Flow Structure of a Coaxial Circular Jet with Axisymmetric and Helical Instability Modes*

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Abstract
The flow structures in a coaxial jet with axisymmetric and helical instability modes for a comparatively low Reynolds number were investigated. The flow visualization and the measurements of velocities were carried out in an open water tank. In addition, three-dimensional numerical simulation of a coaxial jet was also performed using the commercial CFD software FLUENT 6.3. It was confirmed that the helical vortex was shed for the range of ratios of velocity of the inner to that of the outer jet from 0.5 to 1.0. Two characteristic flow regimes, i.e., axisymmetric and helical instability modes, were simulated in the flow field. For the coaxial jet with helical instability, the axial velocity along the centerline of the jet decreased more than that of the coaxial jet with axisymmetric instability. The axial velocity fluctuation at the centerline of the jet was small near the nozzle exit. However, the radial velocity fluctuation at that location increased. The convection velocity of vortices in the inner shear layer was larger than that for the outer shear layer. The convection velocity of vortices with the helical instability was slightly larger than that of the vortices with the axisymmetric instability. Consequently, the variation of velocity fluctuations and the convection velocity was associated with the arrangement of the vortex street in the shear layer.

Key words: Jet, Coaxial Circular Nozzle, Vortex, Axisymmetric Instability Mode, Helical Instability Mode, PIV, Numerical Simulation, Flow Visualization

1. Introduction
The coaxial jet (1)-(4) is a simple way by which two fluid streams can be mixed. Coaxial jets have applications in various fields such as combustion, chemical reactors and jet propulsion. The streams issuing from the annular nozzle and the central circular nozzle of a coaxial circular jet are referred to as the outer and the inner jets, respectively. The two shear layers that are present in coaxial jet flows correspond to the interfaces between the inner jet and the outer jet and between the outer jet and the ambient flow. These shear layers undergo interaction and mixing due to the Kelvin-Helmholtz instability. The mixing characteristics of the coaxial jet are determined by the dynamics and interactions of the vortical structures.
in these shear layers. The flow structures in the near field of such jets are complex and three-dimensional. The presence of vortical structures in the mixing regions of the coaxial jet had been reported by a number of researchers (e.g., Dahm \textit{et al.}(5); Rehab \textit{et al.}(6); Tang & Ko(7); Ko & Au(9); Au & Ko(10); Kiwata \textit{et al.}(11)). These investigations showed that the mean velocity ratio, which was defined as the ratio of the inner jet exit velocity to the outer jet exit velocity, governed the development of the coaxial jets. The results from the measurements of an unexcited coaxial jet with tabs performed by Kiwata \textit{et al.}(12) suggested that the mixing of the two fluid streams was related to the interactions between the axisymmetric and the streamwise vortices.

Three-dimensional vortical structures in developing circular jets were investigated by a number of researchers (e.g., Drubka \textit{et al.}(13); Kusek \textit{et al.}(14); Corke \textit{et al.}(15)(16); Toyoda \textit{et al.}(17)). In particular, the existence of the axisymmetric and the helical instability modes in an unexcited circular jet was confirmed by Drubka \textit{et al.}(13); Kusek \textit{et al.}(14) and Corke \textit{et al.}(16). The effects of the velocity ratio on the three-dimensional vortical structures in the near-field of coaxial jets were studied by Au & Ko(10). They showed that these structures had similar characteristics as the wake vortices in the basic annular jet and in the axisymmetric body (18). Although Kiwata \textit{et al.}(18) indicated the existence of the helical instability in a coaxial jet at a low Reynolds number by the flow visualization, the details of vortical structure and flow characteristics of two instability modes were not clear.

In addition, vortical structures in the wakes of axisymmetric bodies, such as rings, straight cylinders and spheres, have been investigated numerically and experimentally (e.g., Fuchs \textit{et al.}(19); Leweke & Provansal(20); Kiya \textit{et al.}(21); Johnson & Patel(22); Mittal & Najjar(23); Constantinescu & Squires(24)). Particularly, the wake and the three-dimensional vortical structures past the sphere have been investigated by many researchers (e.g., Cannon \textit{et al.}(25); Jang & Lee(26)). The existence of the helical structures in the wake of a sphere was experimentally demonstrated by Taneda(27). Hairpin-like vortices were also found in the wake of a sphere by Achenbach(28). Sakamoto and Haniu(29) classified the wake patterns of a sphere according to the Strouhal number in the range of the Reynolds numbers from $Re = 3 \times 10^3$ to $4 \times 10^4$. The shear layer instability was manifested as the higher branch of bifurcation of the Strouhal number plotted against the Reynolds number. The lower branch corresponded to the frequencies of the helical or the axisymmetric mode. Makita \textit{et al.}(30) also determined that the structure of the wake of a sphere depended on the Reynolds number, and that helical and alternating vortical modes existed for $Re \leq 2 \times 10^3$.

