Aerodynamic Characteristics of a Tumbling Plate under Free Flight*

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Abstract

In this study, we deal with the tumbling, which is a rotating motion with the axis perpendicular to the falling direction. Our purpose is to reveal the fundamental aerodynamic characteristics of the tumbling, experimentally. Regarding a test plate, we consider a prism with a rectangular cross section with a depth-to-width ratio λ of 0.3. The results are as follows. The reduced terminal rotating rate Ω^* , the lift coefficient C_L , the drag coefficient C_D and the lift-to-drag ratio C_L/C_D are independent of the aspect ratio AR , when AR is greater than 10. As the inertia moment ratio I^* increases from zero to 50, Ω^* , C_L and C_D increase. However, Ω^* , C_L and C_D become almost constant, at I^* greater than 50. We propose the empirical formulae to predict them. At low I^* , the tumbling shows a dominant periodicity of 360 deg.

Key words: Autorotation, Flat Plate, Free Flight, Free Fall, Tumbling

1. Introduction

We often see rotational motions of leaves and cards falling in air. When a solid body falls in fluid, its motion is usually complicated and sometimes experiences the autorotations, namely, various kinds of self-induced rotational motions sustained by fluid force.

Among the autorotations such as the tumbling, the flat spin and the corning (for example, see refs.^{(1) & (2)}), we consider the tumbling, which is a rotational motion of a solid body in fluid with the axis perpendicular to the falling direction. The tumbling is important not only for pure academic interests in leaves' and cards' motions, but also for aeronautics, space engineering, ballistics and meteorology. For example, when a satellite re-enters into the atmosphere, its structure collapses into many fragments which disperse in the wide range of the earth-ground surface. Some fragments fall in the tumbling motions. In such cases, the precise knowledge on the tumbling is important to predict the dispersible range.

There have existed some previous researches on the tumbling. In 1949, Dupleich⁽³⁾ conducted experiments in water and air, concerning flat plates with low aspect ratio AR . In 1953, Smith⁽⁴⁾ experimentally researched the tumbling of an aircraft nose. Bustamante & Stone (1969)⁽⁵⁾ conducted high-altitude-free-flight tests from 600m – 3000m. They observed that various plates and cylinders can tumble, being independent of their initial attitudes. Besides, they measured angles and angular velocities in subsonic and supersonic wind-tunnel tests with bearing-support models and in supersonic free-flight tests with gun-launch models. And, they confirmed that flat plates, cylinders, disks, and triangular plates can autorotate. Smith (1971)⁽⁶⁾ conducted wind-tunnel tests at Reynolds numbers $Re = 1.3 \times 10^3 - 2.8 \times 10^5$, and measured lift L , drag D and terminal rotating rate n concerning

*Received 27 Jan., 2009 (No. T2-08-0582)
Japanese Original : Trans. Jpn. Soc. Mech.
Eng., Vol.75, No.749, B (2009),
pp.28-39 (Received 26 June, 2008)
[DOI: 10.1299/jfst.4.168]

some tumbling flat plates and elliptic cylinders. He revealed that reduced terminal rotating rate Ω^* is independent of both inertia moment ratio I^* and AR at $I^* \geq 1$ and $AR > 3$, and that Ω^* increases with increasing Re . Moreover, he addressed that the delay of stall takes part in important role on the tumbling, by the observation of instantaneous flow field. Iversen (1979)⁽⁷⁾ analysed previous experiments which are described using different parameters by different researchers, and proposed empirical formulae. He denied the relation between Re and Ω^* found by Smith (1971), and insisted that Ω^* is intrinsically independent of Re . According to Iversen, the increase of Ω^* with increasing Re is due to the friction of support bearings. Poreh & Wray (1979)⁽⁸⁾ carried out free-flight tests in air and in water, using rectangular-cross-section prisms, and investigated the effects of I^* and AR upon Ω^* , lift coefficient C_L and drag coefficient C_D . Yoshinaga et al. (1984)⁽⁹⁾ discussed the energy transfer between a tumbling body and fluid, by means of the trajectory analyses on a phase-space diagram. Their test models are flat plates and rocket-shaped cylinders. As well as experimental approaches, numerical ones have been challenged, and contributed especially toward the understanding of spatial flow structures. (Ishida (1982)⁽¹⁰⁾ and Oshima et al. (1983)⁽¹¹⁾ calculated tumbling plates and elliptic cylinders, respectively, using discrete vortex methods. They showed vorticity distributions, streamlines together with C_L and C_D . Using finite-difference methods, the full Navier-Stokes equations were solved by Lugt (1980)⁽¹²⁾ and recently by Andersen et al. (2005)⁽¹³⁾ and Yasuda & Takano (2006)⁽¹⁴⁾ together with experiments.)

As the aforesaid, there have been some past researches about the tumbling. However, we cannot get the fundamental relations between such important physical quantities as Ω^* , C_L , C_D and C_L/C_D , and governing parameters. (As shown later, the relations reported in past studies are not accurate enough, because they are based on the experiments at small $AR = 0.25 - 6.0$, and because all the governing parameters such as I^* are not specified completely.) For example, though Ω^* is considerably effective upon the vicinal flow field and then upon the other aerodynamic characteristics, it is difficult to know the precise value of Ω^* in enough terminal condition by wind-tunnel experiments (owing to supporting systems), and by computations (owing to the strong non-linearity and three-dimensionality related with flow separations).

