Numerical Prediction of Cavitation Erosion Intensity in Cavitating Flows around a Clark Y 11.7% Hydrofoil*

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Abstract
A numerical prediction method of cavitation erosion is proposed. In this method, the analysis of bubbles in cavitating flows is performed and the intensity of cavitation erosion is evaluated by the impact pressure induced by spherical bubble collapse. In the present study, two-dimensional cavitating flow around the Clark Y 11.7% hydrofoil is used to examine the proposed numerical prediction method. The proposed numerical method predicts that the intensities of cavitation erosion in noncavitating, attached cavitating and pseudo-supercavitating flows are far weaker than the intensity of cavitation erosion in a transient cavitating flow, and the intensity in the vicinity of the sheet cavity termination is high. These results correspond well to experimental results, and it is confirmed that systematic erosion characteristics are generally captured by this method. Furthermore, the velocity dependence of cavitation erosion is examined, and it is found that the exponent $n$ in the relation between the intensity $I$ and main flow velocity $U_{in}$ ($I \propto U_{in}^n$) becomes large when the bubble radius is large and ranges between 4.3 and 7.0 in the present study. According to the bubble dynamics, the ambient pressure and the rate of increase in pressure increase as the main flow velocity, and the maximum internal pressure increase. Therefore, it is thought that smaller bubbles cause cavitation erosion when the main flow velocity is large.

Key words: Cavitating Flow, Intensity of Cavitation Erosion, Numerical Prediction, Homogeneous Model, Bubble Dynamics

1. Introduction
Cavitation is an abrupt phase change phenomenon and occurs in various high-speed liquid flows when the local static pressure in the liquid is lower than the saturation vapor pressure. This phenomenon causes several problems such as performance degradation of fluid machinery, noise, vibration and erosion on the material surfaces.

Among these problems, cavitation erosion causes material fracture of the fluid machinery because of the violent collapse of a cavity bubble or bubble cluster upon arrival at a region where the ambient pressure recovers. The development of a method by which to numerically predict the intensity or the amount of cavitation erosion in a cavitating flow is desired from an industrial point of view that it demands to evaluate cavitation erosion practically without long time experiment of cavitation erosion.

Franc proposed an erosion model that assumes that work hardening occurs without mass loss when a stress acting on a material is between the yield stress and the ultimate tensile strength of the material, and that mass loss starts when the cumulative energy of the material...
reaches the energy corresponding to the ultimate tensile strength. Dular et al.\textsuperscript{(2)} reported a relationship between cavitation erosion and cavity aspects and evaluated the erosion of a hydrofoil surface, assuming that the shock wave strength associated with cloud cavity collapse is proportional to the velocity of the change in cloud volume. Fukaya et al.\textsuperscript{(3)} proposed a numerical prediction method of cavitation erosion to evaluate the cavitation intensity using the internal pressure and the number density of bubbles. However, a practical numerical method has not yet been proposed because cavitation erosion is related to a complex cavitation phenomenon owing to microscopic bubble or bubble cluster collapse.

The pressure wave radiating from a rebounding bubble (e.g., Hickling and Plesset\textsuperscript{(4)} and Fujikawa and Akamatsu\textsuperscript{(5)}) and the microjet generated from a collapsing bubble near the material surface (e.g., Naude and Ellis\textsuperscript{(6)} and Plesset and Chapman\textsuperscript{(7)}) cause cavitation erosion. When fluid machinery operates for a long time under cavitation, serious problems, such as material fractures, occur. In our previous paint removal test\textsuperscript{(8)} in the cavitating flow around the Clark Y 11.7 % hydrofoil, the paint separation is caused by the collapse of an isolated traveling bubble due to three dimensionality of the flow and by circulating bubbles in the sheet cavity. These indicate that microscopic phenomenon of bubble collapse yields macroscopic result of material damage. Therefore it is necessary to analyze cavitating flow field and the bubble behavior in the flow for prediction of cavitation erosion.

In the present study, a numerical prediction method of cavitation erosion is proposed. The analysis of individual spherical bubbles in cavitating flow is performed using the Eulerian-Lagrangian method, in which the macroscopic flow field is treated using Eulerian mechanics and individual microscopic bubbles are treated using Lagrangian mechanics (Farrell et al.\textsuperscript{(9)} and Kantani et al.\textsuperscript{(10)}). The impact pressure and energy generated by the pressure wave induced by the spherical bubble collapse are used as the evaluation parameters of intensity of cavitation erosion. The present numerical calculations to predict intensity of cavitation erosion are limited to the sheet and cloud cavitation condition. Two-dimensional cavitating flows around a Clark Y 11.7 % hydrofoil at several cavitation numbers are used to examine the proposed numerical prediction method. In addition, the velocity dependence on cavitation erosion is analyzed by the proposed numerical prediction method.

**Nomenclature**

- \( c \) : chord length
- \( D_a \) : density ratio of air in gas phase
- \( e \) : total energy per unit volume
- \( I \) : intensity of cavitation erosion
- \( p \) : pressure
- \( P_w \) : impact pressure
- \( R \) : bubble radius
- \( R_\infty \) : standard bubble radius
- \( Re \) : Reynolds number
- \( t \) : time
- \( T \) : temperature
- \( Y \) : mass fraction of gas
- \( u_i \) : velocity vector
- \( \alpha \) : void fraction
- \( \rho \) : mixture density
- \( \sigma \) : cavitation number
- \( \alpha_{in} \) : initial
- \( a \) : air
- \( b \) : bubble
- \( g \) : gas
- \( in \) : main flow
- \( l \) : liquid
- \( m \) : gas-liquid mixture phase
- \( v \) : vapor

