department of Mechanical Engineering, Tokyo University of Science,
1-14-6 Kudankita, Chiyoda-ku, Tokyo 102-0073, Japan
E-mail: yamamoto@rs.kagu.tus.ac.jp

Abstract
Sand erosion is a phenomenon whereby solid particles impinging on a wall cause serious mechanical damage to the wall surface. The performance and lifetime of various machines, such as airplanes, ships, gas turbines, and pumps, are severely degraded by sand erosion. This phenomenon is a typical gas-particle two-phase turbulent flow and can be considered as a multi-physics problem in which the flow field, particle trajectory, and wall deformation interact. However, neither the change of the flow field nor the related particle trajectory during the erosion process has been taken into account in conventional simulations. This treatment is physically unrealistic. Hence, we have developed a numerical procedure by which to investigate the sand erosion phenomenon, including the temporal change of the flow field and the wall shape. In the present study, we simulate sand erosion of a 90-degree bend with a square cross-section. Bend erosion is the typical subject of sand erosion experiments and is useful for the verification of numerical simulations. The numerical results are compared with experimental data and it is confirmed that the developed code can capture the sand erosion phenomenon reasonably.

Key words: Erosion, Gas-Solid Two-Phase Flow, Three-Dimensional Flow, Turbulent Flow, Pipe Flow

1. Introduction
Sand erosion is a phenomenon whereby solid particles impinging on a wall cause serious mechanical damage to the wall surface. This phenomenon is a typical gas-particle two-phase turbulent flow and can be considered as a multi-physics problem in which the flow field, particle trajectory, and wall deformation interact. In industry, sand erosion has two primary meanings. One is that the performance and lifetime of various machines are severely degraded due to sand erosion, and the other is that sand erosion is used constructively in manufacturing technology applications. In general, the particle concentration is low for the former and high for the latter.

Basic research on sand erosion started in Germany during the 1930s. Finnie(1) attempted a theoretical analysis of sand erosion based on Hertz contact theory. However, Finnie model cannot precisely predict the weight loss for large-angle impacts. Then, Bitter(2)(3) suggested the mechanism of sand erosion, which consists of deformation wear and cutting wear. Bitter model gives the sufficient prediction for any impinging angle for both ductile and brittle materials, but this model is too complex to be applied to industrial machines. Therefore, Neilson and Gilchrist(4)(5) modified the model to be easily applied for practical calculation.
based on the Bitter’s concept. A group at Cincinnati University has also actively studied sand erosion. The primary subject of their sand erosion research is turbomachinery. The erosion model used in their recent studies (for example, Ref. (6)) was basically developed simultaneously but independently with the model proposed by Finnie, Bitter, and Neilson et al. As far as we know, there are no new effective erosion models.

In recent years, sand erosion has been simulated numerically in order to protect industrial machines from mechanical damage. For example, Habib et al. (7)(8) performed sand erosion simulations of a pipe with abrupt contraction and a shell-and-tube heat exchanger using the fairly strict form of the Basset-Bousinesq-Ossen (B-B-O) equation to predict particle movement in a gas-solid two-phase flow. In these simulations, however, the change of the flow field and the related particle trajectory during the erosion process were not taken into account. This treatment is physically unrealistic. In other words, since these simulations consider only the one-way effects from the continuous-phase to the dispersed-phase and from the particle trajectories to the wall deformation, the time development of the wall surface damage due to sand erosion cannot be reproduced precisely. Hence, we have developed a numerical procedure to investigate sand erosion, including the temporal change in the two-phase flow field related to the wall deformation (9)(10). The proposed procedure can reproduce any wall deformation by resolving the wall shape by computational grids. In addition, the interactions can be simulated by repeating a cycle that consists of three steps: the continuous phase, the dispersed phase, and the wall deformation due to sand erosion. This concept is applicable for any sand erosion phenomenon. We can select the appropriate equations for the continuous phase, the dispersed phase, and the wall deformation according to the computational objective.

