Development of a Hot-Wire Probe with Two Parallel Wires Placed Closely Together *

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Abstract

We manufactured a new hot-wire probe with two parallel wires placed closely together for measuring a transitional boundary-layer flow. The main feature of this new probe is that it is much smaller than a conventional multi-sensor probe, such as an X-type probe. Thus, the new probe can be regarded as a sensor similar in configuration to a single normal probe and is expected to provide high spatial resolution for fluctuating velocities in a transitional boundary layer. The measurement principle is very simple, as the streamwise and the normalwise velocity components are simultaneously obtained from the sums and the differences of the linearized outputs of two hot-wire circuits with the aid of the look-up-matrix method. It was found that the complicated relations between the hot-wire-circuit outputs and the flow angle can be simplified by limiting the flow angle to within $\theta = \pm 30$ deg. The new hot-wire probe was used to measure the velocity profiles in a low Reynolds number turbulent boundary layer. The results revealed that the probe is a useful tool for measuring velocities in a transitional boundary layer having a thickness of only several mm.

Key words: Velocimetry, Hot-Wire Probe, Boundary Layer, Turbulent Flow, Low Reynolds Number

1. Introduction

Because of its better frequency response than a Pitot-tube velocimeter, a hot-wire anemometer is generally an effective tool for measuring fluctuating velocity in a turbulent flow. It is also cheaper than a laser Doppler velocimeter and a particle image velocimeter that have been widely used in making flow measurements. However, a single normal probe such as the I-type probe usually only provides information on a streamwise velocity component, though it has very high spatial resolution as a small sensor.

For this reason, either an X-type probe or a split hot-film probe has been used for measuring velocity components in a transitional and a turbulent boundary layer. An X-type probe consists of two inclined wires placed closely together to form an X. However, the use of an X-type probe can lead to significant problems related to spatial resolution. A split hot-film probe consists of a very small cylinder (150 $\mu$m in diameter) coated with a thin platinum film (1000 Å in thickness) and is split longitudinally into two separate sensor elements. This probe is assumed to respond to velocity components contained in a plane perpendicular to the sensor and to have an insignificant response to tangential velocity components along the sensor elements. This makes the applicability of the probe similar to that of an X-type probe. Its main advantage in measuring shear flows is its much smaller size in the direction of the mean shear, which minimizes spatial averaging in regions of a severe velocity gradient and permits measurements closer to a surface than with an X-type probe.
However, a split hot-film probe has a more complex and poorer frequency response to airflows, primarily due to the substrate on which the film is deposited. Consequently, this probe cannot be used for measuring velocity components in a transitional boundary layer where fluctuating velocities contain high-order frequency components. The purposes of the present study are (1) to examine in detail the output characteristics of a newly developed hot-wire probe versus changes in the flow direction and (2) to devise a calibration method and a data acquisition system for the probe.

2. Nomenclature

\[ Q_1, Q_2 : \text{heat transfers from two wires to fluid flow} \quad [\text{J}] \]
\[ \Sigma Q, \Delta Q : \text{sum and difference of the heat losses of two wires} \quad [\text{J}] \]
\[ E_{HW1}, E_{HW2} : \text{outputs of two hot-wire circuits} \quad [\text{V}] \]
\[ \Sigma E_{HW} : E_{HW1} + E_{HW2} \quad [\text{V}] \]
\[ \Sigma E_o, \Delta E_o : \text{sum and difference of the linearized outputs} \quad [\text{V}] \]
\[ x : \text{streamwise distance measured from the origin} \quad [\text{m}] \]
\[ y : \text{wall normal distance measured from the surface of a flat plate} \quad [\text{m}] \]
\[ z : \text{spanwise distance measured from the center line of the flat plate} \quad [\text{m}] \]
\[ \theta : \text{flow direction} \quad [\text{deg}] \]
\[ U_o : \text{free-stream velocity} \quad [\text{m/s}] \]
\[ U_x, U_y : \text{x-, y-components of velocity} \quad [\text{m/s}] \]
\[ V : \text{magnitude of velocity} \quad [\text{m/s}] \]
\[ u', v' : \text{x-, y-components of the fluctuating velocity} \quad [\text{m/s}] \]
\[ \delta : \text{boundary layer thickness} \quad [\text{m}] \]
\[ \delta_1 : \text{displacement thickness of the boundary layer} \quad [\text{m}] \]
\[ \delta_2 : \text{momentum thickness of the boundary layer} \quad [\text{m}] \]
\[ \nu : \text{kinematic viscosity of the fluid} \quad [\text{m}^2/\text{s}] \]
\[ R_h : \text{Reynolds number based on the momentum thickness} \quad (= U_o \delta_2/\nu) \quad [\text{m}] \]
\[ c_f : \text{coefficient of the local skin friction} \]
\[ \Pi : \text{wake parameter} \]
\[ u_c : \text{friction velocity} \quad (= U_o \sqrt{c_f / 2}) \quad [\text{m/s}] \]

abbreviations

HW1, HW2: two hot-wire sensing elements
CTC1, CTC2: constant temperature hot-wire circuits for HW1 and HW2