This paper presents the results of an experimental and numerical study of a coaxial jet. The effects of the vortical structure corresponding to axisymmetric and helical instability modes on the flow mixing and development of the coaxial jet were investigated at a relatively low Reynolds number. Flow visualization was performed using laser-induced fluorescence (LIF). In addition, particle image velocimetry (PIV) and hot-film probes were employed to measure the mean and the fluctuating flow velocities. A three-dimensional numerical simulation of the coaxial jet was also performed using the FLUENT 6.3 software(31) at the flow conditions corresponding to those of the experiment.

\textbf{Nomenclature}

\begin{itemize}
  \item $b$ : thickness of inner nozzle
  \item $D_i$ : inner diameter of inner nozzle
  \item $D_o$ : outer diameter of outer nozzle
  \item $f$ : vortex shedding frequency
  \item $L$ : length of outer nozzle
  \item $Q$ : second velocity gradient tensor
  \item $Re$ : Reynolds number based on $D_o$ and $U_o$
\end{itemize}
r : radial coordinate
St : Strouhal number based on $D_o$ and $U_{ave}$
t : time
$U_{ave}$ : average of the mean velocities ($U_i+U_o)/2$
$U_i$ : mean exit bulk velocity of the inner jet
$U_o$ : mean exit bulk velocity of the outer jet
$U_{omax}$ : maximum exit velocity of the outer jet
$U_i/U_o$ : mean velocity ratio
$u$ : axial velocity
$\bar{u}$ : mean axial velocity
$u'$ : axial velocity fluctuation
$u'_{rms}$ : r.m.s. value of axial velocity fluctuation
$\bar{u}_c$ : mean axial velocity along centerline of the jet
$u_{c,0.25}$ : mean axial velocity along centerline of the jet at $x/D_o=0.25$
$u_{conv}$ : convection velocity
$\bar{u}_{conv}$ : averaged convection velocity
$u'^u'$ : Reynolds stress
$v'$ : radial velocity fluctuation
$x$ : axial coordinate
$\zeta$ : non-dimensional vorticity
$\theta$ : angular coordinate
$\phi$ : phase of the vortex shedding frequency
$\omega_z$ : vorticity

2. Apparatus and techniques

2.1 Experimental Setup

A schematic of the experimental apparatus is shown in Fig. 1. The experimental setup consisted of a coaxial axisymmetric water jet discharging into an open tank (435 mm × 435 mm × 1000 mm), where the fluid (water) was initially at rest. Two constant head tanks were used to supply water flow to the outer and the inner nozzles. The flow rate was maintained at a prescribed value by valves and electro-magnetic flowmeters. A schematic diagram of the coaxial nozzle is shown in Fig. 2. The outer (annular) and the inner (circular) nozzles were made of acrylic resin. The outer nozzle had a 23.5:1 contraction with an outer diameter $D_o$ of 40 mm at the jet exit. The inner nozzle had an inner diameter $D_i$ of 16 mm. The pipe forming the inner nozzle was sufficiently long to yield a fully developed velocity profile at the exit plane of the inner jet. The length of outer nozzle was varied from $L/D_o = 0$ to 2. Since the present coaxial nozzle with the protruded outer nozzle is modeled on the

![Fig.1 Schematic diagram of experimental apparatus](image)
main nozzle of an air jet loom, the present coaxial jet consists of the annular jet with a potential core and the central circular jet with a fully developed velocity profile. The experiments were conducted with a mean exit bulk velocity $U_o$ of the outer jet of approximately 80 mm/s. The Reynolds number based on the outer diameter, $D_o$, and the mean exit bulk velocity, $U_o$, of the annular jet was equal to $Re = 3.0 \times 10^3$. The mean velocity ratio of the inner and the outer jets was varied in the range of $0.1 \leq U_i/U_o \leq 1.0$, where $U_i$ is the mean exit bulk velocity of the inner jet. The value of the boundary layer displacement thickness at the inner surface of the outer nozzle, $\delta^*$, was approximately 1.4 mm.