So, in this study, we methodically carry out a series of free-flight tests in air in order to determine Ω^* , C_L and C_D and C_L/C_D in terminal condition, and attempt to reveal the relations of them with governing non-dimensional parameters. Beside, we observe the details of the tumbling motions in free-flight tests in water.

2. Nomenclature

AR	: Aspect ratio, $\equiv l/w$	
C_D	: Drag coefficient, $\equiv D/(0.5\rho_f U_\infty^2 lw)$	
C_L	: Lift coefficient, $\equiv L/(0.5\rho_f U_\infty^2 lw)$	
d	: Plate depth	[m]
D	: Drag	[N]
g	: Gravity acceleration	[m/s ²]
I	: Inertia moment	[kg·m ²]
I^*	: Inertia moment ratio, $\equiv 32I/(\rho_f w^4 l)$	
l	: Plate span	[m]
L	: Lift	[N]
n	: Terminal rotating rate	[s ⁻¹]
Re	: Reynolds number, $\equiv U_\infty w/\nu$	
$Re(V)$	: Reynolds number based on V , $\equiv Vw/\nu$	
t	: Time	[s]

U_{∞}	: Mean translation velocity	[m/s]
w	: Plate width	[m]
x	: Horizontal coordinate perpendicular to the rotation axis	[m]
y	: Vertical coordinate	[m]
z	: Horizontal coordinate parallel to the rotation axis	[m]
λ	: Depth-to-width ratio, $\equiv d/w$	
ν	: Kinematic viscosity	[m ² /s]
ρ_f	: Fluid density	[kg/m ³]
ρ_s	: Plate density	[kg/m ³]
Ω^*	: Reduced terminal rotating rate, $\equiv \pi n w / U_{\infty}$	

3. Experimental method

3.1. Model

Fig. 1 shows a test plate, namely, a prism with a rectangular cross section. The plate has such dimensions as $l = 0.3 \times 10^{-1} - 3.0 \times 10^{-1}$ m, $w = 1.5 \times 10^{-2}$ m and $d = 4.5 \times 10^{-3}$ m (in air), or as $l = 2.0 \times 10^{-1}$ m, $w = 2.0 \times 10^{-2}$ m and $d = 6.0 \times 10^{-3}$ m (in water), where l , w and d denote span, width and depth of the plate, respectively. Then, non-dimensional geometrical parameters, namely, the depth-to-width ratio λ and the aspect ratio AR correspond to 0.3 and $2 - 20$ (in air), or 0.3 and 10 (in water), respectively. The plates are made of foamed polystyrene, balsa wood, acrylic, aluminum, copper and lead. We can control the plate's inertia moment I by choosing such materials.

We have to consider the condition of the plate's ends, because it influences the three-dimensionality of flow. In the present study, we do not put any end plates which ensure the two-dimensionality of flow. Instead, we keep enough large AR ($= 20$) through all the tests, after the confirmation of the AR effect (as shown later). In addition, we conducted other preliminary experiments where the end-plate effect were researched for $AR = 10$. However, the end-plate effect is related with complicated three-dimensional flow near the plates' ends, then it seems difficult to explain the results plainly. In fact, we have confirmed that the terminal rotating rate n tends to become small with large end plates, and that the plate does not rotate with too large end plates.

3.2. Coordinates

Fig. 2 shows the present coordinate system. x and z represent horizontal coordinates perpendicular and parallel to the rotation axis, respectively. y represents a vertical coordinate.

The three-dimensional motion of a plate is detected by a set of stereo cameras, whose foci are fixed near the prospected orbit of the plate. In order to record the motion in terminal condition, we choose the space enough below the launched position as a test space. On the other hand, the test space is enough above to avoid the ground effect. Of course, we check whether or not the recorded motion is terminal, prior to their analyses (see 4.1). Incidentally, for the calibration before measurements, we use a transparent acrylic cube, which is put in the recorded space.

3.3. Parameters

Governing parameters are classified into two; namely, geometrical and kinematic ones. At first, we consider the former. Referring to Fig. 1, we choose the depth-to-width ratio λ and the aspect ratio AR as the geometrical governing parameters. The definitions of λ and AR are given by the following equations.

$$\lambda \equiv \frac{d}{w}. \quad (1)$$

$$AR \equiv \frac{l}{w}. \quad (2)$$

Second, we consider the kinematic governing parameters. We need two non-dimensional parameters to identify the concerning phenomenon. Conventionally, we select the inertia moment ratio I^* and the Reynolds number Re (for example, see Smith⁽⁶⁾). The definitions of I^* and Re are as follows:

$$I^* \equiv \frac{32I}{\pi\rho_f w^4 l}, \quad (3)$$

and

$$Re \equiv \frac{U_\infty w}{\nu}. \quad (4)$$

I denotes the inertia moment of the plate given by

$$I \equiv \frac{\rho_s w d l (w^2 + d^2)}{12}, \quad (5)$$

where ρ_f , ρ_s , U_∞ and ν denote fluid density, plate density, plate's translation velocity and kinematic viscosity, respectively.