3. Configuration of the new hot-wire probe

![Fig. 1 Schematic diagram of the new hot-wire probe]

The configuration of the new hot-wire probe is shown schematically in Fig. 1. The
probe consists of two tungsten wires of 2.5 µm in diameter and about 2 mm in length. The wires are copper-plated except the sensing elements that are about 0.4 mm in length and are spaced several µm apart. The wires are soldered on the tips of the prongs on both sides. One notable feature of this probe is that two parallel wires are placed closely together. The gap between the wires at the sensing elements is filled with glue to avoid aerodynamic interference between the wires and to increase the sensitivity to the flow direction. Because this kind of glue is good for electric insulation and heat conductivity, the wires are coupled thermally. Thus, the probe is expected to have minimal thermal resistance. A sine-wave test showed that the frequency responses of both wires was over 10 kHz.

4. Principle of velocity measurement with the new hot-wire probe

Figure 2 shows the definition of the sensor plane and its position facing the flow direction. The sensor plane is defined along the axes of HW1 and HW2. It is assumed that HW1 and HW2 respond only to the velocity component normal to the sensor plane, similar to a split hot-film probe. The flow direction is measured from an axis (θ = 0 deg) normal to the sensor plane through the middle of axes of HW1 and HW2 (probe axis).

![Fig. 2 Definition of the sensor plane and its position facing the flow direction](image)

HW1 and HW2 are operated by CTC1 and CTC2, respectively. The following assumptions are made: HW1 and HW2 maintain thermal equilibrium conditions; the time constant of the sensor is smaller than the time scale of the fluctuating velocity in a fluid flow. The following two relations are thus obtained:

\[ \Sigma Q = Q_1 + Q_2 = f(U_\theta, \theta) \]  
\[ \Delta Q = Q_1 - Q_2 = g(U_\theta) \cdot \Theta(\theta) \]  

Equation (1) shows that \( \Sigma Q \) provides some information on \( U_\theta \), although it is under the influence of the flow direction \( \theta \) because the cross section of the sensor is shaped like an ellipse. Equation (2) shows that \( \Delta Q \) changes with \( \theta \) and that its sensitivity \( g \) is a function of \( U_\theta \). Accordingly, information on \( \theta \) can be obtained using both \( \Sigma Q \) and \( \Delta Q \). By obtaining the functional relations (1) and (2) from calibration tests of the new probe, we calculated the normal component \( U = U_\theta \cos \theta \) and the parallel component \( V = U_\theta \sin \theta \) of the flow velocity to the sensor plane. However, \( Q_1 \) and \( Q_2 \) cannot be treated directly and only the corresponding outputs of CTC can be treated. Thus, it is assumed that the relation between the output \( E \) of CTC and \( U_\theta \) can be represented by the so-called n-th power law similar to an I-type probe:

\[ E^2 = C + BU_\theta^n \]  

where \( B \) and \( C \) are calibration constants that are generally a function of ambient temperature. Furthermore, it is well-known that exponent \( n \) has a value of about 0.45 for an I-type probe.
Figure 3 shows the data acquisition system developed for the new probe. $\Sigma E_o$ corresponding to $\Sigma Q$ is the output after $\Sigma E_{HW}$ is linearized; $\Delta E_o$ corresponding to $\Delta Q$ is the difference of the linearized outputs $E_1$ and $E_2$. $\Sigma E_o$ and $\Delta E_o$ are fed to a personal computer through a 2-channel 12-bit A/D converter.

Figure 4 shows schematically the look-up-matrix method for obtaining velocity components $U$ and $V$ from both $\Sigma E_o$ and $\Delta E_o$. The direct output from the 12-bit A/D converter is only 4096 different integer values. First, we calculated $4096 \times 4096$ different values of $\theta$ using all possible integer values $i$ and $j$ corresponding to $\Sigma E_o$ and $\Delta E_o$, respectively, from their calibration curves shown in Figs. 9 and 10 and created a look-up matrix $\theta[i][j]$ in the computer memory. Second, data were acquired by putting each value of $\theta$ into the look-up matrix using the A/D integer format of the measured values of $\Sigma E_o$ and $\Delta E_o$. The velocity $U_\theta$ was determined using Eq. (10) from both $\theta$ and $\Sigma E_o$ thus obtained. The acquired data were stored in an additional memory region. Finally, $U$ and $V$ were calculated using $\theta$ and $U_\theta$. With this method, the computing time of $\theta$ required for measuring the flow velocity can be omitted, and it is possible to obtain an accurate real-time measurement of $U$ and $V$ at a high sampling speed of 10 kHz.