2.2 Experimental Method

Flow visualization was performed using a laser-induced-fluorescence (LIF) technique. The near field of the jet was illuminated by an argon ion laser (Stabilite2017, Spectra-Physics) equipped with a cylindrical lens. The inner and the outer shear layers were marked with two different laser-fluorescent dyes, i.e., an orange dye (aqueous solution of Rhodamine B) and a green dye (aqueous solution of disodium fluorescence), respectively. The ambient fluid contained no dye and appeared black in the photographs. The photographs presented in this paper were captured from cinema sequences recorded via a digital video camera (DSR-40, SONY) at a frame rate of 30 Hz.

The technique of two- and three-dimensional (stereoscopic) particle image velocimetry (PIV) was employed in order to measure the mean and the fluctuating velocities. The working fluid was seeded with acrylonitrile spherical particles with the mean diameter of 20 µm. A light sheet generated with a double-plused Nd:YAG laser (Big Sky Laser Technologies) illuminated the near field of the jet. A continuous sequence of image pairs (up to 200 frames at a framing frequency of 8Hz) was acquired at a resolution of 1599 x 1185 pixels with the CCD video cameras (FlowScense M2 10bit, Dantec Dynamics). The corresponding spatial resolution of the physical field of view was approximately 0.087 mm/pixel. The CCD cameras and the double-pulsed laser were connected to a personal computer, which controlled the timing of the illumination and of the image acquisition. The image pairs were processed on the computer to yield instantaneous distributions of flow velocity vectors. The time interval between the two images in each pair ($\Delta t = 8.0 \text{ ms}$) was interactively adjusted to minimize the number of erroneous vectors in the calculated velocity field.

In addition, measurements of the mean and the fluctuating velocities were performed with I-type probes (MODEL 1210-60W, TSI) and constant-temperature hot-film anemometers (MODEL 1011, 1013, KANOMAX) with linearized output. The spectrum analyses were obtained using an FFT analyzer (ONO SOKKI CF-5210).

2.3 Computational Techniques

The commercial CFD software, FLUENT 6.3 (ANSYS Japan K.K.)\(^{(31)}\), was used to compute the unsteady three-dimensional flow. The flow was assumed to be viscous, laminar and incompressible. The governing equations were the Navier-Stokes and the continuity equations in cylindrical coordinates. The computational domain was discretized using a finite volume method. The convection terms of the governing equation were discretized using the QUICK scheme, and the other spatial derivative terms were discretized using a second-order central differencing scheme. The second-order implicit scheme was used for the time marching. The implicit algorithm of the PISO (pressure-implicit with splitting of operators) method was applied for the pressure-velocity coupling.

The computational domain and the boundary conditions are shown in Fig. 3. The diameter and the length of the calculation domain were equal to 400 mm ($=10D_o$) and 680 mm ($=17D_o$), respectively. The pipe of the inner nozzle had a length of 80 mm ($=2D_o$). The number of grid points in the $x$, $r$, and $\theta$ directions were 186, 95, and 64, respectively.
of approximately $10^6$ grid points were distributed nonuniformly in the computational domain.

The working fluid in the calculation was water, with the density of 998.2 kg/m$^3$ and the viscosity of $1.003 \times 10^{-3}$ Pa·s. Two values of the Reynolds numbers were considered: $Re = 3.0 \times 10^3$ and $5.0 \times 10^3$. The mean velocity ratio of the inner and the outer jets was varied in the range of $0.4 \leq U_i/U_o \leq 1.0$. The inlet boundary condition of the inner jet corresponded to a Poiseuille velocity profile. Although a uniform velocity profile was specified at the inlet boundary corresponding to the outer jet, the velocity profile at the exit of annular nozzle ($x/D_o = 0.25$) agreed with the velocity profile in the experiment approximately. To ensure stability of the calculation, a uniform velocity with a magnitude of $0.02U_o$ was specified at the upstream boundary of the domain outside of the coaxial jet. The walls of the inner and the outer nozzles corresponded to the no-slip boundary condition. The periphery of the computational domain corresponded to the slip condition, where the gradients of all variables were set to be equal to zero. The pressure outlet condition of $\Delta p = 0$ was specified at the downstream boundaries of the domain. For the helical instability mode, a time-dependent velocity condition was adopted on the inlet boundary of the exit of inner nozzle. For example, to generate the coaxial jet of the helical instability mode and the velocity ratio of $U_i/U_o = 0.65$, as shown in Fig. 4, the initial inlet boundary condition of the coaxial jet corresponded to the velocity profile of $U_i/U_o = 1.0$ because the vortical structure of the helical instability mode always appeared at this ratio. After 0.5 sec, the mean exit bulk velocity, $U_i$, of the inner jet was decreased linearly from $1.0U_o$ to $0.65U_o$, and the vortical structure of the helical instability mode was simulated.

