Experimental Study of Adaptive Control of High-Speed Flow-Induced Cavity Oscillations*


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Abstract
This paper describes progress towards the development of an active suppression system for open cavity oscillations. A leading-edge array of piezoelectric zero-net mass-flux (ZNMF) actuators is used to suppress cavity flow oscillations determined from measured pressure fluctuations near the rear wall of cavities with length-to-depth ratios (L/D) of 5 and 6 at Mach 0.3 and 0.4. The waveform parameters of the actuator array excitation signal are systematically investigated using open-loop control. Closed-loop control using the quasi-static downhill simplex and the dynamic ARMARKOV adaptive disturbance rejection and generalized predictive control (GPC) algorithms are applied and compared to open-loop control. Up to 30% broadband reduction in rms pressure fluctuations over a 4 kHz bandwidth and reduction in multiple Rossiter modes are obtained and compare favorably with optimized open-loop sinusoidal control. Through the investigation of different actuator slot geometries, larger slots which generate larger momentum with a forcing frequency near the resonant frequency of the actuator showed the best performance for suppressing the cavity oscillation.

Key words: Cavity Oscillations, Adaptive Control, Zero-Net Mass-Flux Actuator, ARMARKOV, GPC, Downhill Simplex

1. Introduction

Flow-induced cavity oscillation problems have attracted numerous researchers in both fluid dynamics and controls over the last few decades. These oscillations exist in many practical environments, such as sunroofs in automobiles, landing-gear and weapon bays on aircraft.\(^{(1)}\) The pressure fluctuations can exceed 170 dB (reference pressure of 20 μPa) and can potentially cause fatigue failure of the cavity structure and its contents. In addition, the cavity pressure fluctuations can increase drag.\(^{(2)}\) These pressure fluctuations contain tonal and broadband components, and both need to be suppressed for many practical applications.

The physical mechanisms responsible for flow-induced oscillations in an open cavity are illustrated in Fig. 1. At the leading edge of the cavity, the boundary layer separates and forms a free shear layer. Disturbances in the free shear layer amplify and convect downstream, ultimately growing into large-scale vortical structures. In an open cavity, these coherent structures span the length of the cavity and impinge near the trailing edge corner of the cavity. This interaction region acts as an acoustic source that radiates acoustic waves, which travel upstream and force the shear layer through a receptivity process. The overall dynamics produce resonance with strong cavity tones and large broadband turbulent fluctuations. The tones are often referred to as “Rossiter” modes.\(^{(3)}\)
Cavity flow control methods can be classified as either passive or active. Passive methods utilize geometric modifications such as leading-edge spoilers or fences, while active methods require external energy to control the flow. Active control is further subdivided into open-loop versus closed-loop control, which uses sensor feedback. Closed-loop control can be further classified into quasi-static control, where the time scale of the controller is much larger than the time scales of the oscillations, and dynamic control, where the time scale of the control system (i.e., actuators, sensors, and control algorithm) is commensurate with the dynamics.

Attempts at active control of flow-induced cavity oscillations have been ongoing for the past 40 years. A detailed review of various active control methodologies that have been applied to suppress the cavity oscillations is provided in Cattafesta et al. In some of the control methodologies, three-dimensional excitation is used at the leading edge to alter the cavity shear layer, which in turn affects the aft-wall source characteristics. Kegerize et al. used a piezoelectric bimorph actuator and the GPC control method and successfully suppressed the multiple Rossiter modes. However there was no effect for broadband noise level. Cattafesta et al. suggested that fundamental and comprehensive studies on the cavity flow field and its response to open- and closed-loop control are needed to develop efficient and effective control strategies to suppress both the multiple Rossiter modes and broadband noise level simultaneously.

The goal of this study is to understand the effects of open-loop (OL) and closed-loop (CL) control on cavity oscillation suppression of both tonal and broadband noises, with particular emphasis on three-dimensional forcing strategies. Specifically, open-loop control using sinusoidal excitation signals and adaptive closed-loop control – using quasi-static Downhill Simplex (DS) and dynamic ARMAROCV and GPC algorithms – are explored with a three-dimensional zero-net mass-flux (ZNMF) piezoelectric actuator array to suppress the unsteady pressure oscillations inside the cavity at Mach numbers of 0.3 and 0.4.

