Study of Interaction between Unsteady Supersonic Jet and Shock Waves in Elliptical Cell*

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Abstract
The unsteady behavior of a flow driven by a jet suddenly injected into cells is investigated numerically by solving the axisymmetric two-dimensional compressible Navier-Stokes equations. The system of the calculation is a model of laser ablation of a certain duration followed by a discharging process through the exit hole at the downstream end of the cell. The parameters for the calculations are the diameter of the cell, the exit diameter of the cell and the duration of the injected jet. The velocity monitored at the exit hole is used to evaluate the influence of the shape and the exit diameter of the cell on the shock wave, the plume, and their interactions. As a result, it was found that the position in which the shock wave converges and the focal point of the elliptical cell are almost the same for various parameters. The spreading shock wave that converged at the geometrical focal point reflects from the cell wall and this shock wave alternate between the jet injection and the cell exit. It was also found that the number of peaks and the maximum velocity downstream of the cell exit are roughly determined by the contour of the walls of the elliptical cell.

Key words: Shock Wave, Computational Fluid Dynamics, Jet, Unsteady Flow, Axisymmetric Flow

1. Introduction

Pulsed Laser deposition (PLD) is a promising technique that has been applied to grow high-quality thin films, for example, for high-temperature superconductors. Recently, this technique has been used for the preparation of nano-clusters in the gas phase(1). The thin film is formed by piling up clusters discharged with the pulsed laser. Clusters of uniform size are very important for controlling the nano structure in the manufacturing process of the thin films. After the cluster is generated, the best size cluster is chosen in the conventional PLD (2, 3). On the other hand, there is a method for achieving the best size at the generation of the cluster.

Iwata et al. proposed a new method to produce monodispersed clusters, which is
expected to allow the direct generation of monodispersed clusters. This method uses an interaction phenomenon between the plume and shock wave arising in an elliptical cell following a laser ablation in inert gas. They reported that the size of the cluster was controlled using the interaction between the shock wave and the plume.

Yaga et al. reported the calculation results, such as the Mach number, the duration of the injected jet, and the exit diameter of the elliptical cell, under certain conditions. Their study showed that the wave form of the variations and their levels are roughly determined by the durations of the jet and the diameter of the exit.

The purpose of the present study is to investigate the influence of the cell diameter on the plume and the shock wave in the elliptical cell. For simplicity of the numerical analysis, the laser-ablated plume is replaced with a supersonic jet suddenly injected into the cell. In the calculations, air is injected into the flow field inside the cell.

2. Numerical Procedure and Boundary Conditions

In the numerical calculations, the axisymmetric two-dimensional compressible Navier-Stokes equations are solved using the finite volume method with the TVD scheme in curvilinear generalized coordinates. The inviscid flux is evaluated by the third-order Roe's approximate Riemann solver. The third-order Runge-Kutta scheme is used for time integration. The Jacobian matrices and the metric coefficients related to the transformation between the numerical plane and the physical plane are evaluated in a finite volume manner. All calculations are carried out on a 91 x 51 grid.

The contours of the wall are calculated by the following equation:

$$\frac{(x-c)^2}{a^2} + \frac{y^2}{b^2} = 1$$  \hspace{1cm} (1)

where $2a$ and $2b$ are the lengths of the major and minor axes, respectively, and $c$ is the $x$-coordinate of the center. In all shapes, the diameter of the left side wall and the cell exit were fixed. The focal points are $\left(z, \sqrt{a^2 - b^2}, \pm c, 0 \right)$ and $\left(z, \pm \sqrt{a^2 - b^2}, c, 0 \right)$ for $a > b$ and $b > a$, respectively. Figure 1 shows an example of the elliptical contour using the above equation. Three contours indicate the maximum and minimum diameters of the elliptical cell, and shape in the middle shows the standard elliptical cell (from Ref. 5). Calculations are performed for nineteen cases. The open symbols represent the focal points for $D_m/D_j=6.34$, $D_m/D_j=8.94$, $D_m/D_j=10.5$, $D_m/D_j=15.3$, and $D_m/D_j=15.3$, where $D_m$ and $D_j$ are the diameter of the elliptical cell at $x/D_j=8.71$ and the injected jet diameter, respectively, and $x$ is the horizontal distance from the jet injection. As the cell diameter exceeds $D_m/D_j=10.5$, the focal point diverges from the $x$-axis.