5. Experimental set-up and procedures

5.1 Wind tunnel and flat plate

We investigated experimentally some characteristics of the new probe and measured velocity profiles of a turbulent boundary layer along a flat plate to verify its usefulness.
Experiments were performed in a low-speed wind tunnel of the closed return-circuit type with a test section of $1 \times 0.45 \times 3.1$ m in size as shown in Fig. 5. A flat plate measuring 2.5 m in length and 1 m in width was mounted vertically in the test section. A turbulent boundary layer was formed on the flat plate by a tripping wire of 1.5 mm in diameter placed on the surface of the plate at a position 0.2 m downstream from the leading edge. The streamwise direction is denoted by $x$, the wall-normal direction by $y$ and the spanwise direction by $z$. The origin of the axes is at the center of the tripping wire. The residual turbulence in the free stream was below 0.1% for the free-stream velocity, about $U_0 = 7$ m/s, in the present study.

The probe was installed in the traversing mechanism with a rotating device and could be set to an arbitrary flow direction.

5.2 Calibration tests for the new hot-wire probe

Calibration tests were performed in a free-stream outside boundary layer of the flat plate by rotating the probe within the range from $\theta = -70$ to 70 deg. First, we examined whether Eq. (3) was applicable to the probe at each angle $\theta$. Second, we adjusted the linearizers for $E_{HW1}$, $E_{HW2}$ and $\Sigma E_{HW}$ at $\theta = 0$ deg. After the adjustments, we measured the output $\Sigma E_o$ to a flow velocity $U_0$ by changing $\theta$ and then measured the output $\Delta E_o$ to a flow direction $\theta$ by changing $U_0$ in order to find the functional forms of $f$, $\Theta$ and $g$ in Eqs. (1) and (2).

5.3 Measurements of velocity profiles in a turbulent boundary layer

Measurements were made of mean and fluctuating velocity profiles and some quantities of the boundary layer along the centerline of the flat plate at 0.6 m, 0.75 m and 0.9 m downstream from the tripping wire at $U_0 = 7$ m/s, corresponding to $R_\delta \approx 1000$. We used the method of analysis proposed by Tani and Motohashi\textsuperscript{(1)} to determine $c_f$ and $\Pi$. This is a useful method for evaluating mean and fluctuating velocity profiles simultaneously, if $\delta_1$ and $\delta_2$ are obtained from velocity profiles measured with high accuracy. An outline of this method is given below.

The mean velocity profile of the turbulent boundary layer composed from the logarithmic law and the wake function of Lewkowicz is expressed by

$$
\frac{U}{u_*} = \frac{1}{\kappa} \ln \frac{u_* y}{v} + C - \frac{1}{\kappa} \frac{y^2}{\delta^2} \left( 1 - \frac{y}{\delta} \right) \left( 1 - 2 \frac{y}{\delta} \right) + \frac{2}{\kappa} \frac{y^2}{\delta^2} \left( 3 - 2 \frac{y}{\delta} \right)
$$

(4)
When we let
\[ Z = \frac{U_0}{u_r} = \kappa \frac{2}{\sqrt{c_f}} \quad s = Z - 2\Pi - \kappa C \] (5)
\[ \delta_1 \text{ and } \delta_2 \text{ can be respectively expressed by} \]
\[ \delta_1 = \frac{\nu}{U_0} \xi \exp(s) \] (6)
\[ \delta_2 = \frac{\nu}{U_0} \left( \xi - \frac{\eta}{Z} \right) \exp(s) \] (7)
where \( \xi \) and \( \eta \) are
\[ \xi = \frac{59}{60} + \Pi, \quad \eta = \frac{8437}{4200} + \frac{667}{210} \Pi + \frac{52}{35} \Pi^2 \]
We calculated \( Z \) repeatedly so as to satisfy both Eqs. (6) and (7) using the measured values of \( \delta_1 \) and \( \delta_2 \) and determined \( c_f \) and \( \Pi \). However, because the influence of the viscous sublayer could not be ignored in the turbulent boundary layer of a low Reynolds number, it was necessary to correct the measured values of \( \delta_1 \) and \( \delta_2 \) as follows:
\[ \delta_{1c} = \delta_1 - 50.63 \frac{\nu}{U_0} \]
\[ \delta_{2c} = \delta_2 + \left( 50.63 - \frac{136.47}{\kappa Z} \right) \frac{\nu}{U_0} \]
Thus, we had to recalculate \( Z \) by replacing \( \delta_1 \) and \( \delta_2 \) with \( \delta_{1c} \) and \( \delta_{2c} \) respectively. We compared the data obtained with other experimental and computed results\(^{(2), (3), (5)}\) to confirm the applicability of the new hot-wire probe to a thin boundary-layer flow.