The paper is organized as follows. Section 2 briefly describes the control algorithms used in this study. Section 3 outlines the experimental setup and data analysis procedures. The experimental results and discussion are presented in Section 4. This is followed by a short section that summarizes the findings and offers directions for future research.

2. Control Algorithms

This section outlines the three closed-loop control algorithms used to compare to open-loop control in this study. The algorithms include downhill simplex, ARMAROCV adaptive disturbance rejection and GPC. For each of these applications, the control goal is to minimize the fluctuating surface pressure of the cavity using ZNMF actuators at the leading edge of the cavity.

2.1 Downhill Simplex Algorithm

The DS method is a minimization process that is capable of optimizing N-dimensional
systems. It minimizes an objective function and is similar to extremum seeking (see Maury et al.\textsuperscript{(7)} and the references therein) in that a time-averaged extremum of the cost function is assumed to exist. For an open cavity flow, the objective function is the integrated pressure fluctuation near the trailing edge of the cavity over a prescribed frequency range (i.e., 100 Hz – 4.0 kHz in this study). The algorithm aims to minimize the objective function by changing the relevant waveform parameters of the actuators input such as amplitude and frequency.

The downhill simplex algorithm is a simple method and is attractive in experiments since it only requires evaluation of the cost function and not its derivatives.\textsuperscript{(8)} To understand the algorithm, we first define a simplex as a geometrical structure which consists of $N + 1$ vertices (in $N$ dimensions). For instance, a simplex in two dimensions is a triangle. The downhill simplex algorithm makes use of the geometrical concept of a simplex and searches downhill in a straightforward fashion that makes no prior assumptions about the function. It can thus potentially be slow. However, it is guaranteed to find a local minimum,\textsuperscript{(8)} as shown in Fig. 2. The algorithm modifies the simplex repeatedly according to following procedure:

1) Given initial values of a simplex of $N+1$ vertices for $N$-dimensional vector $\mathbf{x}_K$, which contains the parameters to be optimized, order the points so that

$$f(\mathbf{x}_{n+1}) > f(\mathbf{x}_2) > \cdots > f(\mathbf{x}_n) > f(\mathbf{x}_1).$$  \hspace{1cm} (1)

2) Reflect the vertex associated with the largest function evaluation about the simplex centroid to generate a reflected trial point $\mathbf{x}_r$

$$\mathbf{x}_r = \mathbf{x}_{\text{cen}} + m(\mathbf{x}_{\text{cen}} - \mathbf{x}_{n+1}),$$  \hspace{1cm} (2)

where $\mathbf{x}_{\text{cen}} = \frac{1}{N+1} \sum_{i=1}^{N+1} \mathbf{x}_i$ is the centroid and $m > 0$.

3) Compute $f(\mathbf{x}_r)$.

a. If

$$f(\mathbf{x}_r) < f(\mathbf{x}_1),$$  \hspace{1cm} (3)

i.e., $\mathbf{x}_r$ is the new optimum, then the assumed direction of reflection is correct, so expand the search in this direction.

$$\mathbf{x}_e = \mathbf{x}_r + \alpha(\mathbf{x}_r - \mathbf{x}_{\text{cen}}),$$  \hspace{1cm} (4)

where $\alpha > 0$. If $f(\mathbf{x}_r) < f(\mathbf{x}_1)$, then replace $\mathbf{x}_{n+1}$ by $\mathbf{x}_r$, otherwise, the expansion fails, so replace $\mathbf{x}_{n+1}$ by $\mathbf{x}_r$. Else, proceed to step 3b.

b. If
the reflection fails. So generate a new point by contraction
\[ x_c = \bar{x}_{con} + \beta (\bar{x}_{n+1} - \bar{x}_{con}) \]
where \( 0 < \beta < 1 \). If \( f(\bar{x}_c) < f(\bar{x}_{n+1}) \), then replace \( \bar{x}_{n+1} \) by \( \bar{x}_c \), otherwise contract again.

2.2 ARMARKOV Adaptive Disturbance Rejection Algorithm

The ARMARKOV disturbance rejection algorithm was introduced in Venugopal and Bernstein\(^9\) and had been used by Cattafesta et al.\(^{10}\) for control of cavity oscillations and Tian et al.\(^{11}\) for control of flow separation. The algorithm uses a linear model that is identified simultaneously using the recursive ARMARKOV/Toeplitz system identification algorithm. Readers are referred to Venugopal and Bernstein\(^9\) for details of the derivation of the algorithm.

