Evaluation of Turbulence Kinetic Energy Budget in Turbulent Flows by Using Photobleaching Molecular Tagging Velocimetry*

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Abstract
Understanding turbulence kinetic energy (TKE) budget in gas-liquid two-phase bubbly flows is indispensable to develop and improve turbulence models for the bubbly flows. Simultaneous measurement of velocity and velocity gradients with a spatial resolution smaller than the Kolmogorov scale is required to evaluate the TKE budget experimentally. We therefore proposed a molecular tagging velocimetry based on photobleaching reaction (PB-MTV) and applied it to turbulent flows in a square duct to demonstrate the possibility of evaluation of TKE budget. In this study, we improved PB-MTV in its processing speed by utilizing GPGPU (General Purpose Graphic Processing Unit) to increase sample number in measurements. We measured TKE budget in a turbulent water flow in a square duct by using the PB-MTV at the same turbulent Reynolds number as DNS data provided by Horiuti, and compared the measured data with the DNS data to validate PB-MTV for evaluation of TKE budget. We also measured TKE budget in a bubbly flow in the square duct to examine effects of bubbles on TKE budget. As a result, we found that (1) PB-MTV can accurately evaluate TKE budget in turbulent flows, (2) bubbles affect the production and diffusion rates of TKE and do not affect the dissipation rate so much, and (3) the model proposed by Troshko and Hassan can reasonably estimate the production rate of the bubble-induced pseudo turbulence.

Key words: Turbulent Flow, Gas-Liquid Two-Phase Flow, Bubble, Turbulence Kinetic Energy, Photobleaching Molecular Tagging Velocimetry

1. Introduction
Gas-liquid two-phase bubbly flows are frequently encountered in many industrial systems such as chemical plants, power plants, petroleum transport systems and heat exchangers. Development of numerical methods for accurately predicting bubbly flows has been desired to assist the design and improvement of these systems. Since the bubbly flows are intrinsically fluctuating, numerical methods frequently adopt turbulence models for liquid phase turbulence, most of which are based on a transport equation of turbulence kinetic energy \(^{(1-4)}\). Understanding the turbulence kinetic energy (TKE) budget in the bubbly flows and validation of the turbulence models are therefore key issues to improve accuracy of numerical predictions of bubbly flows. Simultaneous measurement of velocity and velocity gradients with spatial resolution smaller than the Kolmogorov scale is required to evaluate the TKE budget experimentally. Since the available measurement methods hardly measure velocity and velocity gradients with such a high spatial resolution, the dissipation rate of TKE has been evaluated by summing the diffusion and production rates of TKE \(^{(5)}\) or...
by using Taylor’s hypothesis(6), and direct evaluation of the dissipation rate has rarely been carried out. We therefore proposed a molecular tagging velocimetry based on photobleaching reaction (PB-MTV)(7) and applied it to turbulent flows in a square duct to demonstrate the possibility of evaluation of TKE budget(8).

In this study, we measure TKE budget in a turbulent water flow in a square duct by using PB-MTV at the same turbulent Reynolds number as the DNS data provided by Horiuti(9), and compare the measured data with the DNS data to validate PB-MTV. We also apply PB-MTV to a bubbly flow in the square duct to investigate effects of bubbles on TKE budget. A general purpose graphic processing unit (GPGPU) is utilized in the image processing of PB-MTV to reduce the processing time and to improve the measurement accuracy in the statistically averaged quantities by increasing the sample number.

Nomenclature

\[ a_{ij} \] Velocity gradient tensor \([1/s]\)

\[ C \] Cross-correlation \([-]\)

\[ d \] Bubble diameter \([\mu m]\)

\[ d_{32} \] Sauter mean diameter \([\mu m]\)

\[ D \] Diffusion rate of \(k\) \([m^2/s^3]\)

\[ J \] Volumetric flux \([m/s]\)

\[ k \] Turbulence kinetic energy \([m^2/s^2]\)

\[ L \] Elevation from duct inlet \([mm]\)

\[ P \] Production rate of \(k\) \([m^2/s^3]\)

\[ Re \] Liquid Reynolds number \([-]\)

\[ Re_B \] Bubble Reynolds number \([-]\)

\[ Re_{t} \] Turbulence Reynolds number \([-]\)

\[ T_{ir} \] Irradiation time \([s]\)