2. **Numerical Method**

2.1. **Macroscopic Numerical Method of Cavitating Flow**

The gas-liquid two-phase Navier-Stokes equation is solved using a locally homogeneous model of a gas-liquid two-phase medium\textsuperscript{(11),(12)} to efficiently simulate a macroscopic cavitating flow. This model treats the two-phase medium as a pseudo-single-phase medium, which has a locally homogeneous void fraction. The liquid phase is assumed to follow the Tamman type equation of state (\textsuperscript{(13)}), and the gas phase is assumed to follow the equation of state for an ideal gas. In the present study, vapor and air (non-condensable gas) are assumed to be in the gas phase.
It is assumed that an unlimited number of infinitely small bubbles or droplets are distributed homogeneously in the control volume of the gas-liquid two-phase medium. The density of the locally homogeneous medium is expressed as follows by linearly combining the gas phase and liquid phase densities with the void fraction $\alpha$:

$$\rho = (1-\alpha)p_l + \alpha p_g. \quad (1)$$

The equation of state of the homogeneous medium is expressed in terms of $Y$ as follows, assuming local equilibrium of pressure and temperature between the gas phase and the liquid phase, and using the relationship between $Y$ and $\alpha (\rho(1-Y) = \rho(1-\alpha), \rho Y = \rho_g \alpha)$:

$$\rho = \frac{p(p + p_c)}{K_l(1-Y)p(T + T_0) + R_g Y(p + p_c)T}, \quad (2)$$

where $p_c$, $K_l$, $T_0$ and $R_g$ are the liquid pressure constant, the liquid constant, the liquid temperature constant and the gas constant, respectively.

In the present study, the cavitating flow is treated as an air-water-vapor two-component two-phase medium. The governing equations are the continuity equation, the momentum equation, the total energy equation of a compressible two-phase medium, and the continuity equations of the mixture gas and noncondensable gas of air, and are expressed as follows:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial (E_j - E_{v_j})}{\partial x_j} = S \quad (3)$$

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u_i \\ e \\ \rho Y \\ \rho Y D_k \end{bmatrix}, \quad E_j = \begin{bmatrix} \rho u_j \\ \rho u_i u_j + \delta_{ij} p \\ \rho u_j H \\ \rho u_j Y \\ \rho u_j Y D_k \end{bmatrix}, \quad E_{v_j} = \begin{bmatrix} 0 \\ \tau_{ij} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S = \begin{bmatrix} 0 \\ q_j + \tau_{jk} u_k \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

$$\tau_{ij} = (\mu + \mu_0)(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k}), \quad (5)$$

$$q_j = -(\kappa + \kappa_0) \frac{\partial T}{\partial x_j}, \quad (6)$$

$$\mu = (1-\alpha)(1+2.5\kappa) \mu_0 + a\kappa_0, \quad (7)$$

$$\kappa = (1-\alpha)\kappa_1 + a\kappa_0, \quad (8)$$

where $H = (e + p)/\rho$, $\tau_{ij}$, $q_j$, $\mu$, $\kappa$ and $\dot{m}$ are the total enthalpy per unit mass, the stress tensor, the heat flux, the viscosity, the heat conductivity and phase change term, respectively. In addition, $e$ is expressed as follows assuming that the enthalpy per unit mass is a linear function with respect to $T$ ($h = C_{pm} T + h_{0m}$, where $C_{pm}$ and $h_{0m}$ are the specific heat at constant pressure and the enthalpy constant of the two-phase medium, respectively):

$$e = \rho \left( h - \frac{p}{\rho} + \frac{1}{2} u_i^2 \right), \quad (9)$$

The evaporation or condensation rate per unit surface area at the interface $\dot{m}_b$ is expressed by the Hertz-Knudsen-Langmuir equation$^{(14)}$:

$$\dot{m}_b = \frac{C_{e/c} T_b}{\sqrt{2\pi R_g}} \left( \frac{p_c^*}{\sqrt{T}} - \frac{p_c}{\sqrt{T_b}} \right), \quad (10)$$

where $C_{e/c}$, $T_b$, $p_c^*$, $p_c$ and $R_g$ are the evaporation or condensation coefficient at the bubble wall, the temperature in a bubble, the saturated vapor pressure, the partial pressure of vapor and the gas constant of vapor, respectively. The evaporation or condensation rate per unit
volume in a two-phase medium $\dot{m}$ is modeled as follows, assuming $T_b = T$ and using the representative bubble radius $R_{rep}$ and the bubble number density $n$:

$$
\dot{m} = 4\pi R_{rep}^2 n C_e/c \frac{p_c^* - p_v}{\sqrt{2\pi R_{gs} T}}.
$$

(11)

Furthermore, it is approximated that $n = \alpha/(4\pi/3R_{rep}^3)$, where $\alpha \approx 4\pi/3R_{rep}^3$ for $\alpha \approx 0$ and $1 - \alpha \approx 4\pi/3R_{rep}^3$ for $\alpha \approx 1$. Finally, $\dot{m}$ is expressed as follows:

$$
\dot{m} = \alpha(1 - \alpha)^2 4\pi R_{rep}^2 n C_e/c \frac{p_c^* - p_v}{\sqrt{2\pi R_{gs} T}} = \alpha(1 - \alpha)AC_e/c \frac{p_c^* - p_v}{\sqrt{2\pi R_{gs} T}},
$$

(12)

where $A$ is the interfacial area concentration in the gas-liquid mixture and

$$
A = \frac{3}{R_{rep}} = \frac{3}{\frac{4}{3} \pi n} \alpha^{-\frac{4}{3}} (1 - \alpha)^{-\frac{2}{3}} = C_a \alpha^{-\frac{4}{3}} (1 - \alpha)^{-\frac{2}{3}}.
$$

(13)

In the present study, model constants $C_a C_s$ and $C_s C_a$ are set to 1,000 and 1 m$^{-1}$[15]. The saturated vapor pressure of water $p_c^*$ is given by the following empirical formula[16]:

$$
p_c^* = p_k \exp \left[ \left( \frac{T_f}{T} \right) \left( a + (b + cT)(T - d)^2 \right) \right],
$$