Figure 3 shows a block diagram of a standard disturbance rejection problem, where \( w \) is the unknown disturbance input, \( u \) is the control signal, \( y \) is the reference signal, \( z \) is the performance signal and \( G_c \) is the disturbance rejection controller. In addition, for the four transfer matrices, \( G_{zw} \) is the primary path, \( G_{zu} \) is the secondary path, \( G_{yw} \) is the reference path, and \( G_{yu} \) is the feedback path. The secondary path \( G_{zu} \) is required to be identified by online or offline system identification methods. Figure 4 shows the block diagram of the application of the ARMARKOV disturbance rejection algorithm to control cavity oscillations, where system identification uses either band-limited white noise or a periodic chirp signal as the input to the actuator. The signals \( z \) and \( y \) can be measured using unsteady pressure sensors, which are presumed to be linear combinations of an unknown disturbance \( w \) and an actuator control input \( u \). The signal \( y \) is used for feedback, while the signal \( z \) is used for performance evaluation. In this study, both signals are measured by an unsteady pressure sensor which is located near the trailing edge. It is assumed that the actuator signal \( u \) is a linear combination of past actuator inputs and current and past measured unsteady pressure sensor outputs. The goal of the adaptive controller is thus to select optimal weights to minimize the power of the fluctuating pressure. The weights should change slowly compared to the fluid dynamic time scales, so the cost function with respect to the weights is minimized to obtain the adaptive...
control law. The weights are adjusted in an iterative fashion in real time using a gradient descent algorithm. The controller requires the following parameters as inputs: the order of the system $n$, the number of Markov parameters for system identification $\mu$, the order of the controller $n_c$, the number of controller Markov parameters $\mu_c$, the parameter $p$ that controls the memory of the controller to past events, and the sampling rate. Increasing parameters ($n$, $\mu$, etc.) may improve performance but will significantly increase the computational costs.\textsuperscript{(10)} Moreover, the sampling rate and other parameters ($n$, $\mu$, etc.) must be tuned to acquire a balance between temporal resolution and performance.

### 2.3 Generalized Predictive Control (GPC) Algorithm

The GPC algorithm is introduced in Clarke et al.\textsuperscript{(12)(13)} The GPC algorithm is based on system output predictions over a finite time horizon. An appropriate cost function is then defined using these predictions and minimized to determine the control law. The GPC algorithm has been applied to active vibration and noise control. Kegerise et al.\textsuperscript{(5)} also demonstrated the potential for this control approach to suppress cavity oscillations.

Briefly, to develop an adaptive GPC algorithm, a predictive matrix equation that provides future system output predictions is formed

$$y_s(k) = Tu_s(k) + Bu_p(k-p) + Ay_p(k-p),$$  \textsuperscript{(7)}

where $y_s(k)$ is a vector of current and future outputs (superscript $T$ below denotes transpose):

$$y_s(k) = \left( y(k) \quad y(k+1) \quad \ldots \quad y(k+s-1) \right)^T.$$  \textsuperscript{(8)}

$u_s(k)$ is a vector of current and future inputs:

$$u_s(k) = \left( u(k) \quad u(k+1) \quad \ldots \quad u(k+s-1) \right)^T.$$  \textsuperscript{(9)}

$y_p(k-p)$ is a vector of past outputs running from time step from $k-p$ to $k-1$:

$$y_p(k-p) = \left( y(k-p) \quad y(k-p+1) \quad \ldots \quad y(k-1) \right)^T.$$  \textsuperscript{(10)}

and $u_p(k-p)$ is a vector of past inputs running from time step from $k-p$ to $k-1$:

$$u_p(k-p) = \left( u(k-p) \quad u(k-p+1) \quad \ldots \quad u(k-1) \right)^T.$$  \textsuperscript{(11)}

Equation (7) shows that the current and future outputs (with a prediction horizon $s$) are a linear combination of the current and future inputs and past input/outputs. The parameter matrices ($T$, $B$ and $A$) in the equation are determined through system identification in a time-dependent fashion.