\[ u_{t} \] Friction velocity \([m/s]\)

\[ U \] Liquid velocity in x direction \([m/s]\)

\[ \overline{u'^2}, \overline{v'^2}, \overline{w'^2} \] Turbulence intensities in x, y, z directions \([m^2/s^2]\)

\[ \overline{u'v'} \] Reynolds shear stress \([m^2/s^2]\)

\[ W \] Channel width \([mm]\)

\[ y \] Distance from wall \([mm]\)

\[ \alpha \] Void fraction \([-]\)

\[ \delta t \] Time interval between images \([s]\)

\[ \Delta \] Size of tag \([m]\)

\[ \varepsilon \] Dissipation rate of \(k\) \([m^2/s^3]\)

\[ \eta \] Size of pixel \([m]\)

\[ \kappa \] Karman constant \([-]\)

\[ \nu \] Kinematic viscosity \([m^2/s]\)

\[ \rho \] Liquid density \([kg/m^3]\)

Subscripts

\[ B \] Bubble

\[ G \] Gas phase

\[ L \] Liquid phase

Superscript

\(+\) Dimensionless

2. Experimental Apparatus and Method

2.1 Experimental apparatus and condition

A schematic of the experimental apparatus is shown in Fig. 1. The vertical duct was made of acrylic resin except the measurement region in which the duct was made of glass. The size of the cross-section and length of the duct were 50 × 50 mm and 1,600 mm,
respectively. Water was filled in the reserve tank and supplied to the test section by the pump. A wire mesh grid (pitch: about 3 mm) was installed at the duct inlet ($L = 0$ mm) to promote the development of turbulent flow. In the bubbly flow experiments, air was supplied from the compressor (Hitachi, Oil Free Compressor SRL-11API) and injected into the flow from the two opposed sidewalls through capillary tubes (diameter: 50 $\mu$m, number of tubes: 5 x 2 sides) at the duct inlet. Air and water flow rates were measured by a flowmeter and graduated cylinder, respectively. The water temperature was kept at 45 $\pm$ 0.5 $^\circ$C through the experiments.

The liquid volumetric flux $J_l$ was fixed at 0.06 m/s so as to keep the same turbulence Reynolds number $Re_t$ as that in DNS of turbulent channel flow by Horiuti (9) ($Re_t = 180$). The liquid Reynolds number $Re$ and the turbulence Reynolds number $Re_t$ are defined by

$$Re = \frac{J_l W}{\nu}$$  \hspace{1cm} (1)
$$Re_t = \frac{u_r W}{2\nu}$$  \hspace{1cm} (2)

where $\nu$ is the kinematic viscosity, $W$ the channel width and $u_r$ the friction velocity defined by

Fig. 1 Experimental apparatus and measurement planes
where \( u \) is the \( x \) component of the velocity and \( \frac{\partial u}{\partial y} \) is the velocity gradient at the wall, which was evaluated from \( \frac{\partial u}{\partial y} \) measured at the measurement point nearest to the wall (\( y = 0.25 \) mm; \( y^+ = 1.85 \) (single-phase flow), 2.74 (bubbly flow)). Here, \( \overline{\cdot} \) denotes the ensemble average. In the bubbly flow experiment, the liquid volumetric flux \( J_L \) was set at the same value as the single-phase flow experiment to examine effects of bubbles on the liquid phase turbulence, and the gas volumetric flux \( J_G \) was set at \( 5.0 \times 10^{-5} \) m/s.

The velocity and velocity gradients were measured along the centerline of the cross-section at the elevation \( L = 1,500 \) mm from the bottom of the duct using the photobleaching molecular tagging velocimetry (PB-MTV). The number of measurement points in the horizontal direction between the wall and the center of the duct was 14. Since PB-MTV is a two-dimensional measurement technique, the measurements were carried out for \( x-y \) and \( x-z \) planes as shown in Fig. 1 (b) and (c). Measurements using a laser Doppler velocimetry (LDV, Dantec 60x series) were carried out for the same measurement points to examine the accuracy of PB-MTV. The sample number of LDV was 30,000 for each measurement point and the statistical average was converged within \( \pm 1 \% \). Preliminary measurements using LDV confirmed that the flow at the measurement position (\( L = 1,500 \) mm) was fully-developed.