(14)

where $p_k = 22.130$ MPa, $T_f = 647.31 \text{ K}$, $a = 7.21379$, $b = 1.1520 \times 10^{-5} \text{ K}^{-2}$, $c = -4.787 \times 10^{-9} \text{ K}^{-3}$ and $d = 483.16 \text{ K}$. Here, $C_{pm}$ and $h_{gm}$ are expressed as a linear combination of $C_{pl}$ and $C_{pg}$, and $h_{gl}$ and $h_{gg}$ with $Y_s$ and $C_{pg}$, $h_{gg}$, and $R_g$, respectively, are expressed as a linear combination $C_{pg}$ and $C_{pg}$, $h_{gg}$ and $h_{gg}$, and $R_g$ and $R_g$ with $D_s$. The discretization of the finite volume method, the ADI method for time integration, the AUSM type upwind scheme[17] with third-order MUSCL-TVD[18] to evaluate the numerical flux and the Baldwin-Lomax model with Degani-Schiff modification[19] as a turbulent model are used. When the change in the mass fraction of gas $\Delta Y_s$ is large at the cell interface, higher-order numerical flux may yield non-physical oscillations. Therefore, a first-order upwind scheme is used if $2|\Delta Y_{s, i+1}| > |\Delta Y_{s, i+1}|$, $2|\Delta Y_{s, i+1}| > |\Delta Y_{s, i+1}|$, $2|\Delta Y_{s, i+1}| > |\Delta Y_{s, i+1}|$, or $2|\Delta Y_{s, i+1}| > |\Delta Y_{s, i+1}|$ at cell $i + \frac{1}{2}$.

### 2.2. Microscopic Numerical Method of Cavitation Bubble

The bubble shape is assumed to remain spherical and the bubble is assumed not to affect other bubbles. The bubble follows the equation of bubble motion considering the primary forces such as the force due to pressure gradient, drag and virtual mass force, as follows[20]:

$$
\rho_b V_b \frac{du_b}{dt} = -V_b \nabla p + \frac{1}{2} \rho R^2 \rho C_D |u_b - u| (u_b - u) + C_{VM} \rho_b \left( \frac{Du}{dt} - \frac{du_b}{dt} \right).
$$

(15)

where drag coefficient $C_D$ is the value for spherical bubble in a pure system[21] and virtual mass coefficient $C_{VM} = 0.5$.

The equation of bubble oscillation considering liquid compressibility up to the first order in bubble wall Mach number and the phase change at the bubble wall is used to evaluate the bubbles radius[22,23],

$$
RR \left( 1 - \frac{\dot{R}}{C} + \frac{\dot{u}_b}{\rho_b c} \right) + \frac{3}{2} \dot{R}^2 \left( 1 - \frac{1}{3} \frac{\dot{R}}{C} + \frac{2}{3} \frac{\dot{u}_b}{\rho_b c} \right) = \frac{1}{\rho_b} \left( 2 \frac{\dot{R}}{\rho_b} + \frac{\dot{u}_b}{\rho_b c} \right)
$$

$$
- \frac{\ddot{R} \dot{m}_b}{\dot{R}} \left( 1 - \frac{\dot{R}}{C} + \frac{\dot{u}_b}{\rho_b c} \right) = \frac{1}{\rho_b} \left( \frac{1}{R^2} + \frac{R d}{C dt} \right) \left( p_a + p_v - p - \frac{4 \mu_{R_b} R - 2 \sigma_{sat}}{R} \right).
$$

(16)

where $p_a$, $p_v$, $C$ and $\sigma_{sat}$ are the partial pressures of air and vapor in a bubble, the speed of sound and the surface tension coefficient of the liquid, respectively, and the overdot denotes the derivative with respect to time. In the present study, since it is assumed that vapor and air (noncondensable gas) are involved in a bubble microscopically, the continuity equations of vapor and air in a bubble are used to calculate the partial pressures of vapor and air.

$$
\frac{d}{dt} \left( \frac{4}{3} \pi R^3 \rho_v \right) = 4 \pi R^2 \dot{m}_b,
$$

$$
\frac{d}{dt} \left( \frac{4}{3} \pi R^3 \rho_a \right) = 0.
$$

(17)
In order to evaluate the temperature in a bubble, the energy equation of the mixture gas in a bubble is solved (24).

\[
\frac{d}{dt} \left( \frac{4}{3} \pi R^3 \rho_{mg} U_{mg} \right) = -\rho_{mg} \frac{d}{dt} \left( \frac{4}{3} \pi R^3 \right) - 4\pi R^2 \delta U_{mg}.
\] (18)

The specific internal energy of the mixture gas in a bubble is

\[
U_{mg} = (\rho_v C_{v} + \rho_a C_{a}) T_b / \rho_{mg}.
\]

where \( C_v \) and \( C_a \) are the specific heats of vapor and air at constant volume, respectively, and \( \delta U_{mg} \) is the energy flux at the bubble wall. The initial \( p_v \) is assumed to be the saturated vapor pressure, and \( p_v \) is then calculated from the condition in which the force equilibrium at the bubble wall is maintained (initial bubble wall velocity \( \dot{R} = 0 \)). The equations of bubble motion and bubble oscillation are solved using the fourth-order Runge-Kutta method.

### 2.3. Coupling Method Between Macroscopic and Microscopic Analyses

The calculations are performed by one-way coupling algorithm from macroscopic analysis to microscopic analysis as following procedure: (1) The determination of reference values for bubble calculation: The macroscopic physical values of liquid phase at bubble center are determined by interpolating from these values at the vertices of the cell involving bubble center. Each value at the vertices is an average value among the surrounding cell centered values. (2) The macroscopic flow field and the bubble translational motion are calculated. (3) The calculation of bubble oscillation: Since the time scales of the macroscopic flow field and the microscopic bubble oscillation are significantly different, the calculation of the bubble volume oscillation may become unstable with the time step used for the calculation of the flow field. In order to avoid this situation, one hundred bubble calculations are performed for each flow field calculation. (4) The determination of bubble collision with a wall: A bubble is assumed to be perfectly elastic collision when the distance from the bubble center to the wall is smaller than the bubble radius.