6. Experimental results and discussion

6.1 Characteristics of new hot-wire probe

First, the position of \( \theta = 0 \) deg was determined from the variations of \( E_{HW1} \) and \( E_{HW2} \) versus \( \theta \) as shown in Fig. 6. The sensor plane was perpendicular to the free stream \( U_0 \) at this position. Both \( E_{HW1} \) and \( E_{HW2} \) were twice the cycle of the X-type probe and were out of phase. Thus, \( \theta = 0 \) deg was determined as the middle of the angles where \( E_{HW1} \) and \( E_{HW2} \) showed their maximum values.

![Fig. 6 Characteristic curves of each output of CTC to flow direction at \( U_0 = 7 \) m/s](image)

Next, when the relations to \( U_0^\alpha \) of \( E_{HW1}^2 \), \( E_{HW2}^2 \) and \( \Sigma E_{HW}^2 \) were examined, all of them showed good agreement with the n-th power law of Eq. (3) with \( n = 0.4 \).

Figure 7 shows \( \Sigma E_{HW}^2 \) plotted versus \( U_0^\alpha \) at \( \theta = 0, -30 \) and \(-70 \) deg. At \( \theta = 0 \) deg, \( \Sigma E_{HW}^2 \) increased linearly with \( U_0^\alpha \), similar to the case of an I-type probe. As \( \theta \) increased,
the slope of the straight line fitted to the experimental data also became larger. This showed that calibration constants $B$ and $C$ of Eq. (3) were the functions of not only the ambient temperature but also $\theta$. As $\theta$ was further increased, however, the virtual origin of the straight line, $U_r^{0.4} = -C/B$ ($U_r$ is usually called the characteristic velocity), moved to the right on the abscissa. This showed that the linear relationship of the output voltage and the flow velocity did not pass through the origin. In the present study, the output compensation assumed that the virtual origin did not move in relation to a change in $\theta$.

![Image](image.png)

**Fig. 7** Relations of $\Sigma E_{\text{th}}^2$ for $U_r^{0.4}$

Hence, we examined the behavior of $U_r^{0.4}$ versus $\theta$ as shown in Fig. 8. The value of $U_r^{0.4}$ remained constant at about $-2.50$ within the range of $\theta = \pm 30$ deg. Thus, we could use the simple n-th power law of Eq. (3) for the new probe within the limits of this range, which is similar to that of a conventional X-type probe.

![Image](image.png)

**Fig. 8** Variation of $U_r^{0.4}$ versus $\theta$

Figure 9 shows that $\Sigma E_o$ increased linearly with $U_0$ through the origin at each $\theta$. Each straight line was fitted to the experimental data within 0.5% of full scale. These results showed that $\theta$ influenced only the slopes of the linear relations. As a result, $f(U_0, \theta)$ in Eq. (1) was separable to $U_0$ and was expressed as a function of $\theta$ as

$$\Sigma E_o = f(\theta) \cdot U_0$$

within the range of $\theta = \pm 30$ deg.
Fig. 9 Calibration of \( \Sigma E_o \) versus \( U_\theta \)

![Graph showing \( \Sigma E_o \) vs. \( U_\theta \)]

Fig. 10 Calibration of \( \Delta E_o \) versus \( \theta \)

![Graph showing \( \Delta E_o \) vs. \( \theta \)]

Figure 10 shows the variations of \( \Delta E_o \) when \( \theta \) was changed from \(-30\) to \(30\) deg. The data obtained showed that all of \( \Delta E_o \) versus \( \theta \) were approximated by sinusoidal curves. The flow velocity decreased as the amplitudes of the curves became smaller. This shows that the sensitivity \( g \) of \( \Delta E_o \) fell with \( U_\theta \). Thus, \( \Delta E_o \) was expressed by

\[
\Delta E_o = g(U_\theta) \cdot \sin \theta
\]

(9)

Figures 11 and 12 show the variations of \( f(\theta) \) and \( g(U_\theta) \), respectively. From these results, it was found that the function forms of \( f(\theta) \) and \( g(U_\theta) \) were quadratic expressions of \( \theta \) and \( U_\theta \), respectively, as expressed in these figures.
From the results of the calibration test, the relations corresponding to Eqs. (1) and (2) were respectively expressed by

$$\Sigma E_o = (a \theta^2 + b \theta + c) \cdot U_\theta$$

(10)

$$\Delta E_o = (\alpha U_\theta^2 + \beta U_\theta + \gamma \sin \theta)$$

(11)

where \(a, b, c, \alpha, \beta, \gamma\) are calibration constants.