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**Fig.2** Schematic diagram of coaxial nozzle

**Fig.3** Computational domain and boundary conditions

**Fig.4** Time-dependent inlet condition for helical instability mode
3. Results and Discussion

3.1 Flow visualization of the coaxial jet

The three-dimensional flow patterns corresponding to the \(L/D_o = 0\) nozzle for \(U_i/U_o = 0.2\) and 0.8 at \(Re = 3.0 \times 10^3\) are shown in Fig. 5. These images are taken from the downstream of the jet exit, with the camera oriented diagonally with respect to the jet axis. To visualize the flow, two different laser-fluorescent dyes, i.e., a fluorescent yellowish-orange dye (aqueous solution of Rhodamine B) and a fluorescent yellowish-green (aqueous solution of disodium fluorescein), were injected through a narrow slit along the perimeter of the inner and outer nozzles (pipes). For the velocity ratio of \(U_i/U_o = 0.2\), the vortex rings of the axisymmetric instability mode are shed periodically in the inner and outer shear layers. In the downstream, the vortex-paring event occurs in the outer shear layer. On the other hand, for \(U_i/U_o = 0.8\), the vortices of the helical instability mode in the both shear layers are shed continuously, as twisted strings.

Figure 6 show the side view of flow patterns corresponding to two instability modes of the \(L/D_o = 0.25\) nozzle for \(U_i/U_o = 0.65\) at \(Re = 3.0 \times 10^3\). A fluorescent yellowish-green dye was injected into the only inner shear layer. A numerical simulation of the coaxial jet was also performed at the same flow conditions of the experiment. Three-dimensional vortical structures, which are visualized by the iso-surface of second velocity gradient tensor \(Q\), are shown in Fig. 7. As shown in Figs. 6 and 7, two vortical structures for both the axisymmetric and the helical instability modes were appeared at the velocity ratio of 0.65. The flow patterns of the numerical simulation and the experiment are in good agreement. The vortical structures in the inner shear layer began to collapse downstream of \(x/Do = 1.25\) and concentrated around the centerline of the jet. The vortex shedding frequency \(f\) was equal to 2.8 Hz for both the axisymmetric and the helical instability modes at \(U_i/U_o = 0.65\).

3.2 Vortex shedding frequency

The time variations of the axial velocity fluctuations \(u'\) in the mixing regions (\(x/Do = 1.0\)) for both the axisymmetric and the helical instability modes are shown in Figs. 8 and 9. Figure 8 presents the experimental result of the \(L/D_o = 0.5\) nozzle at \(U_i/U_o = 0.5\). Figure 9 presents the simulated result of the \(L/D_o = 0.25\) nozzle at \(U_i/U_o = 0.65\). The axial velocities in the mixing region fluctuate periodically. For the axisymmetric instability mode, the velocities of the two points in the inner mixing region oscillate in the same phase. For the helical instability mode, the velocity fluctuations between the two points are out of phase. The distributions of the frequencies of velocity fluctuations with different velocity ratio at
Re = 3.0 × 10^3 are shown in Fig. 10. The vortex shedding frequency, $f$, increases with the increase of the mean velocity ratio, and do not depend on the outer pipe length $L$. The phase difference of the axial velocity fluctuations at the axisymmetric points in the inner and the outer mixing regions ($x/D_o = 1.0$, $r/D_o = ±0.25$ and $±0.5$) is shown in Fig. 11. The phase difference between two points in the experiment was measured using an FFT analyzer. The result of numerical simulation is also plotted in Fig. 11. The difference in the phase angle between the axisymmetric and the helical instability modes was approximately 180 degrees. It is confirmed by the measurements of phase difference and the flow visualizations that the vortex of the helical instability mode appeared for the velocity ratios $0.5 \leq U_i/U_o \leq 1.0$ ($Re = 3.0 \times 10^3 ~ 5.0 \times 10^3$). The variation of the Strouhal number, $St$, corresponding to the dominant frequency of the vortices with the velocity ratio for the $L/D_o = 0.25$ nozzle is shown in Fig. 12. The Strouhal number was based on the outer nozzle diameter $D_o$ and the averaged velocity of the inner and the outer jets $U_{ave} = (U_i + U_o)/2$. The Strouhal numbers corresponding to the experiment and the numerical simulation are in good agreement. It can be observed that the Strouhal number was maintained at an approximately constant value of about 1.75 for $U_i/U_o = 0.4 ~ 0.8$.