We may consider another Reynolds number $Re(V)$, instead of Re . The definition of $Re(V)$ is given by

Fig. 1 Tumbling plate; a prism with a rectangular cross section.

Fig. 2 Schematic diagram of the tumbling rectangular plate and forces.

$$Re(V) \equiv \frac{Vw}{\nu} . \quad (6)$$

A theoretical terminal velocity⁽¹³⁾ is as follows:

$$V \equiv \sqrt{2 \left(\frac{\rho_s}{\rho_f} - 1 \right) wg} , \quad (7)$$

where g denotes the gravity acceleration.

In many practical aspects, we can easily find the advantage in the use of V instead of U_∞ . That is, we can determine V , if physical properties and geometrical dimensions are given. On the other hand, we need experiments or numerical simulations to determine U_∞ . From another point of view, the use of V inevitably causes some lost information. Fortunately, as shown later, we have experimentally confirmed that the uniqueness in the relation between V and U_∞ are approximately satisfied. Then, V seems more suitable to understand the whole picture than U_∞ . Besides, we will also discuss the lost information later.

From a theoretical point of view, we have to separately investigate each effect of such two governing parameters as I^* and $Re(V)$ (or I^* and Re). However, in free-flight experiments, individual estimations to such kinematic parameters have been technically difficult. This has been just one main reason to prevent the understanding of the concerning phenomenon (for example, in Iversen⁽⁷⁾, each parameter effect does not estimated separately from the other parameter effect, under the condition that the other parameter is kept to be constant).

In the present study, for the individual estimations to kinematic parameters avoiding such a difficulty, we choose the following non-dimensional parameter C as one of the two governing kinematic parameters.

$$C \equiv \frac{\sqrt{gw^3}}{\nu} . \quad (8)$$

From a mechanics point of view, C represents the ratio of (inertia force $\times$ gravity force)^{1/2} to viscous force. And, from a practical point of view, C can be easily controlled in experiments. Then, in the present study, we fix C to a constant value ($= 3.86 \times 10^2$), and investigate the pure effect of another kinematic parameter. As another kinematic parameter, we consider I^* or $Re(V)$. En passant, the relation among C , I^* , and $Re(V)$ is given by

$$\frac{Re(V)}{C} = \sqrt{2 \frac{3\pi}{8} \cdot \frac{I^*}{\lambda + \lambda^3} - 1} . \quad (9)$$

Fig.3 shows the relation between I^* and $Re(V)$ in the present experiments, that is, at $\lambda = 0.3$ and $C = 3.86 \times 10^2$.

As dependent parameters, we consider physical quantities describing the tumbling characteristics in terminal condition, such as the terminal rotating rate n and the lift L and the drag D . They are non-dimensionalised as a reduced terminal rotating rate Ω^* , a lift coefficient C_L and a drag coefficient C_D , respectively. Their definitions are as follows.

$$\Omega^* \equiv \frac{\pi n w}{U_\infty} . \quad (10)$$

$$C_L \equiv \frac{L}{\frac{1}{2} \rho_f U_\infty^2 w l} \quad (11)$$

$$C_D \equiv \frac{D}{\frac{1}{2} \rho_f U_\infty^2 w l} \quad (12)$$

From Equation (10), we know that Ω^* represents the ratio of a plate's tip velocity to a plate's translation velocity in terminal condition. Additionally, we consider another important aerodynamic-characteristics quantity C_L/C_D which is called as the lift-to-drag ratio, together with C_L and C_D .

In summary, we consider physical quantities like Ω^* , C_L , C_D and C_L/C_D as the functions of λ , AR , I^* and C , or of λ , AR , $Re(V)$ and C .

3.4. Experimental apparatus

Figs. 4 and 5 show the schematics of free-flight experiments in air and in water, respectively.

In Fig. 4, a flat plate falls from a launcher in 7m high above the ground floor. This system is placed indoor to avoid various disturbances. The launcher gives the plate an initial rotation. After release, the plate begins to fall tumbling. We take stereo-photographs by two high-speed video cameras synchronised with each other at 2000frames/s, and get the angle and the position of the plate.

In Fig. 5, a flat plate falls into a water tank with a water level of 7.5×10^{-1} m. The interval between experiments is kept to be enough long to settle the disturbances in the water tank. We take synchronised photographs by three high-speed video cameras at 500frames/s, and get the angle and the position of the plate as well.

Specifically speaking, we mark three white points on a longer cross-section centre line on one plate's end surface. One point is on the centre, and the other two points are oppositely outside the centre. The former is for the position measurements, and the latter two are for the angle measurements.

Fig. 3 Relation between inertia-moment ratio I^* and Reynolds number $Re(V)$ based on a theoretical descent velocity V at depth-to-width ratio $\lambda = 0.3$, $C = 3.86 \times 10^2$.

4. Results and discussion

4.1. Falling motion of a tumbling flat plate (a sample)

Fig. 6 illustrates a locus of a falling plate in typical tumbling motion during about one-rotation period, as a sample. Arrows in the figure denote the cross-section directions of the plate at different instants. And, a dotted line in the figure denotes the trajectory of the plate's center. The plate translates from a right upper corner to a left lower corner, with clock-wise tumbling. We can see that both the instantaneous translation velocity and the instantaneous rotation rate fluctuate periodically. Then, we are needed to define n , C_L , C_D and C_L/C_D as time-mean values.