To reject the disturbance at the system output, an appropriate scalar cost function for the GPC algorithm is given by

$$J(k) = \frac{1}{2} \left( y_s^T(k)Qy_s(k) + u_s^T(k)Ru_s(k) \right).$$  \textsuperscript{(12)}

The first term in the cost function is a weighted sum of the squares of current and future outputs. The second term acts as a penalty on the control effort. This term is necessary to avoid large control inputs and actuator saturation. To determine the control law, the cost function is minimized with respect to the control input. Substituting Eq. (7) into (12) and
minimizing the result with respect to \( u_s(k) \) leads to

\[
\left. u_s \right|_{opt} = \left[ H \right] \left[ u_p(k-p)^T \quad y_p(k-p)^T \right]^T,
\]

where

\[
\left[ H \right] = -\left( T^T Q T + R \right)^{-1} T^T Q \left[ B \quad A \right].
\]

The matrix of controller coefficients \( H \) minimizes the cost function. A computationally efficient way to compute \( H \) is to use a gradient-descent algorithm

\[
H(k+1) = H(k) - \mu \frac{\partial J(k)}{\partial H(k)}.
\]

Here, the controller coefficients are updated at each time step (since \( T, B \) and \( A \) vary with time) using the previous set of coefficients and a stochastic estimate of the cost-function gradient, which can be computed using input/output data and the identified process parameters. Similar to the ARMARKOV method, the cost function is the power of the fluctuating pressure signal at the trailing edge of the cavity. If the model order is low enough or if the identified parameters are assumed constant, the matrix inverse can be computed and \( H \) is found using Eq.(14). In practice, only the current input \( u(k) \) is used in Eq.(9).

Several parameters in the GPC algorithm must be tuned to achieve a trade-off between the control performance and stability: the model order \( p \), the prediction horizon \( s \), the control effort penalty \( R \), the sensor weights \( Q \), and the sample rate. The model order \( p \) and the prediction horizon \( s \) are selected through the system identification. The model order \( p \) should be large enough in order to adequately model the relevant open-loop dynamics.

3. Experimental Setup

The experimental setup for this study is briefly described in this section. The cavity oscillation control experiments are conducted in a subsonic blowdown wind tunnel with a test section composed of an integrated cavity model and a piezoelectric ZNMF actuator array at its leading edge.

![Schematic of the test section](image_url)

Fig. 5 Schematic of the test section and cavity model.
1.7%. The flush-mounted acoustic treatment plate which enables simulation of the unbounded cavity flow encountered in a weapons bay is used as the wind tunnel cavity ceiling. The acoustic treatment consists of a porous metal sheet (MKI BWM series, Dynapore P/N 408020) backed by 50.8 mm thick bulk fiberglass.

Pressure fluctuations in the cavity floor near the leading edge (LE) and the trailing edge (TE) are measured using two flush-mounted unsteady pressure transducers (Kulite XT-190 series). For spectral analysis, the signals from the transducers are acquired using a NI PXI-1042Q data acquisition system at a sampling rate of 65536 Hz. The spectra shown in this paper are computed using $\Delta f = 16$ Hz, 75% overlap with a Hanning window, and 100 block averages.

The piezoelectric ZNMF synthetic jet actuator array mounted at the leading edge of the cavity consists of five identical cells as shown in Fig. 6. Details of the actuator design are provided in Arunajatesan et al.\textsuperscript{(14)} Each cell contains a commercially available parallel operation bimorph piezoelectric disc (APC 850 from APC Inc.) separating two cavities with slightly different volumes. The advantage of such a design is that it avoids the dc pressure imbalance on the two sides of the diaphragm during the experiment. Three slot geometries for the synthetic jet are investigated for their control performance considering the jet output velocity and interactions with the boundary layer on the leading edge. Each geometry is tabulated in the Table 1.
Table 1: Geometry of each slot in the top plates (Units: mm).

<table>
<thead>
<tr>
<th></th>
<th>Plate 1</th>
<th>Plate 2</th>
<th>Plate 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (l)</td>
<td>3.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Width (w)</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Depth (d)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In order to implement the open-loop control, a signal with specified waveform parameters is generated by an Agilent 33220A Function/Arbitrary Waveform Generator and is used to excite the actuator array through a Trek PZD350 Piezo Driver/Amplifier. In addition to sinusoidal excitation, amplitude modulation and burst modulation excitation waveforms\(^{(15)}\) are also used. Each input waveform is described as follows

**a) Sinusoidal**

\[ V(t) = A \sin(\omega t) , \quad (16) \]

**b) AM**

\[ V(t) = A \sin(\omega t) \sin(\omega_m t) , \quad (17) \]

**c) BM**

\[ V(t) = A \sin(\omega t) \text{Mod}(\omega_m) , \quad (18) \]

where the amplitude \( A \) has unit of Volts, and the carrier frequency \( f_c \) (\( = \omega_c / 2\pi \)) and modulation frequency \( f_m \) are in Hz. The “Mod” function is either a sinusoid or square pulse.