### 3.3 Mean velocities and velocity fluctuations

In order to compare the flow characteristic of the coaxial jet with the axisymmetric and the helical instability modes, the flow field of the coaxial jet was investigated at the mean velocity ratio of 0.65 for the $L/D_o = 0.25$ nozzle. Figure 13 shows profiles of the experimental and the simulated mean axial velocity $\bar{u}/u_o$ for the axisymmetric and the helical instability modes. The axial velocities were normalized by the mean bulk velocity of the outer jet $U_o$. The experimental data were obtained by two-dimensional PIV. In the case of the helical instability mode, the mean axial velocity decreased near the centerline of jet at the downstream position of $x/D_o = 0.75 ~ 1.0$. This decrease in the axial velocity near the centerline of jet was not observed in the case of the axisymmetric instability mode. The difference between centerline velocities corresponding to the axisymmetric and the helical instability modes is also presented in Fig. 14, where $\bar{u}_c$ and $\bar{u}_{0.25}$ are defined as the mean
profiles of the root-mean-square (r.m.s.) values of the axial and radial velocity fluctuations \(u'_{\text{rms}}/U_o\) and \(v'_{\text{rms}}/U_o\) corresponding to the axisymmetric and helical instability modes are shown in Figs. 15 and 16. Although the experimental profiles of velocity fluctuation do not exhibit pronounced axisymmetry in comparison with those of mean axial velocity (Fig. 13), the difference in the velocity fluctuations between the axisymmetric and the helical instability modes became apparent near the centerline of jet at \(x/D_o = 0.5 \sim 1.5\). The axial velocity fluctuations corresponding to the helical instability mode decreased at the downstream locations corresponding to \(x/D_o = 0.5~1.5\).
The magnitude of the peak vorticity decreased, as the corresponding vortices were advected from the vorticity distributions of the numerical simulation. In the outer shear layer, the formation and diffusion of vortices in the near field of coaxial jet can be predicted by the distributions of mean axial velocity (Figs. 13 and 14). The experimental results of vorticity distributions are in approximate agreement with these figures.

Moreover, the region of large intensity of velocity fluctuations assumed a ring-like shape in the case of the helical instability mode for the experiment. Thus, the rolled-up axisymmetric vortex rings in the shear layers induces the axial velocity fluctuation near the center of jet. This phenomenon is related to the difference in the vortical structures corresponding to the axisymmetric and the helical instability modes, which are discussed in the next section.

3.4 Vortical structures for the axisymmetric and the helical instability modes

In this section, the difference of the vortical structure between the axisymmetric and the helical instability modes will be discussed. Instantaneous vorticity contours in the $x-r$ cross sectional plane of the coaxial jet at $U_i/U_o = 0.65$ are shown in Fig. 18. Well-defined peaks of vorticity can be observed in the shear layers at the nozzle exit. Single vortex street formed in the outer mixing region, and two vortex streets formed in the inner mixing region from the lip of inner nozzles, respectively. The formation and diffusion of vortices in the near field of coaxial jet can be predicted by the distributions of mean axial velocity (Figs. 13 and 14). The experimental results of vorticity distributions are in approximate agreement with the vorticity distributions of the numerical simulation. In the outer shear layer, the magnitude of the peak vorticity decreased, as the corresponding vortices were advected from the vorticity distributions of the numerical simulation.
downstream. The vortices in the inner mixing region interacted with those in the outer mixing region. This interaction resulted in a rapid decrease of the magnitude of the vortices in the inner mixing region at the downstream locations corresponding to $x/D_o < 1.5$. While the cases of the axisymmetric and the helical instability modes were qualitatively similar in