### 2.4. Evaluation Method of Intensity of Cavitation Erosion

In the present study, the primary cavitation erosion characteristics are predicted by the impact pressure on the wall surface owing to the propagating pressure wave induced by bubble collapse. Here, we assume that the pressure field in the vicinity of a bubble that collapses and induces the pressure wave is that of the potential flow induced by the spherical motion of the bubble. Next, the equation of motion is expressed in terms of the velocity potential \( \phi \) of the flow field, as follows (25):

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{\partial \phi}{\partial r} \frac{\partial^2 \phi}{\partial r^2} = -\frac{1}{\rho_l} \frac{\partial p}{\partial r}.
\] (19)

where \( \phi \) is the solution of the wave equation for propagation in an ambient liquid and considers liquid compressibility up to the first order of \( 1/C \) as follows (25):

\[
\phi = \frac{1}{r} f \left( t - \frac{r - R}{C} \right) = \frac{1}{r} \left( -R^2 \dot{R} + \frac{1}{C} \left( R^3 \ddot{R} + 2R \dot{R}^2 \right) \right).
\] (20)

Substituting \( \phi \) expressed by Eq. (20) into Eq. (19) and integrating the resulting equation from \( r = r \) to \( r = \infty \), the pressure \( p_r \) at radial position \( r \) from the center of the bubble is expressed as follows:

\[
p_r = p - p_0 \left( f' + \frac{1}{2} \frac{f^2}{r^2} + \frac{2}{C} \frac{f f'}{r^3} + \frac{1}{C^2} \frac{f'^2}{r^4} \right),
\] (21)

where a prime indicates a differential with respect to \( t - (r - R) / C \).

The impact pressure acting on the wall surface \( P_w \) is calculated by Eq. (21) by substituting the distance between the position of bubble collapse and the surface for \( r \) in Eq. (21).

Soyama et al. (26) calculated the individual impact energy \( E_i \) as follows:

\[
E_i = I_r A_i = \frac{P_i^2}{2\rho_i C} r_i A_i, \quad [J]
\] (22)

where \( I_r \) is the moment of inertia of the bubble, \( A_i \) is the area of the bubble, \( P_i \) is the pressure at the bubble center, \( \rho_i \) is the density of the bubble, and \( C \) is the speed of sound in the bubble.
where $I_i$, $\tau_i$, $A_i$, and $P_i$ are the acoustic energy, the impact duration, the affective area and the impact pressure, respectively. Fukaya et al.\(^3\) defined that the cavitation intensity is proportional to the square of the bubble internal pressure. In the present study, the intensity of cavitation erosion $I$ is defined as follows:

$$I = \sum \frac{P^2 \tau}{2 \rho C} / t_{\text{cal}},$$

[\text{W/m}^2] \tag{23}

where $t_{\text{cal}}$ is the total calculation time. Here, $I$ is calculated assuming that $\tau = 0.1 \mu s$ which is the same order of magnitude of half width of pressure wave induced by a bubble with $R_0 = 1.0$ mm\(^2\), and $\rho_l$ and $C$ are the physical values of the liquid phase in the main flow.

### 3. Calculation Condition

The cavitating flow around the Clark Y 11.7% (chord length: 70 mm) two-dimensional hydrofoil is analyzed. Figure 1 shows the calculation area. A C-type grid system with $513 \times 85$ (streamwise $\times$ perpendicular direction) grid points is used. The main flow temperature and void fraction are 293.15 K and 1%, respectively, and the angle of attack is 7 deg. It is adequate to investigate the influence of cavitation number on the intensity of cavitation erosion in either case of main flow void fraction 1 or 0.1 %, since the time average characteristics and cavity aspects in both cases are confirmed to be similar by our preliminary calculations. Main flow void fraction 1 % is used because of the shorter calculation time. The wall boundary condition is the non-slip condition and $\partial Q / \partial n = 0$ at boundary 1, where $n$ is the direction perpendicular to the boundary, and $\rho$, $u$, $p$, $T$, $Y$ and $D_a$ are constant at boundary 2.

In order to investigate the influence of grid resolution, calculations are performed using three different grid resolutions ($257 \times 85$, $513 \times 85$ and $769 \times 85$). Figure 2 shows the calculated and experimental\(^2\) surface pressure distributions at $U_{\text{in}} = 10$ m/s, angle of attack = 8 deg. and $\sigma = 3.0$. The cavitation number $\sigma$ is defined as follows:

$$\sigma = \frac{p_{\text{in}} - p^*}{\frac{1}{2} \rho u^2 \text{in}}.$$

(24)

The main flow pressure $p_{\text{in}}$ is calculated by Eq. (24) with arbitrary $\sigma$. At $\sigma = 3.0$, the flow field is in the noncavitating condition. The differences among the calculated surface pressure distributions are very slight, and these distributions agree well with the experimental surface pressure distributions. On the other hand, at $\sigma = 1.1$, the calculation with $257 \times 85$ predicts a different cavitating condition (the steady attached cavitating condition) from the calculations with $513 \times 85$ and $769 \times 85$ (unsteady transient cavitating condition). Therefore, the grid system with $513 \times 85$ is used for all subsequent calculations involving the cavitating condition although $257 \times 85$ is sufficient to simulate the noncavitating condition.