Bubble diameters \( d \) and void fractions \( \alpha \) in the turbulent bubbly flow in the duct were measured by an image processing method based on the Sobel filter and the Hough transform(10) at the same measurement points as the velocity measurements. Figure 2 shows the probability density function (PDF) of \( d \) and the volume fraction of bubbles in each bubble size class to the total gas volume. The Sauter mean diameter of bubbles \( d_{32} \) was 690 \( \mu m \). The bubble diameter in the bubbly flow ranged from 300 to 1,500 \( \mu m \). Although the number of bubbles larger than 800 \( \mu m \) is much lower than that of the smaller bubbles, the volume fraction of the larger bubbles is about half of that of the smaller bubbles. Hence, the effects of the larger bubble on the flow might not be negligible despite its small number.

![Fig. 2 Bubble size distribution](image-url)

### 2.2 Measurement method

The photobleaching molecular tagging velocimetry (PB-MTV) was used to measure velocity and velocity gradients of liquid phase to evaluate TKE budget. Figure 3 shows a schematic of PB-MTV. The photobleaching is an irreversible photodegradation of a fluorescent dye induced by intense illumination, and it changes the fluorescent dye to non-fluorescent dye. Uranin (fluoresceine sodium salt) was solved in water as the fluorescent dye, and the concentration of the dye was about \( 10^{-6} \) mol/m³. The dye was photobleached by an intense laser beam, which was emitted from an Ar-ion laser.
(Spectra-Physics, Stabilite 2017, beam diameter: 1.4 mm, wave length \( \lambda = 488.0 \) nm) and focused on the measurement plane through the focusing lens (focal length \( f = 36 \) mm). By illuminating the fluorescent dye using the low intensity laser sheet (\( \lambda = 514.5 \) nm), the tagged region formed by the intense beam appeared as dark regions, i.e., non-fluorescent regions, as shown in the photographs in Fig. 3. The image of the tagged region was recorded by the CCD camera (DANTEC, HiSense Mk II, 1,344×1,012 pixels) with a macro lens (VS Technology Co., VS-TC6-65). Time sequences of the intense laser beam and shutter of the camera are also shown in Fig. 3. First, the intense laser beam was irradiated during the irradiation time \( T_{ir} (= 1 \) ms) to form the tag. The irradiation time was determined so as to obtain a high contrast tag image and controlled by using the shutter. After the irradiation of the intense laser beam, the CCD camera recorded two consecutive images with the time interval \( \delta t \) (\( \sim 5 \) ms). A delay generator (Stanford Research Co. Ltd., DG-535) was used to synchronize the shutter and the CCD camera. The size \( \Delta \) of the tag was 40 – 100 \( \mu \)m and about 70 pixels in the recorded image. The tag size was smaller than the Kolmogorov scale of the flow (about 200 \( \mu \)m), and was 1/3 of minimum bubble size (300 \( \mu \)m) so that it was also smaller than the length scale of bubble induced pseudo turbulence.

Liquid velocities on the plane can be evaluated from the displacement of the center of the tagged region between two consecutive images. The center positions of tagged regions were calculated from the intensity distributions in the tagged regions. Then, the displacement vector of the center was divided by \( \delta t \) to obtain the two-dimensional components \((u_0, v_0)\) of the liquid velocity. The deformation of the tagged region reflects the velocity gradient tensor of the flow field. If the flow is two-dimensional and the velocity gradient tensor \( a_{ij} \) is constant in the tagged region, the difference \( \delta\mathbf{R} \) of displacement vectors of two points in an arbitrary fluid particle in the tag can be expressed as

\[
\delta \mathbf{R} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix} \delta t
\]

where \((\delta x, \delta y)\) is the displacement vector from the tag center. A simple searching method based on cross-correlation was adopted to evaluate the constants \( a_{ij} \). The measured shape of the tagged region was transformed by using Eq. (4) with the initial guess values of \( a_{ij} \). The cross-correlation between the resultant shape and the tag shape measured at \( t = \delta t \) was calculated as shown in Fig. 4. Then, the values of \( a_{ij} \) were changed from \( a_{ij} - \delta \) to \( a_{ij} + \delta \) to

![Fig. 3 Schematic of measurement system](image)
obtain the cross-correlation distribution $C(a_{ij})$ against the value of $a_{ij}$. Here, $\delta$ was a small value determined by a rule of thumb, and the increment of $a_{ij}$ was set at about one-pixel resolution in the image ($\eta/\delta t \Delta t$, $\eta$: the size of pixel). The correct values of $a_{ij}$ can be found as the value at which the cross-correlation $C(a_{11}, a_{12}, a_{21}, a_{22})$ takes the peak. Note that the peak was evaluated by using a sub-pixel interpolation based on the Gaussian distribution like the peak detection of correlation in PIV.