We usually observe the loci in an enough wide space to record more than three plate's rotations (with about $50w$ in both the x and y directions). By comparing three values of the U_∞ averaged over each one rotation period, we judge the terminal condition for further analyses.

Fig. 4 Experimental apparatus in air.

Fig. 5 Experimental apparatus in water.

Table 1 Symbols and corresponding parameters, in Figs. 7 - 10.

Symbol	References	λ	AR	I^*	Ω^*	C	Experimental method
●	present	0.3	2.0 - 20	7.73	0.27 - 0.50	380	free flight
○	present	0.3	2.0 - 20	24.4	0.15 - 0.23	380	free flight
□	Dupleich (1949)	unknown (0.003 - 0.01)	1.0 - 2.5	1.60	0.46 - 0.80	6500	free flight
◇	Dupleich (1949)	unknown (0.003 - 0.01)	2.0 - 2.5	2.70	0.53 - 0.63	3000	free flight
△	Dupleich (1949)	unknown (0.003 - 0.01)	2.5 - 5.0	3.50	0.60 - 0.75	1100	free flight
■	Dupleich (1949)	unknown (0.003 - 0.01)	2.5 - 5.0	4.50	0.53 - 0.70	1400	free flight
----	Bustamante & Stone (1969)	unknown (thin)	1.7 - 6.0	unknown	Eq. (13)	unknown	wind tunnel
—	Iversen (1979)	unknown (0.0054 - 0.5)	0.25 - 4.0	more than about 10	Eq. (14)	unknown	wind tunnel (Glaser & Northup, 1971) free flight (Bustamante & Stone, 1969)

4.2. Effect of aspect ratio AR

The tumbling is inherently a two-dimensional phenomenon. But, the actual tumbling is not perfectly two-dimensional, because the existence of plate ends. By the wind-tunnel experiments at $Re = 1.3 \times 10^3 - 2.8 \times 10^5$, Smith⁽⁶⁾ described that there is no influence on Ω^* , in the condition that AR is larger than 3. On the other hand, according to Iversen⁽⁷⁾, we can observe that Ω^* increases with increasing AR , in the wind-tunnel experiments at $AR < 4$. In these studies, λ is much smaller than 1, as well as the present study at $\lambda = 0.3$.

Now, we investigate the effects of AR upon Ω^* , C_L , C_D and C_L/C_D , in the range of $AR = 2 - 20$, in Figs. 7 - 10. Namely, Fig. 7 shows the AR effect upon Ω^* , and Figs. 8, 9 and 10 show the AR effects upon C_L , C_D and C_L/C_D , respectively. Table 1 summarises the symbols and their corresponding parameters used in these figures. Tested values of I^* are 7.73 and 24.4. Then, the corresponding $Re(I)$ are 2.83×10^3 and 5.08×10^3 , respectively. The tested range of Re becomes $2.0 \times 10^3 - 3.4 \times 10^3$.

At first, we see Fig.7. The results for $I^* = 7.73$ and 24.4 show the same trend, not quantitatively but qualitatively. That is, as AR increases from zero to about 10, Ω^* increases. At $AR > \text{about } 10$, Ω^* has a constant value, being independent of AR . Furthermore, we can see that the effect of I^* upon Ω^* is not so small, because the constant value at $AR > \text{about } 10$ depends upon I^* significantly.

Fig. 6 Locus of a falling plate ($\lambda = 0.3$, $AR = 10$, $I^* = 3.29$, $Re(I) = 1.79 \times 10^3$, ($Re = 1.8 \times 10^3 - 2.3 \times 10^3$), $C = 3.86 \times 10^2$). Arrows and a dotted line denote the cross-section direction of a plate and the trajectory of a plate's centre, respectively.

Besides, in Fig.7, we show the previous results by other researchers, for comparison. Their experimental conditions are also summarised in Table 1. Although we recognise that Ω^* monotonously increases with increasing AR for all the results, we can see distinctive discrepancies among them from a quantitative point of view. These discrepancies become large, as AR becomes large. Therefore, when we consider more ideal tumbling at large AR expecting the two-dimensionality of flow, we should note that the past results at large AR are poor as show in Table 1.

Apropos, we can point out some trivial problems in previous researches, in addition to the above smaller- AR problem. Dupleich⁽³⁾ conducted well-controlled experiments concerning the governing parameters. However, the value of λ is unknown. Bustamante & Stone⁽⁵⁾ carried out various kinds of experiments such as free-flight and wind-tunnel tests. However, their empirical formula on Ω^* which is given by

$$\Omega^* = 1 - e^{-0.2AR} \quad (13)$$

is based on wind-tunnel experiments, supposing $\Omega^* = 1$ at $AR = \infty$. Furthermore, the values of I^* and λ are unknown, although λ is described as ‘thin.’ Iversen⁽⁷⁾ show the following empirical formula.

$$\Omega^* = (0.329 \ln \lambda^{-1} - 0.0246 \ln \lambda^{-1})^2 \left\{ \frac{AR}{2 + (4 + AR^2)^{\frac{1}{2}}} \right\} \left[2 - \left(\frac{AR}{AR + 0.595} \right)^{0.76} \right]^{\frac{2}{3}}. \quad (14)$$

This formula is based on the data with wide-ranged parameters, except for AR . However, each parameter effect is not estimated separately from the other parameter effects, because he uses other researcher’s experimental data.