For bubble calculations, the initial positions are located at five points $(x, y) = (-3, 1.00)$, (-3, 2.75), (-3, 4.50), (-3, 6.25) and (-3, 8.00) mm ($x$ is the hydrofoil chord direction, and...
is the direction perpendicular to \( x \). The origin of this coordinate system is located on the leading edge of the hydrofoil), and five bubbles are set at five initial positions for each time interval \( 2/U_{in} \) ms. The initial bubble radius \( R_0 \) is determined by following a procedure similar to the technique of Fukaya et al.\(^{(3)}\) The standard radius \( R_\infty \), which is the radius when the force equilibrium condition is attained under standard ambient pressure (0.1 MPa in the present study), is decided arbitrarily. The initial bubble radius \( R_0 \) is assumed to be the radius when the bubble with \( R = R_\infty \) moves from the position of the standard ambient pressure to the initial position quasistatically.

4. Results and Discussion

4.1. Analysis of Bubble Behavior and Prediction of Intensity of Cavitation Erosion in a Transient Cavitating Flow (\( U_{in} = 10 \) m/s, \( \sigma = 1.1 \) and \( R_\infty = 100 \mu m \))

In this section, numerical analysis of a transient cavitating flow accompanied by the generation and collapse of a cloud cavity is performed. We select \( \sigma = 1.1 \) to simulate a transient cavitating flow. In addition, \( U_{in} = 10 \) m/s (The Reynolds number based on \( U_{in} \) and \( c \) is \( Re = 6.7 \times 10^5 \), \( R_\infty = 100 \mu m \)), and the total calculation time is 100 ms.

Figure 3 shows the time evolution of the pressure distribution, the isoline of the 10% void fraction of the flow field, and the bubble positions and sizes. The macroscopic cavitation region is considered to exist inside the isoline of the 10% void fraction. In Fig. 3(i), the macroscopic sheet cavity attaches to the hydrofoil surface and grown bubbles exist in the sheet cavity. In Fig. 3(ii), the sheet cavity breaks off. In Fig. 3(iii), the sheet cavity begins to flow downstream, and the bubbles in the macroscopic cavity grow. In Fig. 3(iv), the next sheet cavity develops. After that, the sheet cavity detaches from the hydrofoil and flows downstream as a cloud cavity (Fig. 3(v)). The cloud cavity shrinks gradually in Figs. 3(vi) and 3(vii). At this time, the bubbles near the interface of the cloud cavity collapse violently. In Fig. 3(viii), the sheet cavity attaches to the hydrofoil surface, and the aspect of the flow field is similar to that of Fig. 3(i). At this \( \sigma \), as shown in Fig. 3, the sequence of the flow field repeats periodically.

Next, the representative bubble behavior, which generates a high impact pressure acting on the hydrofoil surface is analyzed. Figure 4 shows the time evolution of the pressure distribution, the isoline of the 10% void fraction of the flow field and the bubble trajectory. The
red lines in Fig. 4 denote the bubble trajectories. Figure 5 shows the time histories of the ambient pressure and radius of the bubble. Figure 6 shows the local maximum internal pressure, which is higher than 0.1 MPa, and the impact pressure of the bubble. The bubble entrains into the sheet cavity (Fig. 4(i)) and grows in the low-pressure region (Fig. 5). The bubble shrinks near the sheet cavity termination (Fig. 4(ii)). However, the local maximum internal pressure, which is lower than 0.01 MPa, is still weak, and does not appear in Fig. 6 because the local maximum radius is not so large. The damping effect of liquid viscosity on the bubble oscillation becomes large when the bubble radius before collapse is small \(^{(29)}\). Therefore, bubble oscillation is suppressed by a strong damping effect, and the maximum internal pressure is weak when the local maximum radius is not too large. The bubble is pushed back into the cavity region by the re-entrant jet, which develops near the hydrofoil surface (Fig. 4(iii)), and the bubble begins to grow again. The bubble flows downstream as a part of a cloud cavity (Figs. 4(iv) and 4(v)). Then, the bubble radius is the largest in the cloud cavity (Fig. 5). At the time of the image in Fig. 4(v), the bubble has a high internal pressure (higher than 100 MPa) near the cavity interface (Fig. 6). However, the induced impact pressure, approximately 1 MPa, is very low because the position at which the bubble has a high internal pressure is far from the hydrofoil surface, and the pressure wave attenuates greatly. The bubble is exposed to a rapid increase in ambient pressure near the cavity interface when the cloud cavity shrinks, and the bubble collapses violently. At approximately 67 ms, the internal pressure of the bubble is higher than 4 GPa, and the impact pressure acting on the hydrofoil surface is higher than 250 MPa (Figs. 4(vi) and 6). The bubble then rebounds and collapses several times. The analysis of the bubble behavior confirms that the bubble induces a high impact pressure when the radius is large before collapse and is near the hydrofoil surface.

The intensity of cavitation erosion \(I\) in the flow is investigated by counting high impact pressure. Figure 7 shows the spatial distribution of \(I\), and the average and range of the maximum sheet cavity length. Figures 8 and 9 show the time variations of the number of impact pressure higher than 0.1 MPa and the spatial average of \(I\) for 1.0 ms time intervals from 55 to 70 ms, respectively. According to Figs. 3 and 7, the sheet cavity length is approximately 0.75 \(c\), and \(I\) is maximum in the vicinity of the sheet cavity termination. This result corresponds well with the experimental results reported by Knapp \(^{(30)}\), in which several erosion pits are observed in the vicinity of the sheet cavity termination. According to Fig. 9, \(I\) is large between 63 and 69 ms, and is very large for 67 and 68 ms. In addition, the impact pressure is low \((\leq 10\) MPa\)) from 55 to 63 ms (Fig. 8), whereas \(I\) is very low (Fig. 9). The total number of impact