We prepared the matrix \(\theta [i][j]\) in the computer memory because \(U_\theta\) was eliminated from Eqs. (10) and (11). Thus, \(U_\theta\) and \(V\) were simply calculated, because we measured \(\theta\) and \(U_\theta\) using the measured value of \(\Sigma E_o\) and the method of the look-up matrix.

### 6.2 Mean velocity profiles in low-Reynolds number turbulent boundary layer

Figure 13 shows the streamwise mean velocity profiles represented by the inner scale \(y^+\) of a turbulent boundary layer.

The solid lines are the calculated velocity profiles of Eq. (4) in the wall and wake.
regions presented by Tani and Motohashi and of the equation in the buffer region proposed by Spalding\textsuperscript{(4)}. The measured and calculated profiles were in good agreement. From these results, it was found that, like an I-type probe, the new probe can accurately measure mean velocity profiles in the vicinity of a wall surface less than $y^+ = 10$ where there is a viscous sublayer. In comparison with the data reported by Erm and Joubert\textsuperscript{(2)}, it was also made clear that the new probe is superior to the X-type probe, which cannot measure velocity profiles near a wall surface.

The parameter $c_f$ that characterizes the inner region of a boundary layer is plotted versus $\log R_{\delta}$ in Fig. 14. For comparison, the data obtained with the Kármán-Schoenherr formula (dashed line) and the data reported by Purtell et al.\textsuperscript{(5)} are shown together with that of the present I-type probe.

![Fig. 14 Variation of skin friction coefficient $c_f$ versus $R_{\delta}$](image)

The parameter $\Pi$ that characterizes the outer region of a boundary layer is also plotted versus $\log R_{\delta}$ in Fig. 15. Both $c_f$ and $\Pi$ were in good agreement with the results of the present I-type probe as well as the data reported by other researchers. Therefore, it is concluded that, like the I-type probe, the new hot-wire probe is capable of measuring the mean velocity accurately in a thin, low-Reynolds-number boundary layer.

![Fig. 15 Variation of wake parameters $\Pi$ versus $R_{\delta}$](image)

6.3 Distributions of velocity fluctuations and Reynolds shear stress

Extensive tests have been performed to verify whether the new probe can measure fluctuating velocities with high accuracy. Figures 16 and 17 show the distributions of fluctuating velocities $u'/u_\tau$, $v'/u_\tau$ and of Reynolds-shear stress $-\overline{uv}/u_\tau^2$, respectively, represented by the outer scale $y/\delta$ together with the experimental results of Erm and Joubert and the DNS results of Spalart\textsuperscript{(3)}. At every $R_{\delta}$, the results obtained with the new probe were somewhat smaller than Spalart's results, though they were in good agreement with the
results of Erm and Joubert and the present I-type probe.

These results show that the new probe is also capable of measuring fluctuating velocities accurately. From the profiles of $u'/u$, represented by the inner scale $y^+$ as shown in Fig. 18, it was found that the probe is capable of measuring fluctuating velocities even in a region dominated by viscous stress.
7. Conclusions

We manufactured a new hot-wire probe and examined some of its features experimentally. The new probe was used to measure the velocity in a turbulent boundary-layer flow at a low Reynolds number. The results obtained are summarized below.

1. The velocity-output relation of the new probe matched the simple $n$-th power law similar to that of the I-type probe within the limits of $\theta = \pm 30$ deg.

2. A data acquisition system was constructed without greatly improving the hot-wire circuit for the I-type probe by applying the look-up-matrix method with the aid of a PC.

3. The calibration curves of Eqs. (10) and (11) were accurately obtained with an easy calibration method. As a result, the velocity components $U$ and $V$ were measured within 0.5% of full scale.

4. The new probe has the same spatial resolution as the I-type probe. It provides simultaneous measurements of two-velocity components near a wall surface which have so far been difficult to measure. The results obtained showed that the new probe is an effective tool for measuring mean quantities in a transitional boundary layer having a thickness of only 6 mm.

References