Next, we see Figs. 8 - 10. Fig. 8 shows that C_L has such a similar trend as Ω^* . That is, as AR increases from zero to about 10, C_L increases. At $AR >$ about 10, C_L has a constant value, being independent of AR . And, the I^* effect upon C_L is not small. One outstanding difference of Fig. 8 from Fig. 7 is a smaller scattering manner of the data.

In Fig. 9, C_D has a different trend from those of Ω^* and C_L in Figs. 7 and 8. That is, C_D keeps a constant value, even at small AR . However, the I^* effect upon C_D is not small, as well as Ω^* and C_L .

In Fig. 10, C_L/C_D has such a similar trend as Ω^* and C_L in Figs. 7 and 8, again. That is, as AR increases from zero to about 10, C_L/C_D increases. At $AR >$ about 10, C_L/C_D has a constant value. This is consistent with the results of C_L and C_D in Figs. 8 and 9. However, the I^* effect upon C_L/C_D is rather small, in contrast with Ω^* , C_L and C_D .

Fig. 7 Reduced rotating rate Ω^* of a tumbling plate versus aspect ratio AR at depth-to-width ratio $\lambda = 0.3$, $C = 3.86 \times 10^2$.
 ● $I^* = 7.73$, $Re(V) = 2.83 \times 10^3$ ($Re = 2.0 \times 10^3 - 2.7 \times 10^3$).
 ○ $I^* = 24.4$, $Re(V) = 5.08 \times 10^3$ ($Re = 2.6 \times 10^3 - 3.4 \times 10^3$).

Fig. 8 Lift coefficient C_L versus aspect ratio AR at depth-to-width ratio $\lambda = 0.3$, $C = 3.86 \times 10^2$.
 ● $I^* = 7.73$, $Re(V) = 2.83 \times 10^3$ ($Re = 2.0 \times 10^3 - 2.7 \times 10^3$).
 ○ $I^* = 24.4$, $Re(V) = 5.08 \times 10^3$ ($Re = 2.6 \times 10^3 - 3.4 \times 10^3$).

Fig. 9 Drag coefficient C_D versus aspect ratio AR at depth-to-width ratio $\lambda = 0.3$, $C = 3.86 \times 10^2$.
 ● $I^* = 7.73$, $Re(V) = 2.83 \times 10^3$ ($Re = 2.0 \times 10^3 - 2.7 \times 10^3$).
 ○ $I^* = 24.4$, $Re(V) = 5.08 \times 10^3$ ($Re = 2.6 \times 10^3 - 3.4 \times 10^3$).

Fig. 10 Lift-to-drag ratio C_L/C_D versus aspect ratio AR at depth-to-width ratio $\lambda = 0.3$, $C = 3.86 \times 10^2$.
 ● $I^* = 7.73$, $Re(V) = 2.83 \times 10^3$ ($Re = 2.0 \times 10^3 - 2.7 \times 10^3$).
 ○ $I^* = 24.4$, $Re(V) = 5.08 \times 10^3$ ($Re = 2.6 \times 10^3 - 3.4 \times 10^3$).

Table 2 Symbols and the corresponding I^* , $Re(V)$ and Re , in Figs. 11 - 20
($\lambda = 0.3$, $AR = 10$, $C = 3.86 \times 10^2$).

Symbol	I^*	$Re(V)$	Re
●	3.29	1790	$1.8 \times 10^3 - 2.3 \times 10^3$
●	7.73	2830	$2.0 \times 10^3 - 2.7 \times 10^3$
○	24.4	5080	$2.6 \times 10^3 - 3.4 \times 10^3$
■	48.7	7200	$3.0 \times 10^3 - 3.8 \times 10^3$
■	55.9	7720	$3.4 \times 10^3 - 4.2 \times 10^3$
□	61.9	8120	$3.6 \times 10^3 - 4.6 \times 10^3$
▲	65.7	8370	$3.2 \times 10^3 - 3.8 \times 10^3$
▲	71.9	8760	$3.7 \times 10^3 - 4.3 \times 10^3$
△	89.5	9780	$4.1 \times 10^3 - 4.7 \times 10^3$
◆	103	10500	$4.9 \times 10^3 - 5.1 \times 10^3$
◆	108	10800	$5.0 \times 10^3 - 5.9 \times 10^3$
◇	120	11300	$5.2 \times 10^3 - 6.2 \times 10^3$
*	141	12300	$4.6 \times 10^3 - 6.4 \times 10^3$

In summary, it is turned out that $\mathcal{Q}^*$, C_L , C_D , and C_L/C_D are all independent of AR , provided that AR is larger than about 10. Then, from now on, we consider only the results at $AR = 10$, as we can ignore the AR effects.