![Fig. 4 Time evolution of pressure distribution and isoline of 10% void fraction, and trajectory of a bubble \((\sigma = 1.1, R_\infty = 100\mu m, Re = 6.7 \times 10^5)\) \(\text{(Red lines indicate bubble trajectories.}\)
Fig. 5  Time histories of ambient pressure and bubble radius ($\sigma = 1.1, R_\infty = 100 \mu m, Re = 6.7 \times 10^5$)

Fig. 6  Time histories of local maximum internal pressure and impact pressure

pressure higher than 0.1 MPa for 68 and 69 ms is greater than that for 67 and 68 ms, whereas $I$ for 68 and 69 ms is far lower than that for 67 and 68 ms. Therefore, it is thought that the low impact pressure is not a significant factor contributing to cavitation erosion. At 67 and 68 ms, the macroscopic cavity flows downstream, and the cloud cavity collapses. In addition, several bubbles in the cloud cavity collapse violently (Figs. 3(v), 3(vi) and 3(vii)). The value of $I$ is large at $0.7 \leq x/c \leq 1.0$, downstream of the sheet cavity termination (Fig. 7) because the bubbles in the cloud cavity induce a high impact pressure. Therefore, the collapse of the cloud cavity is found to be a significant factor contributing to cavitation erosion corresponding with the experimental results reported by Reisman and Brennen(31).

4.2. Prediction of Intensity of Cavitation Erosion for Various Cavitation Conditions ($U_{in} = 10 m/s$ and $R_\infty = 100 \mu m$)

In this section, the intensity of cavitation erosion is predicted at various cavitation numbers with the same main flow velocity in order to investigate the relationship between cavitation erosion and cavitation number. With the exception of the cavitation number, the calculation conditions are the same as those in the previous section.

Figures 10 and 11 show the maximum sheet cavity length, the standard deviation of the sheet cavity length and the spatial average of $I$. The total calculation time for the transient cavitating flow is 100 ms, which is longer than the times of three cycles of transient cavitating flow, and that for the other flows is 20 ms, which is sufficient for the bubbles set at the initial time to pass through the cavity region. According to Fig. 10, the sheet cavity length is zero at $\sigma = 2.0$, which indicates a noncavitating flow, and $I$ is also zero. The intensity of the cavitation erosion in the present study in a noncavitating flow is confirmed to be zero. The
maximum sheet cavity length is approximately 0.4 \( c \) at \( \sigma = 1.6 \). The standard deviation of the sheet cavity length is approximately zero, and the flow is an attached cavitating flow. At this \( \sigma \), \( I \) is not zero but is very low, and the intensity of cavitation erosion is found to remain weak. At \( 0.6 \leq \sigma \leq 1.2 \), the standard deviation of the sheet cavity length is large, and the flows are transient cavitating flows. The standard deviation becomes large when \( \sigma \) becomes small at these values of \( \sigma \). The values of \( I \) for the transient cavitating flow are far larger than those for the noncavitating and attached cavitating flows. In particular, at \( \sigma = 1.2 \) and 1.1, the values of \( I \) are very large. At \( \sigma = 0.3 \), the maximum sheet cavity length is approximately 1.9 \( c \), and the standard deviation of the sheet cavity length is approximately zero. This indicates a pseudo-supercavitating flow. Like the values for noncavitating and attached cavitating flows, the value of \( I \) is very low at this \( \sigma \). There are three characteristics of cavitation erosion in Fig. 11, namely, extremely weak intensity for noncavitating, attached cavitating and pseudo-supercavitating flows, very strong intensity for a transient cavitating flow (\( \sigma = 1.2 \) and 1.1), and moderate intensity for a transient cavitating flow (\( 0.6 \leq \sigma \leq 1.0 \)).

The tendency of cavitation erosion with respect to \( \sigma \) is similar to the experimental tendency (e.g., He and Hammitt(32)). Next, the reasons for the existence of these three characteristics are discussed.

First, the flow fields having extremely weak intensity of cavitation erosion are analyzed.
The flow fields in the noncavitating, attached cavitating and pseudo-supercavitating flows are pseudo-steady flows. Therefore, the representative bubble behavior is thought to characterize the intensity of cavitation erosion. Figure 12 shows the radius and ambient pressure histories of representative bubbles in these flows. The abscissa of Fig. 12 shows the x coordinate of the bubble position. At \( \sigma = 2.0 \), the ambient pressure is far higher than the saturation vapor pressure (approximately 2,300 Pa for 293 K) and the growth rate is very low (the maximum radius is smaller than twice the initial radius) (Fig. 12(i)). In addition, the bubble does not collapse violently because the pressure increase is moderate. Therefore, the intensity of cavitation erosion for the noncavitating flow is extremely weak. At \( \sigma = 1.6 \), the bubble in the cavity region (low-pressure region) can grow larger than the bubble in the noncavitating flow, and the bubble in the cavity region collapses more violently near the sheet cavity termination (0.2 \( \leq x/c \leq 0.4 \)) than the bubble in the noncavitating flow due to the pressure increase being more rapid than that in the noncavitating flow (Fig. 12(ii)). However, the impact pressure is lower than 0.5 MPa and is far lower than the impact pressure occurring in the transient cavitating flow (Figs. 6 and 8). This is because the maximum radius is not too large (approximately four times greater than the initial radius). At \( \sigma = 0.3 \), the maximum radius is more than seven times greater than the initial radius (larger than the bubble at \( \sigma = 1.6 \)). However, the maximum internal and impact pressures of the bubble in Fig. 12(iii) are very low (approximately 1.2 and 0.016 MPa) because the pressure increase is moderate and the position of the bubble collapse is far from the hydrofoil (\( x/c \geq 1.2 \)).