**Fig. 4** Schematic of cross-correlation of tag shapes

This image processing requires long CPU time, and therefore, it is not easy to increase the sample number. We therefore made use of a general purpose graphics processing unit (GPGPU; NVIDIA, Tesla C1060, Number of processing cores: 240, Processor core clock: 1.296 GHz, Memory: 4GB) to reduce the processing time. We parallelized the image processing program of PB-MTV, and succeeded in over 90% reduction of the processing time. This reduction enables us to increase the sample number in measurements. Figure 5 shows dependences of the statistical quantities, that is, turbulence intensities, $u'^2$ and $v'^2$, and Reynolds shear stress $\bar{u'}\bar{v'}$ on the sample number. The solid lines indicate the averaged values for 5,000 samples, and the dashed lines represent the error levels of $\pm$ 10%. When the sample number is larger than 800, each averaged value converges to within $\pm$ 5% of the value averaged for 5,000 samples. Hence, the sample number in the present experiments was set at 1,000 for each measurement point. Uncertainties in measured velocity and velocity gradient were $3.12 \times 10^{-5}$ m/s and 0.143 s$^{-1}$, respectively. The details of the measurement method and its accuracy can be found in Hosokawa et al.\textsuperscript{7,8}. There are...
some sources of degrading images, i.e., (1) difficulty in visualization of tag due to cutoff of
the laser sheet by bubbles, (2) blur due to bubbles locating between the camera and the
measurement point, and (3) decreasing in accuracy due to unclearness of tag caused by
reflection of laser sheet at gas-liquid interface. The number of degraded image sets, from
which velocity cannot be evaluated, was less than 1 % of the total number of image sets,
since the maximum local void fraction was less than 0.2 % in the present experimental
condition. Large deformation or breakup of a tag due to bubbles was not observed in the
recorded images because of a short time interval $\delta t$ between images.

3. Results and Discussion

3.1 Turbulence kinetic energy budget in single-phase flow

PB-MTV was applied to the single-phase flow and the measured data were compared
with the results of DNS for a turbulent channel flow by Horiuti(9) to examine its accuracy.
The mean velocity measured by PB-MTV is plotted in Fig. 6 together with that measured by
LDV and the DNS result. The axial mean velocity $U$ and the distance from the wall $y$ are
normalized by the friction velocity $u_\tau$ and $\nu/u_\tau$, respectively. The friction velocity $u_\tau$ in the
PB-MTV measurement was calculated from the measured velocity gradient at the
measurement point nearest to the wall. The following universal velocity distribution(11) is
also plotted in Fig. 6.

$$\frac{\bar{u}}{u_\tau} = U^+ = \frac{y}{v} = y^+$$ for linear sublayer (5)

$$\frac{\bar{u}}{u_\tau} = U^+ = \frac{1}{k} \ln y^+ + C = 2.44 \ln y^+ + 5.5$$ for logarithmic region (6)

The velocity distribution measured by PB-MTV agrees well with the data measured by
LDV, and the discrepancy between the measured $U^+$ and the DNS result or the universal
velocity distribution is also small. The small discrepancy in the core region ($y^+ > 30$) might
be caused by the difference in channel geometry (experiment: the square duct, DNS: a
two-dimensional parallel channel). Figures 7 – 11 show comparisons of the three
components of the turbulence intensity, the Reynolds shear stress $\overline{uu'}$ and TKE $k$
normalized by $u_\tau$. The DNS results for turbulent flow in a square duct ($Re_\tau = 300$) by
Gavrilakis(12) are also plotted in Figs. 7 – 10. The turbulence intensity, $\overline{uu'}$ and $k$
measured by PB-MTV agree not only with the data measured by LDV but also with the
DNA results. These results clearly demonstrate that PB-MTV can accurately measure mean
velocity and statistic values of turbulent flows.