In order to implement the downhill simplex control, the rms TE pressure fluctuation integrated over a prescribed frequency range (i.e., 100 Hz – 4.0 kHz in this study) is normalized by the freestream dynamic pressure and is used as the objective function. The control algorithm aims to minimize the objective function by changing the relevant waveform parameters: amplitude \( A \), carrier frequency \( f_c \), and modulation frequency \( f_m \).

For the ARMARKOV adaptive disturbance rejection control and generalized predictive control (GPC), the signal from the TE transducer is pre-amplified and low-pass filtered using a Kemo multi-channel filter (Model VBF 35). The cutoff frequency is set to be 4 kHz for a sampling frequency of 10.24 kHz. This signal is then sampled with a 5-channel, 16-bit simultaneous sampling dSPACE ADC (model DS2001). The control algorithms are coded in SIMULINK and s-functions and are compiled via Matlab/Real-time workshop (RTW). These codes are then uploaded and run on the dSPACE digital control system, which collect inputs/outputs from DS2001ADC boards and compute the control signal once per time step. The output signal from dSPACE is passed through Kemo filter to smooth the zero-order hold signal from the DAC. The signal from this filter is then fed through the Trek amplifiers to drive the actuator array.

A z-type mirror based Schlieren system was used for flow visualization. Details are described in Ref. (15). The light source is a strobe light (Perkin Elmer Inc.). The beam diameter of the light from the beam expander which is neighboring the light source is 25 mm. A f4/100 mm condenser lens focuses the light on an adjustable rectangular aperture. This aperture is placed at the focal point of the first parabolic mirror. Two parabolic mirrors of 152.4-mm-diameter with focal length of 609.6 mm and 1524 mm are used to produce a parallel beam passing through the test section. A second parabolic mirror focuses the light on the knife-edge position. The knife-edge direction is horizontal so as to visualize the transverse density gradient of the flow. In order to reduce the light intensity entering the camera, a neutral density filter is attached in front of the camera lens. The Schlieren image is captured by a CCD camera (Powerview plus 2M, TSI Inc.) through a lens (Nikon AF
4. Experimental Results and Discussion

4.1 Actuator Characterization

The measured velocity output for three peak-to-peak voltage (V_{p-p}) inputs of the actuator are shown in Fig. 7, where the centerline velocity (amplitude) of the third cell (in the middle of the actuator array) for the slot plate 1 is presented. Due to the differing cavity size on either side of the piezoelectric bimorph, the jet velocity for each slot in a pair differs. The result demonstrates broadband output and a peak velocity greater than 100 m/s near 1100 Hz in the faster slot. Figure 8 compares the centerline velocity and momentum coefficient at the slot in the slow slot of the third cell for the three slot plates listed in Table 1. The momentum coefficient is calculated by assuming incompressible flow and normalized by the momentum flux in the boundary layer which spans the leading edge: \( \frac{1}{2} \rho \omega^2 W_c \delta \), where \( \rho \) is the density of freestream, \( \omega \) is the freestream velocity, \( W_c \) is cavity width, and \( \delta = 2.96 \text{ mm} \) is the measured boundary layer thickness at the leading edge of the cavity for Mach 0.3. Each plate has a similar trend of increasing velocity with input voltage and has a similar shape in the frequency response with the maximum velocity occurring near 1100 Hz for each plate. The high aspect ratio slot, plate 2, shows the highest output velocity, while the smallest slot area, plate 3, shows higher output velocity than slot plate 1. However the reference slot, plate 1, generates the highest output of momentum, while the smallest slot, plate 3 produces much less momentum.

![Fig. 7 Centerline velocity of the third cell of the slot plate 1 actuator array.](image)

![Fig. 8 Comparison of centerline velocity and momentum coefficient in the low-speed slot for three slot plates (input voltage: 75 V_{p-p}).](image)
4.2 Flowfield Investigation

Figure 9 shows typical instantaneous Schlieren images which indicate the representative flowfields for Mach 0.3 and 0.4 for the acoustically-treated tunnel ceiling. The flow direction is from left to right, and the cavity walls are highlighted with black solid lines. The cavity length-to-depth ratio is 5.0. Despite the relatively weak density gradients at these low freestream Mach numbers, the vortical flow structures are clearly observed and are indicative of the third Rossiter mode which is estimated from the relationship between the vortex spacing and the mode number of cavity oscillations. The region where the shear layer structures interact with the trailing edge of the cavity is clearly seen as a region of intense fluctuations in the density gradient field.