![Fig. 12 Radius and ambient pressure histories of the representative bubble (\( R_{\infty} = 100 \mu m, Re = 6.7 \times 10^5 \))](image)

(i) Noncavitating condition (\( \sigma = 2.0 \))

(ii) Attached cavitating condition (\( \sigma = 1.6 \))

(iii) Pseudo-supercavitating condition (\( \sigma = 0.3 \))

Next, the calculations of various transient cavitating flows are analyzed in order to discuss the characteristics of cavitation erosion of the transient cavitating flow. Figure 13 shows the spatial distributions of local maximum internal pressures above 1 GPa at \( \sigma = 1.2, 1.1, 1.0 \) and 0.8. The solid line indicates the hydrofoil, and the positions and sizes of the circle indicate the locations at which bubbles show high local maximum internal pressures and the magnitudes of the internal pressures, respectively. A comparison of the spatial distributions reveals that the bubbles that have high local maximum internal pressures at \( \sigma = 1.2 \) and 1.1 are concentrated near the hydrofoil surface as compared to those at \( \sigma = 1.0 \) and 0.8, and the number of bubbles at \( \sigma = 1.2 \) and 1.1 is greater than the number of bubbles at \( \sigma = 1.0 \) and 0.8. Therefore, the values of \( I \) at \( \sigma = 1.2 \) and 1.1 are far greater (Fig. 11). The spatial distribution becomes extensive when \( \sigma \) is small. This is because a larger cloud cavity occurs and the position of cloud collapse moves downstream when \( \sigma \) is small. At larger \( \sigma \), in the transient cavitating flow, the cloud cavity collapses near the hydrofoil, and the intensity of the cavitation erosion is very strong.
4.3. Velocity Dependence of Cavitation Erosion

The intensity of cavitation erosion varies with the flow conditions. In particular, the flow velocity has a significant influence on the intensity of cavitation erosion. The pitting rate, the intensity of cavitation erosion, and the amount of cavitation erosion are proportional to the fourth to ninth power of the flow velocity\(^{(30),(32)}\). The intensity of cavitation erosion in cavitating flows around the Clark Y 11.7% hydrofoil for various main flow velocities are predicted in order to investigate the velocity dependence. The initial bubble radius \(R_0\) is thought to influence the velocity dependence because \(R_0\) has a significant influence on the bubble translational motion and oscillation. Therefore, the simulations with various \(R_\infty\) are performed. A case of \(\sigma = 1.2\) is picked up in the calculations of all conditions, since the cloud cavity collapses near the hydrofoil at \(\sigma = 1.2\).

Figure 14 shows the relationship between the spatial average of \(I\) and \(U_\infty\) for various \(R_\infty\) at \(\sigma = 1.2\). The power approximation curves for each \(R_\infty\) (the relation \(I \propto U_\infty^n\)) are also described in Fig. 14. It is found that \(I\) is high when \(U_\infty\) and \(R_\infty\) are large and \(I\) is largely dependent on \(U_\infty\) at all \(R_\infty\). The exponents of the power approximation curves are from 4.3 to 7.0 and are near the experimental values\(^{(30),(32)}\). The exponents for each \(R_\infty\) are different, and the exponent is large when \(R_\infty\) is small. For each flow velocity, the rate of increase in \(I\) from \(R_\infty = 80 \mu m\) to \(R_\infty = 100 \mu m\) is lower than that from \(R_\infty = 60 \mu m\) to \(R_\infty = 80 \mu m\) and that from \(R_\infty = 40 \mu m\) to \(R_\infty = 60 \mu m\). This tendency is remarkable at large \(U_\infty\) (30 and 40 m/s). Therefore, it is thought that the increase in \(I\) owing to the increase in \(R_\infty\) converges. In addition, the convergence occurs at smaller \(R_\infty\) when \(U_\infty\) is larger.

Figure 15 shows the relationship between the maximum impact pressure \(P_{wmax}\) and \(U_\infty\) at various values of \(R_\infty\). The power approximation curves for each \(R_\infty\) are also described. The exponents of the curves are listed in Table 1. The maximum impact pressure is high when \(U_\infty\) and \(R_\infty\) are large as well as \(I\) (Fig. 14). However, the exponents are smaller than those of \(I\) because \(I \propto P_w^2\) and \(I\) is influenced by the number of impact pressure. In addition, the inverse tendency, whereby the maximum impact pressure for the case in which \(R_\infty = 100 \mu m\) is lower than that in the case of \(R_\infty = 80 \mu m\) in \(U_\infty = 30\) and 40 m/s, also exists. This inverse tendency indicates that the impact pressure has a local maximum value for large \(U_\infty\).

The impact pressure is dependent on the internal pressure during bubble collapse. Figure 16 shows the relationship between the maximum internal pressure of the bubble \(P_{max}\), which induces the maximum impact pressure, and \(U_\infty\) for various values of \(R_\infty\). The power approximation curves (Fig. 16), and the exponents (Table 1) are described. According to Figs. 15 and 16 and Table 1, the velocity dependence of the internal pressure is smaller than that of the
impact pressure, except for the case in which $R_o = 40 \mu m$, whereas the velocity dependence of the internal pressure at $R_o = 40 \mu m$ is large.

In order to investigate the oscillation characteristics, the single spherical bubble collapse was calculated using Eq. 16, and the relationship between the ambient pressure acting on the bubble and the internal pressure of the bubble are analyzed. The ambient pressure increases linearly with time from 3 kPa to the maximum ambient pressure $p_{am}$. Figure 17 shows the maximum internal pressure of the bubble, $p_{bmax}$ at various rates of pressure increase. Here, $p_{bmax}$ increases rapidly with the increase in the rate of pressure increase in a certain range for all of the calculation conditions. The rate of pressure increase is influenced by main flow velocity and bubble translational motion (the bubble with smaller $R_o$ follows well velocity field of macroscopic flow). Comparing the cases for which $R_o = 1,000 \mu m$ and $p_{am} = 0.1$ and 1.6 MPa reveals that the range of increase in $p_{bmax}$ at $p_{am} = 1.6$ MPa is greater than that at $p_{am} = 0.1$ MPa, and the maximum value of $p_{bmax}$ at $p_{am} = 1.6$ MPa is higher than that at $p_{am} = 0.1$ MPa. This indicates that $p_{bmax}$ is high when $p_{am}$ is high if the rate of pressure increase is sufficiently large. It is thought that $p_{am}$ is proportional to $U_{in}^2$, assuming that the ambient pressure acting on a bubble is proportional to the energy of the flow. Specifically, $p_{am}$ at $U_{in} = 40$ m/s is 16 times greater than that at $U_{in} = 10$ m/s, and, according to Fig. 17, $p_{bmax}$ at $U_{in} = 40$ m/s is more than 10 times greater than that at $U_{in} = 10$ m/s. The increase