![Profiles of r.m.s. value of axial velocity fluctuations at x-r cross section](image1.png)

Fig. 15 Profiles of r.m.s. value of axial velocity fluctuations at x-r cross section for $U_i/U_o = 0.65$, $L/D_o = 0.25$ nozzle

![Profiles of r.m.s. value of radial velocity fluctuations at x-r cross section](image2.png)

Fig. 16 Profiles of r.m.s. value of radial velocity fluctuations at x-r cross section for $U_i/U_o = 0.65$, $L/D_o = 0.25$ nozzle

![Contours of r.m.s. value of axial velocity fluctuations at r-θ cross section](image3.png)

Fig. 17 Contours of r.m.s. value of axial velocity fluctuations at r-θ cross section for $U_i/U_o = 0.65$, $L/D_o = 0.25$ nozzle
terms of the corresponding vortical structures, in the former case, well-defined concentrations of vorticity in the outer shear layer persisted farther downstream from the jet exit. The vortical structures in the inner and outer mixing regions are denoted by ‘I’ and ‘O’, and the ‘+’ and the ‘−’ signs represent the counterclockwise and the clockwise direction of fluid rotation, respectively.

The trajectories of the vortices in the inner and the outer mixing regions of the coaxial jet by the numerical simulation are plotted in Fig. 19 at intervals of 0.1 sec. The positions of the vortices were obtained by tracking the corresponding local peaks of vorticity in the outer mixing region (Vortices ±O) and in the inner mixing region (Vortices ±I1, ±I2 ±I2a and ±I2b). In the outer mixing region, the vortex trajectories corresponding to the axisymmetric instability mode were similar to those corresponding to the helical instability mode. In the inner mixing region, the trajectories of the vortices ±I1 corresponding to the axisymmetric and the helical instability modes diverged slightly, and then at $x/D_o > 1.0$, the vortex with the helical instability moved inside in comparison with the trajectory of the vortex with the axisymmetric instability. In the helical instability mode, the vortices ±I2 bifurcates into two smaller structures, i.e., Vortices ±I2a and ±I2b, and the vortices ±I2b are convected across the centerline of the jet. At $x/D_o = 1.25$, the vortices ±I2b decay and disappear. The increments of the radial velocity fluctuation near the centerline of coaxial jet with the helical instability are related to the behavior of the bifurcated vortices ±I2b.

Fig. 18 Vorticity contours in the coaxial jet at $x-r$ cross section for $U_i/U_o=0.65$, $L/D_o=0.25$ nozzle
The time variations of the axial positions of vortices by the numerical simulation are shown in Fig. 20. The axial position of the vortices increased almost linearly. The variations of the convection velocity and the local peak of vorticity of the vortices with the axial position by the numerical simulation are shown in Figs. 21 and 22. The convection velocities of vortices $\pm O$ in the outer mixing for both the axisymmetric and the helical instability modes were lower than those of vortices in inner mixing region downstream of $x/D_o > 0.5$. The vortices $\pm I_1$ and $\pm I_2$ with the axisymmetric instability in the inner mixing region accelerated rapidly at $x/D_o = 1.2$. These vortices exhibited peak convection velocities near $x/D_o = 1.5$. At $x/D_o = 0.8$, the vorticity in the inner mixing region exhibited a local maximum, while the vorticity in the outer mixing region decreased gradually with increasing $x/D_o$. The average convection velocities, which were normalized by the maximum exit velocity of the outer jet $U_{omax}$, observed in the region from $x/D_o = 0.25$ to 2.75 are shown in Table 1. For both instability modes, the average convection velocities in the inner mixing region were greater than those in the outer mixing region. In the inner mixing region, the average convection velocities for the helical instability mode were slightly greater than those corresponding to the axisymmetric instability mode. It is for this reason that the vortex of the helical instability mode moves inside. It should be noted that the values of the convection velocity of the coaxial circular jet with the axisymmetric instability at a comparatively low Reynolds number are similar to those of the coaxial jet at a high Reynolds number (Au & Ko 1987).
Instantaneous three-dimensional vortical structures by the numerical simulation, i.e.,
the iso-surfaces of the second velocity gradient tensor $Q$ are shown in Fig. 23. In the case
of the axisymmetric instability mode, the two vortices (vortices $-I_1$ and $+I_2$) in the inner
mixing region expanded axially at $x/Do = 1.0 \sim 1.5$. At this point, vortex $-I_1$ conveys with
the jet exit velocity (as shown in Fig. 18) due to the interaction with the counterclockwise
vortex $+O$ in the outer mixing region. Farther downstream, the vortical structure splits into
several smaller structures. As shown in Fig. 22 (a), the decay rate of vorticity in the inner
mixing region was greater than that in outer mixing region. In the case of the helical
instability mode, the three vortices (vortices $+O$, $-I_1$ and $+I_2$) were found in the outer and
the inner mixing regions. The convection velocity of vortex $-I_1$ increased due to the