For the boundary conditions, the non-slip conditions are applied to the cell wall, except for the position of the jet injection and the cell exit, as shown in Fig. 1. The position of the jet injection is set at $x/D_j=0.0$. The cell exit with the diameter $D_e$ connects inside and outside of the cell, through which the flow passes during the propagation of the shock wave and the pressure waves.
During the focusing process of the propagating shock wave, the interaction between the converging shock wave and the plume plays an important role in the growing cluster size. The suddenly injected jet is shut off after a certain period so that the calculation can be used for a basic reference for PLD techniques. The parameters for the calculations are the cell diameter of $x/D_j = 8.71$, the diameter of the cell exit, and the jet duration. The fixed conditions are as follows: $p_e/p_j = 1.5$, $T_e/T_j = 0.5$, and $M = 4$, where $p_e$ and $T_e$ are the initial ambient pressure and the temperature inside and outside the cell, respectively, $p_j$ and $T_j$ are the jet exit pressure and the temperature, respectively, and $M$ is the injected jet Mach number. The Reynolds number based on the velocity and the diameter of the jet is about 1000. Based on the standard elliptical cell diameter $D_{m}/D_j = 8.94$, the diameter of the elliptical cell was varied from $D_{m}/D_j = 6.34$ to 15.3.

3. Results and Discussion

3.1 Verification of calculation result

The phenomenon whereby the shock wave discharged from a round axisymmetric
two-dimensional hole is calculated to verify the validity of the calculated results. We compared the calculated results with the experimental results of Ishii et al.\(^7\) and the calculated results of Irie et al.\(^8\) The density contours of the numerical results were found to be consistent with those of the experimental results and so are not suitable for the quantitative comparison. Figures 2(a) and 2(b) show the barrel shock wave and vortex confirmed in the calculation\(^8\), which are also observed in the experiment\(^7\). In order to discuss these phenomena more quantitatively, the vortex ring positions \(x_v/D, y_v/D\) produced by the sudden injection of the jet are plotted in Fig. 3. The open and solid symbols represent the vortex center position of data of Ishii et al. and of Irie et al., respectively. The results of the radial coordinate \(y_v/D\) in our research are slightly higher than those calculated by Irie et al. However, the present calculation results are consistent with the experimental results of Ishii et al. The present calculation results for the axial coordinate \(x_v/D\) are in good agreement with those of Irie et al. and Ishii et al. Therefore, the present calculation is sufficiently reliable for these types of flows.

The grid dependency of the elliptical cell in case of \(D_m/D_j = 8.94\) was also checked with a coarse mesh (91x51) and a fine mesh (361x201). Figures 4(a) and (b) show the variations in the calculated \(x\)-component velocity at \((x/D_j = 8.71, y/D_j = 4.01)\) and \((x/D_j = 16.1, y/D_j = 2.45)\), respectively. Although there is slight difference between them, both figures suggest that the accuracy of the coarse mesh is high enough to perform calculations in the following discussions. Accordingly, the coarse mesh (91x51) was adopted to reduce the calculation time in this paper.

### 3.2 The case of the standard elliptical cell

Figures 5(a) ~ 5(f) show the calculated density contours using the following parameters: cell diameter \(D_m/D_j = 8.94\), jet duration \(t_{jet} = 1.3\), and cell exit diameter \(D_e/D_j = 1.0\), where \(D_m\) and \(D_e\) are the diameter of the cell at \(x/D_j = 8.71\) and the diameter of the cell exit, respectively. The time shown in each figure denotes the non-dimensional elapsed time from the moment of the jet injection. The arrows indicate the direction of the traveling shock wave. Figure 5(a) shows the injected jet in the cell and the shock wave generated by the sudden injection of the jet. The shock wave propagates toward the right wall of the cell together with the jet plume. The distance between the shock wave and the plume head has increased in Figs. 5(b) and 5(c). The plume has decelerated while the
shock wave continues moving toward the cell exit hole. The propagating shock wave is reflected from the upper wall of the cell and changes its direction toward the focal point. Figure 5(d) shows the moment at which the shock wave is almost focused at the focal point of the cell and, at the same time, the plume front is located at approximately this point. After the shock wave has converged at the focal point, it begins to spread out again. Most of the spreading shock wave, except for the left traveling shock wave, impinges on the wall again, as shown in Fig. 5(e). On the other hand, the shock wave traveling along the direction indicated by the arrows is due to, so called, the secondary shock wave of the blast wave (9), (10). It shows that the causes of the shock wave generations are the sudden injection and cut off of the jet. In Fig. 5(f), the compression wave indicated by arrows is the shock wave that reflects from the right side wall, reflects from the left side wall, and then propagates toward the cell exit again.