4.3. Effect of inertia-moment ratio I^*

According to Smith's free-flight experiments at $Re = 10^4$, it is clear that both C_L and C_L/C_D are independent of I^* , if I^* is larger than 10.⁽⁶⁾ And, according to Iversen⁽⁷⁾, $\mathcal{Q}^*$ is independent of I^* at $I^* > 10$. The Iversen's conclusion is supposed to be at $AR = 0.5$, $\lambda = 0.0156$. In the present study, we try to investigate the I^* effects up on $\mathcal{Q}^*$, C_L , C_D and C_L/C_D in such a wide parameter range as $I^* = 10^0 - 10^2$. Specifically speaking, Figs. 11, 12, 13 and 14 show the I^* effects upon $\mathcal{Q}^*$, C_L , C_D and C_L/C_D , respectively, in the range of $I^* = 3.3 - 141$. Table 2 summarises the symbols and their corresponding parameters used in these figures. A chain line in each figure corresponds to an approximation equation described in the figure.

In Fig. 11, $\mathcal{Q}^*$ increases with increasing I^* at $I^* < \text{about } 50$. At $I^* > \text{about } 50$, $\mathcal{Q}^*$ seems to asymptote to a constant value between 0.4 and 0.6, while there exist some data scatterings. Then, the effect of I^* upon $\mathcal{Q}^*$ becomes negligible at $I^* > \text{about } 50$.

For reference, we draw Equation (14) proposed by Iversen as a solid straight line. In contrast with the Iversen's conclusion, we cannot ignore the I^* effect upon $\mathcal{Q}^*$ at $I^* \approx 10$. In addition, the value of $\mathcal{Q}^*$ predicted by his formula is much smaller than the present experiments.

Next, we see Figs. 12 - 14. Fig. 12 shows that C_L has such a similar trend as $\mathcal{Q}^*$. That is, as I^* increases from zero to about 50, C_L increases. At $I^* > 50$, C_L seems to asymptote to a constant value of 0.9. One outstanding difference of Fig. 12 from Fig. 11 is a smaller scattering manner of the data.

In Fig. 13, C_D has such a similar trend as C_L in Fig. 12, not only qualitatively but also quantitatively. And, we can also observe a small scattering manner of data as well as Fig. 12.

In Fig. 14, according to both Figs. 12 and 13, C_L/C_D tends to have a constant value of a unity. (Strictly speaking, C_L/C_D decreases slightly with increasing I^* .)

Incidentally, in Fig. 13, we draw the experimental data by Dupleich.⁽³⁾ We can see remarkable discrepancies between the present and his results, even from a qualitative point of view. To settle this inconsistency, further investigations are required. Specifically speaking, we estimate the values of I^* by two kinds of ways, because of his insufficient

descriptions of governing parameters. Bar symbols in Fig. 13 denote the values assuming that λ equals zero. The corresponding ranges of AR and C are wider than those in Table 1, because of the difference of the data used. Cross symbols in Fig. 13 denote the values quoted by Poreh & Wray⁽⁸⁾ from Dupleich. As we can not know which of Dupleich's data they use, such governing parameter as λ , AR and C are possibly not constant, but vary with some ranges, as well as bar symbols. Thus, it is not easy to discuss further in the present stage.

4.4. Effect of Reynolds number $Re(V)$

In the late sub section, we have considered I^* and C as two governing kinematic parameters, and investigated the effects of I^* upon such quantities as Ω^* , C_L , C_D , and C_L/C_D under the condition that C keeps constant. As a result, we can successfully propose simple empirical formulae to predict those quantities with enough accuracies.

On the other hand, it is possible to choose other sets of governing kinematic parameters. Now, we attempt to investigate another set, that is, $Re(V)$ and C instead of I^* and C .

Complementarily speaking, as C keeps constant through the present study, the relation between Re and $Re(V)$ can be approximately determined as a function with uniqueness. Fig. 15 shows this relation, although there exist some dispersions. Here, the symbols in the figure correspond to those in Table 2.

The Re effects in the past studies are as follows. According to Lugt,⁽¹⁾ although he did not show explicit data, he addressed that there is no Re effect upon Ω^* at $Re > 1.5 \times 10^3$, and that $\Omega^* = 0.8 - 1.0$ at small λ and at large AR . As well, Iversen⁽⁷⁾ assumed the small Re effect upon Ω^* , when he considered the results by Smith.⁽⁶⁾ He also did not show explicit data, lacking quantitative discussion about those.

In the present study, we try to investigate the $Re(V)$ effects upon Ω^* , C_L , C_D and C_L/C_D in such a wide parameter range as $Re(V) = 1.79 \times 10^3 - 1.23 \times 10^4$. Specifically speaking, Figs. 16, 17, 18, and 19 show the $Re(V)$ effects upon Ω^* , C_L , C_D and C_L/C_D , respectively. Again, Table 2 summarises the symbols and their corresponding parameters used in these figures. A chain line in each figure corresponds to an approximation equation described in the figure, together with a dotted line corresponding to the approximation equation proposed in the last subsection based on I^* instead of $Re(V)$.

In Figs. 16 – 18, Ω^* , C_L and C_D increase with increasing $Re(V)$ at $Re(V) < \text{about } 7 \times 10^3$, respectively. At $Re(V) > \text{about } 7 \times 10^3$, Ω^* , C_L and C_D seem to asymptote to constant values, respectively. Then, the effects of $Re(V)$ becomes negligible at $Re(V) > \text{about } 7 \times 10^3$. Concerning data scatterings, those on Ω^* are much larger than those on C_L and C_D . In Fig. 19, according to both Figs. 17 and 18, C_L/C_D tends to have a constant value of a unity.