On the other hand, the erosion of a surface by solid particles in a fluid stream is perhaps the dominant factor that makes industry reluctant to install pneumatic conveying systems for handling abrasive materials. Erosion is more severe for sudden changes in flow direction, for example, bends, cyclones, and valves of conveying systems. The bend erosion is the typical target of sand erosion experiments and is useful for the verification of numerical simulations. Inconveniently, the secondary flow that occurs in such a flow field, including the streamline curvature, cannot be satisfactorily reproduced by the standard k-ε model (KEM). Precise prediction of secondary flow driven by the cross-sectional shape requires the consideration of the nonlinearity of Reynolds stress. To predict complex flow fields, large eddy simulation (LES) or the Reynolds stress equation model (RSM) is generally employed. Since time consuming LES is not necessary for our purpose, RSM is a better choice. The most general RSM is the Gibson-Launder model (11), which is called the “basic” model. To consider low Reynolds number effects, Shima’s model (12)(13) is often used. These models have been widely used with success to calculate various turbulence flows. However, these models contain wall reflection redistribution terms that require the wall normal vector and the distance from the wall. Speziale et al. (14) proposed second moment closure (SMC) without wall reflection redistribution terms. On the other hand, attempts to extend the applicability of the eddy-viscosity model (EVM) have been active since the 1980s. Yoshizawa (15), Shih et al. (16), and other researchers have proposed a nonlinear eddy-viscosity model (NLEVM). These models take into account quadratic terms of strain and vorticity to estimate Reynolds stress, which are applicable to rotating channel flow, for example. These quadratic models can predict the effect of rotation, but not anisotropic Reynolds stress due to streamline curvature. Therefore, Craft et al. (17) proposed the NLEVM k-ε model including the cubic stress-strain relation and successfully predicted the flow field for a rotating pipe, a curved channel, and a turbulent jet impinging onto a heated flat plate. Moreover, in order to improve the prediction performance, Craft et al. (18) developed the NLEVM k-ε-A2 model, which solves the A2 transport equation in addition to k and ε equations.
The purpose of the present study is the numerical simulation of sand erosion on a square-section 90-degree bend. Initially, we use the Launder-Spalding (Std.) model, the Launder-Sharma (L-S) model, the Craft-Launder-Suga (C-L-S) model, the Gibson-Launder (G-L) model, the Speziale-Sarkar-Gatski (S-S-G) model, and the Shima model. Note that Std. and L-S are linear EVM KEMs, C-L-S is a NLEVM KEM, and G-L, S-S-G, and Shima are RSMs. Compared with the experimental study by Kim and Patel (19), the performance of each model for predicting the bend flow field is presented. In addition, based on the considerations revealed in the model validation study, we apply our three-dimensional sand erosion prediction code with Std., L-S, and S-S-G to the 90-degree bend with a square cross-section. These results are compared with experimental data by Mason and Smith (20). It is demonstrated that our simulation quantitatively reproduces the outer and the inner wall deformations due to sand erosion.

2. Numerical Procedures

2.1. Assumptions

The problem treated in the present study is mechanical damage of a bend. Therefore, the particle concentration in the flow field is small enough to ignore particle-particle collisions and interactions with the flow field from the particle-phase. Normally, since the sand erosion phenomenon requires a long period and the time scale is much longer than that of the flow field, the change of the flow field can be regarded as a quasi-steady state. Therefore, steady-state or time-averaged flow distributions are thought to be valid for each eroded geometry at every instance. This means that in the present study, the sand erosion phenomenon is mimicked as a series of quasi-steady states. Then, the computational procedures for the prediction of the sand erosion phenomenon are as follows: (1) calculate the turbulent flow field, (2) calculate the particle trajectories, and (3) change the wall shape. These procedures are repeated iteratively until the computational time reaches the prescribed terminal time.