![Fig. 9 Instantaneous Schlieren photographs of flow over the cavity. Flow direction is from left to right.](image)

Figure 10 shows a spectrogram of the trailing-edge unsteady pressure measurement (normalized by the dynamic pressure of the freestream $Q$) for the cavity with acoustic treatment on the tunnel ceiling. Rossiter mode frequencies are also superimposed in both figures. The predictions are given by Howe:\(^{(17)}\):

$$f_L = \frac{n - \alpha}{U} \frac{1}{1 + M_\infty} \sqrt{1 + (y - 1)M_\infty^2} / 2,$$  \hspace{1cm} (19)

where $M_\infty$ is freestream Mach number, $U$ is the freestream velocity, $\alpha = 0.25$, $k = 0.70$ (for $L/D = 5$) and 0.65 (for $L/D = 6$) is the ratio between the convective velocity and the freestream velocity, and $n = 1, 2, \ldots$ is the Rossiter mode index. For both $L/D$ cases, the predictions match the experimental results within a reasonable degree of accuracy.

![Fig. 10 Spectrogram of TE unsteady pressure measurement with acoustically-treated tunnel ceiling (ref dynamic pressure of the freestream).](image)
Figure 11 shows the probability density function for Rossiter mode 2 in the baseline TE pressure signal at \( M = 0.3 \). For the plot the fluctuating pressure measurements are bandpass filtered around each Rossiter mode using a KEMO VBF-35 analog filter. In each case, the bandpass frequencies are set to ±10 Hz of the Rossiter mode frequency. The values of mean, standard deviation (STD), skewness, and kurtosis for Rossiter mode 2 are summarized in Table 2 and results for modes 3 to 5 are summarized in the table as well. The results show that the probability density functions for mode 2 and the other modes, which are not shown here, are nearly Gaussian. As described in Rowley et al.,(18) this is an indication of a lightly damped, linearly stable system that is constantly excited by external disturbances in the upstream boundary layer. These disturbances are amplified, causing oscillations at the resonant frequencies of the cavity. If the disturbances are removed, however, the oscillations would no longer be present. This implies the cavity oscillations may therefore be characterized as a forced, linearly stable system. Therefore, the use of system identification to the open-loop system, and adaptive feedback control algorithms (e.g., GPC and ARMARKOV) is justified for use to suppress the cavity oscillations in this study.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mean (Pa)</th>
<th>STD (Pa)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 2 (880 Hz)</td>
<td>0.00</td>
<td>271</td>
<td>0.01</td>
<td>3.04</td>
</tr>
<tr>
<td>Mode 3 (1280 Hz)</td>
<td>0.00</td>
<td>242</td>
<td>0.01</td>
<td>3.04</td>
</tr>
<tr>
<td>Mode 4 (1820 Hz)</td>
<td>0.00</td>
<td>241</td>
<td>0.00</td>
<td>3.04</td>
</tr>
<tr>
<td>Mode 5 (2320 Hz)</td>
<td>0.00</td>
<td>232</td>
<td>0.00</td>
<td>3.03</td>
</tr>
</tbody>
</table>

### 4.3 Flow Control Results

Figure 12 shows the results for the OL sinusoidal control at Mach 0.3 and 0.4. In particular, spectra of the TE fluctuating pressure \( (P_{fl}) \) normalized by the averaged freestream dynamic pressure \( (Q) \) are presented. The peaks seen in the figures for the baseline uncontrolled cases are due to the well-known Rossiter modes. Note that the sharp peaks at 1100 Hz and 2200 Hz for the controlled cases are associated with the acoustic signal produced by the actuator array. The input voltage used here is 50 V \( _{pp} \), and the result of the higher input voltage case of 75 V \( _{pp} \) is plotted as well for comparison. For all OL sinusoidal control cases, maximum suppression corresponds to forcing at 1100 Hz. This is presumably because this frequency is close to the resonant frequency of the actuator but sufficiently removed from a cavity resonance frequency to initiate lock-on resonance to the forcing frequency. While broadband suppression is achieved at \( M = 0.4 \), tonal suppression is reduced, indicating a larger jet velocity is required for control. The reduction of overall power and percent reduction of TE pressure fluctuation for both Mach number cases are tabulated in Table 3. Higher input voltages, which produce larger output jet velocities as seen in Fig. 7, show better control performance up to a point. These results suggest the existence of either an optimal value (at Mach 0.3) or saturation (at Mach 0.4) of the voltage.
amplitude for the range of input voltages considered. Note that sufficiently high input voltage ultimately leads to failure of the piezoceramic discs.