For the interaction between the shock wave and the jet plume, it is important to determine the effect of the shock wave on the velocity at the cell exit. Figure 6 shows the velocity variations at \( x/D_j = 20.6 \) and 28.7 for the same conditions as for Fig. 5, which correspond to the exit and far downstream of the cell. The solid line indicates the \( x \)-component velocity variation at the cell exit inside the wall at \( x/D_j = 20.6 \), and the dotted line indicates the \( x \)-component velocity variation at the downstream position \( x/D_j = 28.7 \). In Fig. 6, the time indicated by (e) corresponds to that of Fig. 5(e). There are three peaks at \( x/D_j = 20.6 \). The first peak indicated by (e) indicates the moment at which the shock wave focusing at the focal point of the cell reaches the cell exit. The second peak observed at \( t = 23.6 \) when the shock wave generated by stopping the jet arrives at the cell exit during the period of the plume passage to the cell exit. The third peak at \( t = 38.5 \) shows the moment at which the propagating shock reaches the cell exit again, after the shock wave focuses at the focal point. This propagating shock wave is induced by the suddenly injected jet and is reflected from the left side wall. At the cell exit downstream \( x/D_j = 28.7 \), peaks appear at \( t = 29.7, 38.5, \) and \( 48.5 \). These three peaks at the cell exit downstream correspond to the peak of the cell exit, and the peaks at the cell exit downstream occur after a certain time.

### 3.3 Effect of cell diameter on the flow in the cell

Figures 7(a) ~ 7(d) show the calculated density contours for diameters \( D_m/D_j = 6.34 \), where the remainder of the conditions are the same as those for Fig. 5. Figure 7(a) shows that the shock wave generated by the sudden injection of the jet travels toward the right wall of the cell. The straight shock wave that reflected from the upper wall propagates toward the focal point of the cell, as shown in Fig. 7(b). This shock wave penetrates the plume, converges at the focal point, and propagates toward the right wall again, as shown in Fig. 7(d). At the same time, the spreading shock wave reflects from the right side wall, reflects from the left side wall again, and reaches the cell exit. The reflected shock wave increases the velocity of the cell exit. The influence of the reflected shock wave on the flow is found to be the same as in the case of the standard elliptical cell.
For the investigation of the influence of the large diameter on the flow, the calculated density contours for the cell diameter $D_m/D_j=12.9$ are shown in Figures 8(a) ~ 8(f). The elapsed time in which the reflected shock wave interacts with the plume is long because the cell diameter is large, and Fig. 8 shows a different trend for the cell diameter $D_m/D_j=12.9$. After the reflected shock wave is focused at the focal point in the cell, it starts to pass through the plume from the upper side as a result of the change in the focal point far from the center axis $x/D_j=10.0$, $y/D_j=8.34$. Figure 8(a) shows that the shock wave generated by the jet and the contour of the right side wall of the cell are similar. Therefore, the right side of the shock wave reaches the cell wall at approximately the same time in Fig. 8(b), and this shock wave collides with the right side wall of the cell. This right running shock wave reflects from the cell wall, and propagates toward the geometric focal point. Moreover, this shock wave is focused at the focal point and propagates toward the two directions that indicated by the arrows in Figs. 8(c) and 8(d). Accordingly, the shock wave momentarily divides the plume into two portions, when the propagating shock wave reaches the plume. The right running shock wave passes through the plume head and reaches the cell exit. At the same time, the velocity at the cell exit increases. Furthermore, the left running shock wave reflects from the left side wall of the cell and reaches the cell exit. The influence of this shock wave on the velocity at the cell exit is extremely small.