2.2. Gas-phase

Numerical predictions have been carried out with a finite difference technique. The gas-phase is considered to be a continuum phase, while the particle-phase is a dispersed one. As mentioned above, the particle-phase that is assumed to be of low concentration has no influence on the gas-phase (i.e., one-way coupling). The gas-phase flow is assumed to be three-dimensional, compressible and turbulent. It is calculated by the Eulerian approach, based on the Favre-averaged continuity equation, Navier-Stokes equations and energy equation (i.e., RANS approach).

\[
\frac{D \rho}{D t} = 0 \tag{1}
\]

\[
\frac{D}{D t} \left( \rho \bar{u}_i \right) = - \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \bar{e} - \rho \bar{u}_i \bar{u}_j \right) \tag{2}
\]

\[
\frac{D}{D t} \left( \rho \bar{e} \right) = - \frac{\partial}{\partial x_j} \left( \rho \bar{u}_i \bar{u}_j \right) + \frac{\partial}{\partial x_j} \left( \bar{e} - \rho \bar{u}_i \bar{u}_j \right) \tag{3}
\]

where \( t \) is the time, \( x_i \) is location in Cartesian coordinates, \( u_i \) is the velocity in the \( x_i \) direction, and \( \rho, p, T, \) and \( e \) denote the density, static pressure, static temperature, and total energy of the fluid. In addition, \( \left( \bullet \right), \left( \ast \right), \) and \( \left( \ast \ast \right) \) denote the fluctuating component, the Reynolds averaging operation, and the Favre averaging operation, respectively. In the turbulence model, we use three KEMs (Std., L-S, and C-L-S) and three RSMs (G-L, S-S-G, and Shima).

The governing equations were discretized using Yee-Harten’s second-order upwind TVD scheme (1987) for the inviscid terms, the second-order central difference scheme for
the viscous terms, and the four-stage Runge-Kutta method for the time integration. In order to reduce the time required to obtain the steady state solutions, we applied the local time stepping method.

### 2.3. Particle-phase

The particle-phase is treated by the Lagrangian approach, in which particles are tracked over time along their trajectories through the flow field. We assume that the particles are spherical and non-rotating. The strictest momentum equation of spherical particles is the B-B-O equation, but as mentioned above, particle-particle collisions and the interaction with the flow field from the particle-phase are negligible because the solid loading is sufficiently small. In addition, the buoyancy force, the pressure gradient force, the added mass force, and the Basset history force are negligible because the difference between the density of solid particles and the fluid is large and the free-stream velocity is high (approximately 100 m/s). In the future, we will investigate the effects of particle shape and rotation. Under these assumptions, the equations of particle motion are as follows:

\[
\frac{dx_p}{dt} = u_{pi} \quad (4)
\]

\[
\frac{du_p}{dt} = F(u_{pi} - u_{p}) \quad (5)
\]

\[
F = \frac{3C_D p_f D}{4\rho_f D_p} \quad (6)
\]

where \( f \) and \( p \) denote the fluid and particle, respectively, \( D \) is the particle diameter, \( F \) is the drag force, and \( C_D \) is the drag coefficient defined by the particle Reynolds number \( Re_p \) based on the relative velocity between the gas-phase and the particle, as follows:

\[
C_D = \begin{cases} 
\frac{24}{Re_p} & (Re_p < 1000) \\
0.4 & (Re_p > 1000) 
\end{cases} \quad (7)
\]

\[
Re_p = \frac{D |u_{pi} - u_{p}|}{\nu} \quad (8)
\]

where \( \nu \) is the kinematic viscosity of gas-phase. The leapfrog method is applied for the time integration of the above equations. The fluid velocity \( u_f \) is the time-averaged velocity obtained by RANS. In a previous study \( (9) \), we carried out two-dimensional computations considering the effect of the fluctuating velocity and found that the turbulence did not affect the results of the computations. The same tendency is confirmed in the present study.

When the particle impinges on the surface of a wall, the particle velocity after the impact is decreased according to the kinetic energy consumed by erosion. Namely, the velocity components corresponding to the energy consumed by deformation and cutting wear are subtracted from the vertical and horizontal velocity components of the wall surface.