Fig. 11 Reduced rotating rate Ω^* of a tumbling plate versus inertia-moment ratio I^* , for several values of Reynolds number Re at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 12 Lift coefficient C_L versus inertia-moment ratio I^* , for several values of Reynolds number Re at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 13 Drag coefficient C_D versus inertia-moment ratio I^* , for several values of Reynolds number Re at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 14 Lift-to-drag ratio C_L/C_D versus inertia-moment ratio I^* , for several values of Reynolds number Re at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

The chain lines in Figs. 16 – 19 show good agreements with experiments, as well as the dotted lines which represent the approximation equations in Figs. 11 – 14. Then, we can see that those equations are highly accurate, in spite of their simplicities.

Furthermore, we compare the chain lines with the dotted lines. In Fig. 16, the dotted line gives a better approximation than the chain line. In Figs. 17 and 18, on the contrary, the chain lines are better. In summary, the approximation equation based on $Re(V)$ is more suitable for the prediction of Ω^* , while those based on I^* are more suitable for the predictions of C_L and C_D .

Finally, we discuss the lost information induced by the use of $Re(V)$ instead of Re . As shown in Fig. 15, the relation between $Re(V)$ and Re is approximately unique. In addition, at a fixed $Re(V)$ or I^* , the magnitude of data dispersion on Ω^* (as shown in Fig. 16) is much larger than those on C_L and C_D (as shown in Figs. 17 and 18). Then, Ω^* is the most sensitive to the lost information.

Complementarily speaking, the data dispersion on Ω^* is caused by different value of n which are all measured in enough terminal conditions. According to Andersen et al.⁽¹³⁾, the terminal conditions are subject to initial conditions as well as the geometrical and kinematic governing parameters. However, as both I^* and Re in the present study are much larger than those by them, we cannot conclude that the present result corresponds to their result in the present stage.

Fig. 20 shows the reduced rotating rate Ω^* of a tumbling plate plotted against the Reynolds number Re , for several values of inertia-moment ratio I^* . As Re increases from zero to about 4×10^3 , Ω^* seems to have a constant value between 0.4 and 0.6, being independent of Re . And, of course, the approximation equations for Ω^* in Figs. 11 and 16, which are denoted by dotted lines, agree well with them. However, we can find that the data with the same I^* collapse near those approximation equations, not with a random scattering manner, but with a coherent scattering manner on a certain curve. This suggest such a tendency as Ω^* decreases with increasing Re .

To conclude, this tendency is considered to be reflected by a close interaction between C_L and Ω^* . Now, we consider the dynamical equilibrium on a free-flight plate, and we get

$$L^2 + D^2 = (\rho_s l d w g)^2. \quad (15)$$

We suppose that the lift force L is produced mainly by the Magnus effect, then

$$L \propto \rho_f U_\infty n w^2 l. \quad (16)$$

And,

$$D \propto \frac{1}{2} \rho_f U_\infty^2 w l. \quad (17)$$

From Equations (15) – (17), we get

$$\frac{n^2 w^2}{U_\infty^2} = \frac{1}{C_1^2} \frac{g^2 d^2 (\rho_s / \rho_f)^2}{U_\infty^4} - \frac{C_2^2}{C_1^2}, \quad (18)$$

where C_1 and C_2 are the coefficients which are related with Equations (16) and (17), respectively. Next we consider other coefficients A and B instead of C_1 and C_2 , as follows.

$$\frac{1}{C_1} = \frac{64 v^2 (1 + d^2 / w^2)^2}{9 \pi^4 g^2 w^6} A. \quad (19)$$

$$\frac{C_2^2}{C_1^2} = \frac{1}{\pi^2} B. \quad (20)$$

Then, together with Equations (3) – (5) and (10), we get such a non-dimensionalised equation as

$$\Omega^* = \sqrt{A \frac{I^{*2}}{Re^4} - B}. \quad (21)$$

When we determine $A = 10^{10}$ and $B = 0.04$ by experiments, we can draw chain lines in Fig. 20. These chain lines are in good agreements with experiments.

Incidentally, we can get the relation between Re and Ω^* , using Equation (21) together with the relation between $Re(V)$ and Re in Fig. 15 and with the relation between I^* and $Re(V)$ in Equation (9). This also gives us another approximation equation to predict Ω^* from I^* or from $Re(V)$. In Fig. 20, the relation is drawn by a solid line. Unfortunately, the approximation equation based on the relation seems less accurate than the other ones drawn by dotted lines.

Fig. 15 Reynolds number Re versus Reynolds number $Re(V)$ based on a theoretical descent velocity V , for several values of inertia-moment ratio I^* at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 16 Reduced rotating rate Ω^* of a tumbling plate versus Reynolds number $Re(V)$ based on a theoretical descent velocity V , for several values of inertia-moment ratio I^* at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 17 Lift coefficient C_L versus Reynolds number $Re(V)$ based on a theoretical descent velocity V , for several values of inertia-moment ratio I^* at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 18 Drag coefficient C_D versus Reynolds number $Re(V)$ based on a theoretical descent velocity V , for several values of inertia-moment ratio I^* at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 19 Lift-to-drag ratio C_L/C_D versus Reynolds number $Re(V)$ based on a theoretical descent velocity V , for several values of inertia-moment ratio I^* at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

4.5. Periodicity

Figs. 21 and 22 show the angular velocity fluctuations ω of a free-flight plate tumbling during about three rotations in water at $I^* = 2.5$ and 0.75 , respectively. In each figure, the ordinate denotes the dimensionless angular velocity $\omega/\omega_{ave.}$, where $\omega_{ave.}$ is time-mean angular velocity which is equal to $2\pi n$. The abscissa denotes the reduced time tn , where t and n are time and the terminal rotating rate, respectively. In water experiments, C is fixed to 8.86×10^3 .