The calculated pressure contours are shown in Figs. 9(a) ~ 9(d) in order to allow a more detailed examination of the flow field for Figs. 8(c) ~ 8(f), respectively. Figure 9(a) shows the focusing of the shock wave. The interaction regions of the shock wave divide the right and left sides of the focal point, as shown in Fig. 9(b). Figures 9(c) ~ 9(d) clearly show that the right running high-pressure area, which passed through the plume head, reaches the
Moreover, the left running shock wave passes through the tail of the plume. Therefore, the right running high-pressure area causes the pressure difference between the inside and outside of the cell. Hence, the velocity at the cell exit is accelerated by the pressure difference arriving at the cell exit.

3.4 Relation between the cell diameter and the velocity variation

The cell diameter is considered to be one of the important factors in the interaction between the shock wave and the plume in the cell. Figures 10(a) and 10(b) show the velocity variations at $x/D_j = 20.6$ and 28.7, respectively, for the cell diameter $D_{m}/D_j = 6.34$, 8.94 and 12.9 at $x/D_j = 8.71$. In Fig. 10(a), the first sharp peaks, which are caused by the focusing shock wave, occur for $D_{m}/D_j = 6.34$, 8.94 and 12.9 at $t = 10.6$, 13.5, and 17.9, respectively. These peaks seem to decrease with the increase in the cell diameter $D_{m}/D_j$. This is because the focusing shock wave arrives at the cell exit, showing that the larger distance between the focal point and the cell exit produce the lower peak velocity. The velocity increases for $D_{m}/D_j = 6.34$, 8.94, and 12.9 at $t = 17.3$, 19.1, and 25.1, respectively.

![Fig. 10 Velocity variations at $x/D_j = 20.6$ and 28.7 for $D_{m}/D_j = 1.0$, $t_{jet}=1.3$](image-url)
due to the plume reaching the cell exit. Therefore, unlike the focusing shock wave arrival at the cell exit, a rapid increase in velocity does not occur for \( \frac{D_m}{D_j} = 6.34, 8.94, \) or 12.9. However, the second peaks appear at \( t = 24.9, 23.6, \) and 30.3 because the secondary shock wave generated by the cutoff of the jet arrives at the cell exit during the passage of the plume. Moreover, almost the same peak value as the first peak occurs at \( t = 34.1 \) in the case of \( \frac{D_m}{D_j} = 6.34. \) This acceleration occurs because the shock wave reflecting from left side wall arrives at the exit of the cell during the passage of the plume. Furthermore, the shock wave reflected from the left side wall arrives at the cell exit for the other two diameters \( \frac{D_m}{D_j} = 8.94 \) and 12.9, although the influence of this shock wave on the velocity is not so strong. These slight increases occur at \( t = 38.5 \) and 47.0.

Figure 10(b) shows the velocity variations outside the cell at \( x/D_j = 28.7. \) Outside the cell, the characteristics of the velocity variation are similar to those of the attenuating wave form of the velocity variation at the cell exit. Therefore, the two peaks might merge into a single peak after the plume, and the shock wave is discharged into the downstream chamber. This is because the third peak is larger than the second peak, and, accordingly, the third peak overtakes the second peak. For the case of no combinations of peaks, \( \frac{D_m}{D_j} = 8.94, \) three peaks of approximately the same height are observed at \( t = 30.0, 38.5, \) and 48.5. On the other hand, the second and third peaks at \( t = 38.5 \) and 48.5 for \( \frac{D_m}{D_j} = 8.94 \) merge into a single peak at \( t = 42.7 \) for \( \frac{D_m}{D_j} = 6.34. \) Furthermore, the peak for \( \frac{D_m}{D_j} = 12.9 \) at \( t = 30.3 \) corresponds to the peak at \( t = 43.5 \) in Fig. 10(b).

### 3.5 Relation between the peak value and its arrival time

As discussed in the previous chapter, the diameter of the cell is considered to be a factor affecting the velocity variation peak, as well as its peak value and arrival time. Figures 11(a) and 11(b) show the relation between the velocity peak value and the peak arrival time at downstream of the cell exit with the cell diameter. In each figure, the circle, square, and triangle symbols represent the first, second, and third peaks, respectively. For cell diameters below \( \frac{D_m}{D_j} = 8.94, \) the difference in arrival times between the second and third
peaks decreases with the decrease in the cell diameter and these two peaks merge to a single peak at $D_m/D_j = 6.94$. This is due to the fact that the third peak appears early because of the decrease in the distance between the focal point and the cell exit. In contrast, the difference in arrival times between the first and second peaks decrease as the cell diameter is increased, as a result of the late arrival time of the first peak because the focal point is far from the center axis.