### 2.4. Erosion estimation

Bitter\(^{(2)(3)}\) suggested that sand erosion damage due to particle impacts can be considered as separate mechanisms, namely, deformation wear due to the velocity normal to the surface and cutting wear due to the tangential velocity. The total volume loss \( W_T \) is the sum of the volume losses due to deformation wear \( W_D \) and cutting wear \( W_C \):

\[
W_T = W_D + W_C \quad (9)
\]

However, since Bitter’s theoretical work is exhaustive and extremely intricate, it is too difficult to employ his model in practical applications. Therefore, we apply a simpler relation based on the model proposed by Neilson and Gilchrist\(^{(4)(5)}\), in which the weight losses \( W_D \) and \( W_C \) can be rewritten as
\[ \psi \alpha^2 \sin^2 \frac{1}{KVM} WD = (10) \]

\[ n = \pi = (12) \]

where \( M \) is total mass of particles, \( \alpha \) and \( V \) are the attack angle and impinging velocity of a particle, \( K \) is the threshold value of the velocity component normal to the surface, below which no deformation wear occurs, \( \alpha_0 \) is the attack angle at which the tangential component of the reflection velocity becomes zero (see Fig. 1). \( n \) is constant that depends on the surface material, and \( \psi \) and \( \phi \) represent the energy needed to remove the unit weight of material from the wall by deformation and cutting wear, respectively. The dimensions of \( \psi \) and \( \phi \) are \( m^2/s^2 \). Therefore, the units of Eqs. (10) and (11) are the same as those of Eqs. (13) and (14). Note that \( \psi \), \( \phi \), \( n \), and \( K \) were obtained in preliminary experiments.

However, the geometric information of the eroded surface cannot be obtained from the Neilson-Gilchrist erosion model because Eqs. (10) and (11) describe only the weight loss of the surface material. Thus, the eroded surface geometry damaged by one particle, which consists of deformation wear (volumes of A) and cutting wear (volumes of B), is assumed to be as shown in Fig. 2. Then, \( l_c \) and \( \theta_p \) should be calculated as geometric information, where \( l_c \) is the eroded length due to the cutting wear and \( \theta_p \) determines the depth of the erosion cavity. The weight losses due to deformation wear \( W_D \) and cutting wear \( W_C \) can be geometrically expressed using the symbols in Fig. 2 as follows:

\[ \begin{align*}
W_D &= \frac{1}{2} M(V \sin \alpha - K)^2 \psi \\
W_C &= \begin{cases} 
\frac{1}{2} M V^2 \cos^2 \alpha \sin n \alpha & (\alpha < \alpha_0) \\
\frac{1}{2} M V^2 \cos^2 \alpha \phi & (\alpha \geq \alpha_0) 
\end{cases} \\
\alpha_0 &= \frac{\pi}{2n}
\end{align*} \]

Fig. 1 Schematic diagram of particle impact against wall

Fig. 2 Modeled erosion cavity

Fig. 3 Schematic diagram of the computational grid

Fig. 4 Schematic diagram of the computational grid

\[ W(i) = W_T \times l_c(i) / l_c \]

\[ W_D = \rho \pi \left( \frac{D_p}{2} \right)^3 \left( \cos^2 \theta_p + \frac{2}{3} \cos \theta_p \right) \]

\[ W_C = \rho \pi \left( \frac{D_p}{2} \right)^2 \left( \theta_p - \frac{1}{2} \sin 2 \theta_p \right) \]

where \( W_D \) and \( W_C \) are known. Then, \( l_c \) and \( \theta_p \) are determined using Eqs. (13) and (14).
It is difficult and uneconomical to fully resolve the eroded cavity defined in Fig. 2 for each particle-wall collision. Therefore, we use the Erosion Line approach proposed in Ref. (10). In this approach, the computational grid consists of two regions, that is, a region in a flow field and a region within a solid wall (see Fig. 3). The block height in the wall is uniform and is the same as the minimum height in the flow field. The weight of each node is set prior to calculation. The mass of the wall grid nodes decreased as the erosion progressed. If the mass of the nodes becomes zero, the nodes drop out and are recognized as part of the fluid region. The weight loss of each node is distributed as follows:

1. Draw an erosion line $l_c$ on the surface in the computational domain (see Fig. 4).
2. Calculate the partial length $l_c(i)$ occupied by each grid cell.
3. Calculate the local weight loss based on the ratio of the total length $l_c$ and the partial length $l_c(i)$.
4. Assign the local weight loss of the cell to each node based on the distance to the center of gravity of the partial erosion line.