![Graph](image1.png)

Fig. 12 OL sinusoidal control results, where $f = 1100$ Hz and $V_{p-p} = 50$ V or 75 V.

Table 3: Comparison of control performance against the input voltage for Mach 0.3 and 0.4.

<table>
<thead>
<tr>
<th>Input Voltage $[V_{p-p}]$</th>
<th>Overall power reduction $[dB]$</th>
<th>% reduction in rms pressure fluctuations</th>
<th>Overall power reduction $[dB]$</th>
<th>% reduction in rms pressure fluctuations</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>-2.5</td>
<td>24.7</td>
<td>-1.6</td>
<td>16.7</td>
</tr>
<tr>
<td>75</td>
<td>-3.2</td>
<td>30.4</td>
<td>-2.4</td>
<td>24.4</td>
</tr>
<tr>
<td>100</td>
<td>-1.9</td>
<td>19.6</td>
<td>-2.5</td>
<td>24.7</td>
</tr>
</tbody>
</table>

Figure 13 compares the percent reduction of TE pressure fluctuation ($P_{rms}$) for three different slot geometries in the OL sinusoidal control for Mach 0.3 and 0.4, respectively, for an input voltage is 75 $V_{p-p}$. The reduction in $P_{rms}$ between the baseline and OL sinusoidal case is largest at 1100 Hz and shows the best performance from 1000 Hz to 1200 Hz. These results coincide with the actuator output at resonance, indicated by the centerline jet velocity of the synthetic jet in Fig. 8. Plate 1 performs best (particularly at Mach 0.4). From the results in Fig. 8, plate 1 provided the smallest actuator output velocity but the highest momentum of all plate configurations. The higher momentum alters the shear-layer near the leading edge and consequently influences the flow-induced resonance. However, there are several parameters that may contribute to the control effectiveness. Other relevant parameters beyond the velocity ratio and momentum coefficient include the slot aspect ratio. These preliminary results suggest a more detailed analysis and study of these parameters are warranted.

![Graph](image2.png)

Fig. 13 Comparison of $P_{rms}$ reduction [%] for three different slot geometries in open-loop sinusoidal control. The input voltage is 75 $V_{p-p}$, and frequency is swept from 100 Hz to 2400 Hz in steps of 100 Hz.
Figures 14-16 show the results of downhill simplex control at Mach 0.3. For three waveform types (sinusoidal, AM and BM), the DS algorithm arrives at nearly the same carrier forcing frequency as the OL case but in a small fraction of the time. It should be noted that the DS simplex generally finds a local minimum, and hence the optimized results depend on the initial conditions and constraints. In this case, the forcing frequency is constrained between 500 Hz and 1500 Hz, and this range is chosen to include the resonance frequency of the actuator. The results indicate again that both tones and broadband noise level are mitigated using these schemes. Multiple Rossiter modes are reduced but are not eliminated, and the second Rossiter mode remains the strongest. The broadband pressure fluctuations are reduced over the entire frequency range of interest (~4 kHz). The reason for this reduction may be due to the mean-flow modification provided by the three-dimensional perturbations introduced by the actuator. By comparison, approximately 2.5 dB versus 2.6 dB in overall power reduction which corresponds to 24.7% versus 26.0% reduction of the pressure fluctuation are obtained for the OL and DS sinusoidal cases, respectively. In addition, continuous sinusoidal forcing is better than either the AM or BM case.

Fig. 14 Downhill simplex (sinusoidal) control results, Mach = 0.3, $f_c = 1125$ Hz and $V_{p-p} = 50$ V.

Fig. 15 Downhill simplex (amplitude modulation) control results, Mach = 0.3, $f_m = 42$ Hz, $f_c = 1200$ Hz, and $V_{p-p} = 50$ V.