In Fig. 11(b), for cell diameter of less than $D_m/D_j = 9.90$, when the focal point is on the center axis, the peak value gradually decreases with increasing cell diameter. This is due to the decrease in the difference between the second and third peak arrival times. Then, for cell diameter greater than $D_m/D_j = 9.90$, when the focal point is far from the center axis, the peak value started increasing at 9.90, reached a peak at 12.9, and started to decrease. This increase occurs because the plume and the focusing shock wave generated by the cutoff of the jet arrive at the cell exit at almost the same time. Therefore, these results can be explained by the fact that the height of the peak depends not only on the cell diameter but also on the focal position changed by the diameter of the cell.

3.6 Effect of cell exit diameter and jet duration on the flow in the cell

The wall in the vicinity of the exit hole has a slight effect on the blockage on the plume jet when it is discharged into the downstream chamber. In Fig. 12, the peak value downstream of the cell exit $x/D_j = 28.7$ is plotted against the diameter of the cell exit. In each figure, the circle, square, and triangle symbols represent the peaks value for $D_m/D_j = 6.34$, 8.94, and 12.9, respectively. In the case of $D_m/D_j = 8.94$ and 12.9, the peak value monotonically increases as the cell exit diameter increases. This suggests that the influence of the blockage near the wall of the exit on the plume decreases. In contrast, in the case of $D_m/D_j = 6.34$, the peak value increases with increasing cell exit diameter over the range of $D_j/D_j = 0.5$ to 1.0 and then decreases because the single peak divides into two peaks if the timing of the arrival times of the two peak are different.

Figure 13 shows the relation between the peak value and the jet duration for various
The peak value increases as the jet duration becomes longer, except for $t_{jet}=1.3$. This increase is due to the fact that the plume receives the greater momentum when the jet duration is longer, and the plume maintaining the start velocity reaches the cell exit. For the cell diameter $D_m/D_j=8.94$ and 12.9, the peak values at $t_{jet}=1.3$ and 1.65 are lower than that for $D_m/D_j=6.34$. However, the peak values at $t_{jet}=2.35$ and 2.70 become larger than that for $D_m/D_j=6.34$. These trends are attributed to the relative position of the focal point to the plume head just after the moment of focusing of the shock wave.

In order to explain the trend, the typical examples of these relations are shown in Figs. 14. In Fig. 14(a), the spreading shock wave is considered to decelerate the plume. On the other hand, Figs. 14(b) and 14(c) illustrate the case that the spreading shock wave accelerates the plume head causing higher velocity downstream of the cell exit.

### 4. Conclusions

The suddenly injected jet and its interaction with the shock wave in the cell were numerically studied by solving two-dimensional axisymmetric compressible Navier-Stokes equations. The parameters of calculations are the diameter of the cell $D_m/D_j$, the cell exit diameter $D_e/D_j$ and the jet duration $t_{jet}$, where $D_m$, $D_e$, and $D_j$ are the diameter of the cell at $x/D_j=8.71$, the diameter of the cell exit, and the diameter of the jet, respectively. The conclusions are summarized as follows:

1. The spreading shock wave that converged at the geometrical focal point reflects from the cell wall, and this shock wave alternates between the jet injection and the cell exit.
2. For the cell diameters of $D_m/D_j=6.34$ and 8.94, the shock wave converging on the center axis reaches the cell exit. However, for $D_m/D_j=12.9$, the shock wave focusing away from the center axis divides the plume into two parts.
3. With decreasing cell diameter, the velocity peak appears early and its value increase because of the decrease in the distance between the geometric focal point and the cell exit.
4. The peak value of velocity and the number of the peaks downstream of the cell exit are roughly determined by the focal position of the cell.
5. The peak value increases with the increase in the jet duration except for $t_{jet}=1.3$.

One of the causes of the flow acceleration is found to be the relative position of the focal point to the plume head.

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