An erosion cavity is approximated by a line of length $l_c$, taking into account that the width of the erosion cavity is sufficiently small, as compared with the three-dimensional grid spacing on the wall surface.

In this procedure, when a particle approaches the wall surface, we judge the surface including three grid points that are the nearest nodes from the particle as the wall surface upon which the particle impinges. The inner product of the wall normal vector and the vector, which consists of a node on the surface and the particle, is considered to be the distance from the wall surface to the particle. If the distance is smaller than the particle radius, a collision is considered to have occurred. The angle between the surface including the three nodes and the velocity vector of the solid particle is the impinging angle. Based on the Neilson-Gilchrist model, both the rebound speed and the angle after the collision are determined. Thus, the eroded cavity formation changes the rebound angle of the subsequent impinging particle.

3. Computational Conditions

First, in order to verify the prediction performance of the turbulence models for the flow field of the rectangular cross-section 90-degree bend, experiments by Kim and Patel (19) are simulated. The Reynolds number for this case is $2.24 \times 10^5$ based on the bend height ($H$) and the mean bulk velocity ($u_b$). $R/H = 3.5$, where $R$ is the bend curvature radius. The cross-section of the duct is rectangular with an aspect (height-to-width) ratio of 1:6. There are two reasons why the bend, the aspect ratio of which is 6, is selected for the validation of the models. One of our ultimate goals is to apply the proposed method to sand erosion on cascades of gas turbines, for example. Recent cascades of gas turbines are usually

![Fig. 5 Schematic diagram of the 90-degree bend](image)

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>width of channel</td>
<td>6 H</td>
</tr>
<tr>
<td>entrance length</td>
<td>5 H</td>
</tr>
<tr>
<td>exit length</td>
<td>7.5 H</td>
</tr>
<tr>
<td>radius of curvature</td>
<td>3.5 H</td>
</tr>
<tr>
<td>Grid Number</td>
<td></td>
</tr>
<tr>
<td>coarse</td>
<td>$121 \times 43 \times 81$</td>
</tr>
<tr>
<td>medium</td>
<td>$121 \times 111 \times 161$</td>
</tr>
<tr>
<td>fine</td>
<td>$121 \times 187 \times 251$</td>
</tr>
</tbody>
</table>

Table 1 Setup of the 90-degree bend with a rectangular cross-section
somewhere between fully three-dimensional (aspect ratio of 1) and fully two-dimensional (aspect ratio of greater than 12). Therefore, a validation study with respect to the bend, the aspect ratio of which ranges between approximately 4 and 8, is required. Many validations of square cross-section bends (aspect ratio of 1) have been reported in Refs. (21)-(23) and in the literature, and the prediction performance of turbulence models has been clarified significantly. However, since the available data of validation for the medium aspect ratio is limited, the model performance is little known. Figure 5 shows a schematic diagram of the 90-degree bend. Stations U1-D2 are the sections for which numerical results are compared with experimental data. For the purpose of verification, we computed the grid independency for three grid resolutions. The coarsest grid and the medium grid respectively were confirmed to be sufficient for resolution for high Reynolds number (HRN) models and low Reynolds number (LRN) models. Note that Std., G-L, and S-S-G, are HRN models and L-S, C-L-S, and Shima are LRN models. The conditions are summarized in detail in Table 1. The grid numbers in the table indicate, in order, streamwise, radial, and spanwise. The boundary conditions are imposed as follows. At the inlet boundary, velocity, static temperature, and turbulence quantities are fixed. On the other hand, at the exit, the static pressure is specified. On the walls, slip and adiabatic conditions with the wall function are imposed in the case of HRN models. In the case of LRN models, non-slip and adiabatic conditions without the wall function are imposed to resolve the viscous sublayer.