![Fig. 21 Instantaneous axial convection velocities for $U_i/U_o = 0.65$, $L/Do = 0.25$
nozzle (CFD)](image)

(a) Axisymmetric instability mode
(b) Helical instability mode

![Fig. 22 Attenuation of the local peaks of vorticity for $U_i/U_o = 0.65$, $L/Do = 0.25$
nozzle (CFD)](image)

(a) Axisymmetric instability mode
(b) Helical instability mode

Table 1 Average convection velocities from $x/Do = 0.25$ to 2.75

<table>
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<th>Instability mode</th>
<th>Mixing region</th>
<th>$u_{conv}/U_{max}$</th>
<th>Present</th>
<th>Au &amp; Ko (1987)</th>
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<td>Inner mixing region</td>
<td>0.682</td>
<td>0.6 ~ 0.7</td>
<td></td>
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<tr>
<td></td>
<td>Outer mixing region</td>
<td>0.534</td>
<td>0.5 ~ 0.6</td>
<td></td>
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<td>Helical</td>
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<td></td>
<td></td>
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<td></td>
<td>Outer mixing region</td>
<td>0.549</td>
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interaction with the counterclockwise vortex +O in the outer mixing region. The maximum convection velocity of the helical instability mode in the inner mixing region was not larger than that of the axisymmetric instability mode. However, the average convection velocity of the helical instability mode in the inner mixing region was larger than that of the axisymmetric instability mode (as shown in Table 1). In the downstream, a phenomenon of vortex coalescence was observed in the flow patterns of the helical instability mode, and the vortex structure became very complex. Thus, the mixing characteristics of a coaxial jet were determined by the dynamics and interactions of the three-dimensional vortical structures in the mixing region.

4. Conclusions

The effects of the vortical structures corresponding to the axisymmetric and the helical instability modes on the flow mixing and development of a coaxial jet at relatively low Reynolds numbers were investigated by the experiment and the numerical simulation. As a result, the following conclusions can be drawn:

Fig. 23 Instantaneous three-dimensional vortex structure for $U_i/U_o = 0.65$, $L/D_o = 0.25$ nozzle (CFD)
(1) The coaxial circular jet with the helical instability appeared for the velocity ratios $0.5 \leq U_i/U_o \leq 1.0$ ($Re = 3.0 \times 10^3 \sim 5.0 \times 10^3$). Three-dimensional numerical simulation of the coaxial circular jet was able to obtain both vortical structures corresponding to the two instability modes.

(2) The vortex shedding frequency was in good agreement between the experimental results and the numerical simulation. The Strouhal number based on the averaged bulk velocity had a constant value of about 1.75 for the range of $U_i/U_o = 0.4 \sim 0.8$.

(3) The characteristic difference between the two instability modes was observed in the near-field of the coaxial circular jet. For the helical instability mode, the mean axial velocity decreased in the vicinity of the centerline of the jet than that for the axisymmetric instability mode. For the axisymmetric instability mode, the axial velocity fluctuation magnitude increased in the vicinity of the centerline of the jet. However, for the helical instability mode, the magnitude of the radial velocity fluctuation increased in that region.

(4) The average convection velocities in the inner mixing region were greater than those in the outer mixing region for both instability modes. In the inner mixing region, the average convection velocities for the helical instability mode were slightly greater than those for the axisymmetric instability mode.

(5) It is found that the flow characteristic difference between the two instability modes near the centerline of jet is related to the behavior of the vortices in the inner mixing region. In the axisymmetric instability mode, a vortex is convected parallel to the centerline of the jet approximately. On the other hand, in the helical instability mode, a vortex bifurcates into two smaller vortices and these vortices are convected across the centerline of the jet.

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