Fig. 16 Downhill simplex (burst modulation) control, Mach = 0.3, $f_m = 40$ Hz, $f_c = 1180$ Hz, and $V_{p-p} = 50$ V.

Figures 17 and 18 show preliminary ARMARKOV and GPC control results, respectively. The ARMARKOV and GPC algorithms are somewhat less effective compared to the OL and DS, presumably due to the reduced coherence (not shown) between the input and output signals. The three-dimensional forcing disrupts the spanwise coherence of the shear-layer perturbations, and this disruption reduces the coherence between the control input and pressure output signals. The reduced coherence leads to poorer dynamical models and reduced control performance relative to two-dimensional slot forcing along the entire
leading edge.\textsuperscript{(5)} The reduction in overall sound pressure level (OASPL) and rms pressure fluctuations for all control methods at Mach 0.3 are summarized in Table 4. Comparisons show that non-model based simplex algorithms have demonstrated their efficacy to reduce both the tonal and broadband components for the given test condition. Results for Mach 0.4 cases are summarized in Table 4 as well. Due to the reduced control performance for Mach 0.4 case at 50 V\textsubscript{p-p}, the results are obtained with 75 V\textsubscript{p-p}.

Fig. 17 ARMARKOV control results, Mach = 0.3, model order $n = 6$ and Markov parameter number $\mu = 10$.

Fig. 18 GPC control, Mach = 0.3, model order $p = 20$, and horizon $s = 6$.

Table 4: Summary of performance of different control methods for Mach 0.3 and 0.4 flows.

<table>
<thead>
<tr>
<th>Control Method</th>
<th>Overall power reduction [dB]</th>
<th>% reduction in rms pressure fluctuations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach 0.3 (50 V\textsubscript{p-p})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Open-Loop</td>
<td>-2.5</td>
<td>24.7</td>
</tr>
<tr>
<td>DS – Sinusoid</td>
<td>-2.6</td>
<td>26.0</td>
</tr>
<tr>
<td>DS – AM</td>
<td>-1.3</td>
<td>13.9</td>
</tr>
<tr>
<td>DS – BM</td>
<td>-1.9</td>
<td>19.5</td>
</tr>
<tr>
<td>ARMARKOV</td>
<td>-1.6</td>
<td>16.8</td>
</tr>
<tr>
<td>GPC</td>
<td>-1.6</td>
<td>16.7</td>
</tr>
<tr>
<td>Mach 0.4 (75 V\textsubscript{p-p})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Open-Loop</td>
<td>-2.4</td>
<td>24.4</td>
</tr>
<tr>
<td>DS – Sinusoid</td>
<td>-3.1</td>
<td>30.3</td>
</tr>
<tr>
<td>DS – AM</td>
<td>-1.1</td>
<td>11.5</td>
</tr>
<tr>
<td>DS – BM</td>
<td>-2.2</td>
<td>22.4</td>
</tr>
<tr>
<td>ARMARKOV</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GPC</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5. Conclusions and Future Work

The results of a comparative study aimed at efficient active suppression of flow-induced cavity oscillations are presented. Through the investigation of different actuator slot geometries, larger slots which generate larger momentum with a forcing frequency near the resonant frequency of the actuator showed the best performance for suppressing the cavity oscillation. Up to 30% reduction in the rms pressure fluctuations in a 4 kHz bandwidth are obtained using various open-loop and closed-loop controllers via a zero-net mass-flux actuator array at the cavity leading edge. In addition, multiple Rossiter-mode noise reduction was also demonstrated. Kegerise et al.\textsuperscript{(5)} previously demonstrated multiple Rossiter-mode suppression using active control, but broadband noise suppression was not possible in their study because of the two-dimensional nature of the perturbation inputs. Brès and Colonius\textsuperscript{(19)} showed that three-dimensional spanwise control, in which each actuator has a different phase angle, was potentially more effective than two-dimensional control. Therefore as a next step, a flow visualization study to understand the effects of OL and CL control on the flow physics in order to develop an effective but efficient control scheme is necessary. Hence, future work will focus on acquisition of quantitative Schlieren (Background Oriented Schlieren: BOS) and particle image velocimetry (PIV) measurements.
obtained simultaneously with unsteady surface pressure measurements. These data will help determine the relationships between pressure, velocity, and density and consequently help to develop better control methodologies.

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References