Next, the erosion calculations are executed for the 90-degree bend with a square cross-section measured by Mason and Smith (20). The scales of the bend and grid numbers are listed in Table 2. This bend has a large radius (\(R/H = 9\)). Such a large curvature bend suffers from smaller erosion than the standard radius bend when gas is the carrier fluid. The boundary conditions are similar to the case of the flow field verification. The inlet air velocity is 100.6 m/s. The ratio of the mass flow rate of solid particles to that of air is 0.5. The materials of the solid particles and the wall were Perspex (\(\rho = 3.9 \times 10^3\) kg/m\(^3\)) and alumina (\(\rho = 1.19 \times 10^3\) kg/m\(^3\)), respectively. The particle diameter is 55 \(\mu\)m. Since significant calculation time is required in order to trace all of the solid particles corresponding to the solid loading ratio of 0.5, a number of particles of all sands are calculated, and the total weight loss of the bend is corrected by multiplying the local weight losses by a coefficient. The calculating particle number, which is determined as the particle number, does not affect the numerical results by

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**Table 2** Setup of the 90-degree bend with a square cross-section

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>height of channel</td>
<td>25.4</td>
</tr>
<tr>
<td>width of channel</td>
<td>25.4</td>
</tr>
<tr>
<td>entrance length</td>
<td>685.8</td>
</tr>
<tr>
<td>exit length</td>
<td>685.8</td>
</tr>
<tr>
<td>radius of curvature</td>
<td>228.6</td>
</tr>
<tr>
<td>wall thickness</td>
<td>30.48</td>
</tr>
</tbody>
</table>

**Table 3** Model functions in the erosion model

<table>
<thead>
<tr>
<th>(V) [m/s]</th>
<th>(\psi) [m(^3)/s(^2)]</th>
<th>(\phi) [m(^3)/s(^2)]</th>
<th>(n)</th>
<th>(K) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.8</td>
<td>(1.74 \times 10^6)</td>
<td>(1.01 \times 10^6)</td>
<td>3.91</td>
<td>2.5</td>
</tr>
<tr>
<td>15.7</td>
<td>(1.35 \times 10^6)</td>
<td>(1.32 \times 10^6)</td>
<td>8.18</td>
<td>2.5</td>
</tr>
<tr>
<td>18.0</td>
<td>(5.71 \times 10^5)</td>
<td>(1.23 \times 10^6)</td>
<td>3.91</td>
<td>2.5</td>
</tr>
<tr>
<td>20.0</td>
<td>(6.04 \times 10^5)</td>
<td>(9.78 \times 10^5)</td>
<td>7.96</td>
<td>2.5</td>
</tr>
<tr>
<td>25.0</td>
<td>(3.21 \times 10^5)</td>
<td>(1.02 \times 10^6)</td>
<td>1.99</td>
<td>2.5</td>
</tr>
<tr>
<td>100.0</td>
<td>(1.60 \times 10^7)</td>
<td>(2.31 \times 10^7)</td>
<td>1.51</td>
<td>2.5</td>
</tr>
</tbody>
</table>
preliminary computations with different particle numbers. In these simulations, one million
of particles are investigated. The particles are uniformly distributed at the inlet region of the
computational domain in the initial condition. The computations of the flow field are carried
out every time 1/10 of the particles of the total ingestion are introduced, and the calculations
of the particle trajectories and the wall deformation are then performed using the steady
state solution. Thereby, sand erosion of the bend is simulated by 10 cycle computations.

Since there are few available experimental data on collisions between Perspex and
alumina, the fitted values of erosion parameters ($\psi$, $\phi$, $n$, and $K$) for the experimental data\(^{(24)}\)
that were obtained by similar conditions were used. These parameters are linearly
interpolated from Table 3 as a function of impinging velocity ($V$).