Fig. 14 Relationship between the spatial average of the intensity of the cavitation erosion and the flow velocity ($\sigma = 1.2$)

Table 1 Exponents of the approximation curve ($\sigma = 1.2$)

<table>
<thead>
<tr>
<th>$R_o[\mu m]$</th>
<th>100</th>
<th>80</th>
<th>60</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ for $I \propto U_{in}^m$</td>
<td>4.3</td>
<td>5.5</td>
<td>5.7</td>
<td>7.0</td>
</tr>
<tr>
<td>$n$ for $P_{wmax} \propto U_{in}^m$</td>
<td>1.7</td>
<td>2.7</td>
<td>2.0</td>
<td>2.9</td>
</tr>
<tr>
<td>$n$ for $P_{bmax} \propto U_{in}^m$</td>
<td>0.76</td>
<td>0.96</td>
<td>0.51</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Fig. 15 Relation between the maximum impact pressure and the main flow velocity ($\sigma = 1.2$)

Fig. 16 Relationship between the maximum internal pressure of the bubble that induces the maximum impact pressure and the main flow velocity ($\sigma = 1.2$)
in the maximum ambient pressure is one of the reasons why the maximum internal pressure increases with the increase in $U_{in}$ (Fig. 16).

In order to investigate the more rapid increase of the maximum internal pressure at $R_{\infty} = 40 \mu m$ as compared to other values of $R_{\infty}$, the maximum internal pressure at various values of $R_{0}$ ($= 1,000, 100$ and $10 \mu m$) and $p_{am} = 0.1$ MPa are compared in Fig. 17. The comparison indicates that, when $R_{0}$ is small, the range of the increase in $p_{bmax}$ with the increase in the rate of pressure increase is at a higher rate of pressure increase. Therefore, a bubble with small $R_{0}$ has a high internal pressure only when the bubble is exposed to a higher pressure increase. Considering a bubble with velocity $U_{in}$ moving from the cavity region $x_1$ with pressure $p_1 \approx 0.5 \rho U_{in}^2 C_p + p_{in}$ to pressure recovery region $x_2$ with pressure $p_2 \approx 0.5 \rho U_{in}^2 C_p + p_{in}$, where $C_p$ is pressure coefficient, the pressure gradient from $x_1$ to $x_2$ is $(p_2 - p_1) / (x_2 - x_1) [\text{Pa/m}]$ and the rate of pressure increase acting on the bubble is $(p_2 - p_1) / (x_2 - x_1) U_{in} = 1/2 \rho \left( C_{p2} - C_{p1} \right) / (x_2 - x_1) U_{in}^3 [\text{Pa/s}]$. Therefore the rate of pressure increase is thought to be proportional to $U_{in}^3$. This indicates that the rate of pressure increase acting on a bubble becomes large when $U_{in}$ is large, and smaller bubbles have higher internal pressures. The increase in the maximum internal pressure with the increase in the rate of pressure increase is the reason why the maximum internal pressure at $R_{\infty} = 40 \mu m$ increases more rapidly than at other values of $R_{\infty}$ (Fig. 16). Therefore, it is thought that the smaller bubble is responsible for cavitation erosion when $U_{in}$ is large.

![Fig. 17 Maximum impact pressure and maximum internal pressure of the bubble that induces the maximum impact pressure](image)

5. Conclusion

A method of predicting the intensity of cavitation erosion using a one-way coupling method of analyses of homogeneous cavitating flow field and individual bubbles in the flow is proposed. The intensity of cavitation erosion is defined by the impact pressure induced by the spherical bubble collapse of individual bubbles. The Two-dimensional cavitating flow around a Clark Y 11.7% hydrofoil is used to examine the proposed prediction method. The velocity dependence on the cavitation erosion is analyzed using the proposed numerical prediction method. The results are summarized as follows:

1. In a transient cavitating flow, the bubbles that flow downstream as part of a cloud cavity induce a high impact pressure when the cloud cavity shrinks. The predicted intensity of cavitation erosion is high in the vicinity of sheet cavity termination.

2. The intensities of cavitation erosion in noncavitating, attached cavitating and pseudosupercavitating flows are far weaker than that at the transient cavitating flow. In the transient cavitating flow, the spatial distribution of high impact pressure becomes extensive when $\sigma$ is small, and the intensity of cavitation erosion becomes very strong when the cloud cavity collapses near the hydrofoil.

3. The intensity of cavitation erosion becomes high when the main flow velocity and the bubble radius are large. Exponent $n$ in the relationship between the intensity of cavitation erosion and the main flow velocity ($I \propto U_{in}^n$) becomes large when the bubble radius is small.
In the present study, the exponents range from 4.3 to 7.0 (standard bubble radii range from 100 μm to 40 μm).

(4) The maximum internal pressure of the bubble increases with the increase in the main flow velocity because of the increase in the maximum ambient pressure ($p_{am} \propto U_m^2$).

(5) Smaller bubbles have high internal pressures because of the increase in the rate of pressure increase (the rate of pressure increase $\propto U_m^3$). It is thought that smaller bubbles cause cavitation erosion when the main flow velocity is large.

Acknowledgement

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References