At first, we see Fig. 21. We can observe a typical tumbling which is dominated by a periodicity of 180deg. (a half rotation); namely, the periodicity of 0.5 in tn in the figure. Then, during three rotations of the plate, we see six dominant fluctuations on $\omega/\omega_{ave.}$. In addition, if we observe the figure more carefully, we can also observe a weak but clear periodicity of 360deg. (one rotation); namely, the periodicity of 1.0 in tn . For example, we can see that the peak value of $\omega/\omega_{ave.}$ alternates between about 1.3 and 1.1.

In Fig. 22, we can observe such a periodicity of 360deg., more clearly. In other experiments, we have confirmed that the periodicity of 360deg. becomes clearer, as I^* decreases even at large Re . Of course, the periodicity of 180deg. is always prevailing in such conditions. Fig. 23 shows a locus of a falling plate under the same conditions as Fig. 22. We can see two periodicities of 180deg. and 360deg. The former and the latter represent one cycle a half rotation and one cycle a rotation, respectively. Again, we can confirm the latter on the locus. Namely, the plate translates by a shorter distance in the first half rotation, and by a longer distance in the second half rotation.

Smith⁽⁶⁾ and most of later researchers consider only the periodicity of 180deg., as a dominant fluctuation component. The prevailness of the 180deg. periodicity seems rational, because of the geometrical symmetry of the present model cross section. On the other hand, we can clearly observe the 360deg. periodicity together with the 180deg. periodicity, such as Fig. 22 in some cases (for example, see Andersen et al.⁽¹³⁾). We have also observed the 360deg. periodicity in wind-tunnel experiments at small I^* , even at large Re , where it might be difficult to accept the direct interpretation of the 360deg. periodicity with the period doubling in chaos.

Fig. 20 Reduced rotating rate Ω^* of a tumbling plate versus Reynolds number Re , for several values of inertia-moment ratio I^* at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$, $C = 3.86 \times 10^2$.

Fig. 21 Dimensionless angular velocity $\omega/\omega_{ave.}$ of rectangular plate versus reduced time tn for inertia-moment ratio $I^* = 2.5$, $Re(V) = 3.54 \times 10^4$ ($Re = 1.4 \times 10^3 - 1.6 \times 10^3$), at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$.

Fig. 22 Dimensionless angular velocity $\omega/\omega_{ave.}$ of rectangular plate versus reduced time tn for inertia-moment ratio $I^* = 0.75$, $Re(V) = 1.63 \times 10^4$ ($Re = 7.1 \times 10^3 - 7.7 \times 10^3$), at depth-to-width ratio $\lambda = 0.3$, aspect ratio $AR = 10$.

Fig. 23 Locus of a falling plate ($\lambda = 0.3$, $AR = 10$, $I^* = 0.75$, $Re(V) = 1.63 \times 10^4$, $Re = 7.1 \times 10^3 - 7.7 \times 10^3$). Arrows and a dotted line denote the cross-section direction of a plate and the trajectory of a plate's centre, respectively.

5. Conclusions

We have investigated the aerodynamic characteristics of a tumbling flat plate by free-flight tests in air at $\lambda = 0.3$, $AR = 2 - 20$, and, $I^* = 3.3 - 141$. Through all the tests, C is fixed to 3.86×10^2 . Results are as follows.

- (1) Ω^* , C_L , C_D and C_L/C_D are all independent of AR , if AR is grater than 10.
- (2) Ω^* , C_L , C_D and C_L/C_D are all independent of I^* , if I^* is grater than 50.
- (3) Ω^* , C_L , C_D and C_L/C_D and independent of $Re(V)$, if $Re(V)$ is grater than 7×10^3 .
- (4) We propose simple and accurate empirical formulae to predict Ω^* , C_L , C_D and C_L/C_D .
- (5) Concerning the lost information due to an averaging, namely, the use of $Re(V)$ instead of Re , the coherent scattering manner on Ω^* can be explained by the prevailness of the Magnus effect.

The accuracy of the above results is considered to be satisfactory, in comparison with past studies because two governing parameters (C , I^*) or (C , $Re(V)$) are completely in control. In future, we can expect higher consistency with past studies and higher accuracy in prediction in wider parameters' ranges, by accumulating more experimental data to investigate the C effect and so on.

Beside, we have observed the details of the tumbling in free-flight test in water. As I^* decreases, the tumbling is dominated by the periodicity of 360deg., together with that of 180deg. which is exclusively dominating at large I^* .

Acknowledgement

This study was partly supported by the Academic Frontier Research Project on "Next Generation Zero-Emission Energy Conversion System" of Ministry of Education, Culture, Sports, Science and Technology.

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