Mason and Smith reported that once a pocket formed on the bend wall due to sand
erosion, solid particles accumulate in the eroded pocket and the erosion depth no longer
grows. In order to reproduce this phenomenon, it is essential to consider particle-particle
collisions. However, for simplification, we limit the erosion evolution if the eroded cavity
angle becomes too sharp (Fig. 6).

4. Results and Discussion

4.1. Validation

Figure 7 shows the streamwise velocity ($u$) profile of the midspan. The position and
velocity components are normalized by the bend height and the mean bulk velocity. In
general, the primary flow deflects inward at the start of curvature. Thereafter, the primary
flow deflects rapidly toward the outer wall by centrifugal force. The value of $R/H$ of the
bend is small, and Fig. 7 indicates that the inside velocity is higher than the outside velocity
from station 15 to $D_2$. Our simulations successfully reproduce this tendency. C-L-S and
RSMs improve the velocity profile of the outer wall, as compared with linear EVMs. However, the C-L-S model is numerically unstable, and the results show the unphysical
oscillation. Therefore, stronger numerical viscosity is required. Comparisons of other
surfaces are omitted here. Although it is confirmed that C-L-S and RSMs improve the
predictions of streamwise velocity, as compared with linear EVMs, the differences are not
significant. Moreover, since mean velocity distributions govern erosion, the purpose of this
validation is to confirm the reproductivities of the mean velocity profiles. A previous
study\(^{(30)}\) reported that erosion quantities are not affected, even if the prediction accuracy of

![Fig. 7 Streamwise velocity profile of midspan (symbol: experiment, line: calculation)](image)
the turbulent characteristics is insufficient. Hence, comparisons of the turbulent characteristics are not presented.

It is important to verify that the reproducibility of secondary flow in curved tubes since the reproducibility with KEM is low. Secondary velocity profiles at the 0.25\(H\) section and at the 0.50\(H\) section of station 45 and station D1, respectively, are exhibited in Fig. 8. The results for Std. and S-S-G are shown as representative of KEMs and RSMs, respectively. Here, \(v\) and \(w\) are the velocity components of the bend curvature direction and spanwise direction, respectively. Both results agree well with experimental data. Compared with S-S-G, Std. slightly underpredicts the peak of the curve near the side wall (section 0.25\(H\)). However, the disadvantage of KEM is limited near the side walls. Moreover, the damage due to sand erosion may be not important there. In addition, Std. is a reasonable choice for sand erosion simulation of the 90-degree bend. The anisotropic Reynolds stress of the 90-degree bend is not remarkable, as compared to the 180-degree bend, linear EVMs are available for the present case.

The ratio of the CPU time required for solution convergence is approximately 1 (Std.) : 5 (L-S) : 6.5 (C-L-S) : 1.5 (G-L and S-S-G) : 18 (Shima). The HRN type models give appropriate solutions with very reasonable costs.

4.2. Change in the flow field due to sand erosion

Figure 9 shows the change in the velocity vectors at the midspan before and after sand erosion. Since the results with any turbulence model applied in the present study show a similar tendency, the results for only the representative model (Std.) are presented here. It is confirmed that the low-velocity domain in the cavity formed by the particle impacts. This causes the boundary layer at the downstream region of the bend to become thick, and the primary flow is deflected toward the opposite side wall and to be accelerated.

Figure 10 depicts the change in the static pressure contour before and after sand erosion. Static pressure increases in the cavity and upstream. In other words, the pressure
drop becomes large due to sand erosion. In addition, since the surfaces of the bend wall are rough, as shown later herein, the static pressure distribution becomes irregular.

4.3. Particle trajectories

Representative particle trajectories before and after erosion (60% of total sand ingestion) are shown in Fig. 11. The progression of sand erosion changes the rebound pattern. When erosion does not occur, particles impinge against the outer wall surface with a very low impinging angle. Therefore, the rebound angle of the particles is small, and the particles flow downstream with the repeated impacts on only the outer wall. On the other hand, after the formation of the erosion cavity on the surface of the outside wall, the impinging angle become high, and some particles impact on the inner wall. This leads to the erosion of the inside wall. These findings correspond to the experimental findings of Mason and Smith [20]. In addition, this clearly suggests that the wall deformation due to sand erosion should be considered in the numerical simulation.

The ratio of CPU time required for the calculations of the particle movement to computations of the flow field is approximately 1/100 in the case of the standard KEM. The proportion between the models is approximately 1 (Std.) : 3 (L-S) : 1 (S-S-G).

4.4. Eroded surface

Figure 12 illustrates the time evolution of the eroded surfaces due to sand erosion with bird eye view. The eroded wall boundaries on the outer wall and the inner wall are shown without side walls. Thus, there are the side walls in the white region. A highly rough surface and an irregular streaky erosion pattern can be seen. Initially, the particles ingested in the bend part impact the outer wall at a low impinging angle and form the cavity from 10 degrees to 30 degrees (Fig. 12 (a)). The formation of this primary cavity causes the impinging angle to become large. Consequently, the particles impact the inner wall at 50
degrees. Moreover, particles that rebound from the inner wall impact the outer wall again. This fact causes secondary wear on the outer wall (Fig. 12 (b) and (c)).

The erosion depth at the midspan for the bend angle is shown in Fig. 13. In the figure, the numerical results obtained by three turbulence models and the experimental data are compared. Positive values indicate the erosion of the outer wall surface, and negative values indicate the erosion of the inner wall surface. The primary wear of the outer wall, the primary wear of the inner wall, and the secondary wear of the outer wall mentioned above are also confirmed experimentally. Simulations with any turbulence models quantitatively reproduce the erosion depth. Turbulence models appear not to substantially affect erosion estimation. Fortunately, this means that standard KEM, which requires little CPU time and memory, can be employed to predict sand erosion on the 90-degree bend with a square cross-section. However, S-S-G overpredicts the primary wear of the inner wall. Since RSMs tend to give the large flow deflection of a free stream due to the curvature of the bend and the centrifugal force, the velocity at the curvature over the outer wall exceeds the results predicted by KEMs. Therefore, the primary wear of the outer wall occurs farther downstream, and the impinging angle on the outer wall changes. The difference between the results obtained by KEMs and S-S-G occurs due to these mechanisms. Based on the validation of the rectangular bend, the flow field predicted by S-S-G appears to be more realistic than that predicted by KEMs. Thus, KEMs might overpredict the primary wear of the outer wall and underpredict the primary wear of the inner wall. Since the time development of sand erosion depends on many factors, it is difficult to clearly specify the reason for this. Further investigations, including the erosion parameters shown in Table 3, are required.

5. Conclusions

The purpose of the present study was numerical prediction of sand erosion on a square-section 90-degree bend using the numerical method considering the temporal change of a wall surface and the change of the two-phase flow corresponding to the deformation. We obtained the following results:

1. The turbulent flow field of the 90-degree bend can be reproduced by the six models considered in the present study.
2. RSMs, rather than EVMs, accurately predict secondary flow.
3. The proposed procedure can quantitatively reproduce the outer and inner wall deformations due to sand erosion.
4. The inner wall erosion is caused by changes in the particle trajectories by the outer wall deformation.
5. As turbulence models do not substantially affect prediction performance, HRN models that require little CPU time and memory are a reasonable choice.

The inner wall erosion is attributed to the change in the continuous phase and the
dispersed phase flow resulting from the outer wall erosion. If the two-phase flow does not reflect the effect of wall deformation, the inner wall erosion cannot be reproduced. Therefore, it seems that the proposed procedure is useful for simulating sand erosion phenomena.

In the present study, using the simplified model, we modeled the phenomenon whereby erosion is limited to the eroded cavity generated by the collisions between flowing particles and depositing particles in the pocket. In the future, we will clarify the phenomena that are simplified or neglected in the present study by numerical simulation based on stricter modeling considering particle-particle collisions.

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References

(10) Kuki, J., Toda, K. and Yamamoto, M., Development of Numerical Code to Predict