The turbulence production $P$, diffusion $D$ and dissipation $\varepsilon$ were evaluated by
substituting the measured local instantaneous velocities and velocity gradients into the
following equations:

$$P = -\overline{uu'_j} \frac{\partial U_i}{\partial x_j}$$ (7)

$$D = \frac{\partial}{\partial x_j} \left( \frac{1}{2} \overline{uu'_i uu'_j} \right) + \frac{\partial}{\partial x_j} \overline{v' k}$$ (8)

$$\varepsilon = \overline{\partial u'_i \partial u'_j} \overline{\partial x_j}$$ (9)

Note that the pressure diffusion term was neglected due to difficulty in its measurement.
Figure 12 shows the TKE budget normalized by $u_\tau^3/\nu$. The measured production, diffusion and dissipation rates of TKE agree well with the DNS results. The magnitude of measured $\varepsilon^+$ takes slightly larger values than the DNS results. The measured $P^+$ in $y^+ > 15$ is also slightly larger than the DNS results. These discrepancies between measured and DNS results are not so large, judging from the facts that (1) the channel geometries are different between the experiment and DNS and that (2) the maximum discrepancies in velocity and turbulence properties are 5% and 15%, respectively, as shown in Figs. 6 - 11. The over estimation in measured magnitude of $\varepsilon$ is caused by taking the gradient of fluctuation velocity squared, which changes a negative error in the gradient to a positive error in $\varepsilon$. The sum of $P^+$, $D^+$ and $\varepsilon^+ = P^+ + D^+ + \varepsilon^+$, is not so large, i.e. about 15% of the maximum of $P^+$. Hence, the maximum error in the evaluated TKE budget can be estimated as 15%. The pressure diffusion which is neglected in Eq. (8) is within 5% of the maximum $P$ judging from the DNS results by Horiuti$^{(9)}$. The effect of the pressure diffusion term on the measured TKE budget is therefore small. This is also confirmed from the fact that $D^+$ experimentally evaluated without the pressure diffusion term agrees well with the DNS results including the pressure diffusion term. We therefore conclude that PB-MTV can accurately evaluate TKE budget in turbulent flows with weak pressure diffusion. Note that the accuracy in velocity and velocity gradient measured by PB-MTV correspond to 0.1 pixel in the tag image$^{(8)}$. Increase in the resolution of the tag image improves the accuracy in velocity and velocity gradient, and results in better evaluation of TKE budget.
3.2 Turbulence kinetic energy budget in bubbly flow

PB-MTV was applied to the bubbly flow in the vertical duct to examine the effect of the bubbles on the turbulence properties and TKE budget. The measured void fraction $\alpha$ and axial mean velocity $U'$ normalized by $u_\tau$ are shown in Figs. 13 and 14, respectively. The friction velocity $u_\tau$ ($= 6.65 \times 10^{-3}$ m/s) in the bubbly flow is higher than that in the single-phase flow ($u_\tau = 4.49 \times 10^{-3}$ m/s). The void fraction takes a peak in the near wall region ($y^+ \sim 20$). Although the axial mean velocity in the bubbly flow takes the same trend as the single-phase flow in the near wall region ($y^+ \leq 20$), it takes lower values than the single-phase flow in the core region ($y^+ > 20$). This is because the acceleration of the liquid velocity due to the bubbles in the high $\alpha$ region is large due to the relative velocity comparable to the liquid velocity, and therefore, the liquid velocity in the core region decreases to keep the total liquid flow rate constant.

Figures 15, 16, 17 and 18 show the dimensionless axial and wall-normal components of turbulence intensity, $\bar{u}^2$ and $\bar{v}^2$, and the dimensionless TKE $k'$ in the single-phase and bubbly flows. Compared with the single-phase flow, $\bar{u}^2$ and $k'$ in the bubbly flow are lower in $y^+ < 60$, whereas $\bar{v}^2$ is higher in this region. This implies breakup of the shear-induced turbulence eddies by the bubbles in the near wall region(13). To the contrary, turbulence augmentation due to bubble-induced pseudo turbulence is observed not only in $\bar{v}^2$ but also in $\bar{u}^2$ and $k'$ in the core region ($y^+ > 60$). The Reynolds shear stress in the bubbly flow increases with $y^+$ in $y^+ \leq 20$, and there is no difference between the single-phase and bubbly flows in spite of the presence of bubbles. It, however, decreases as $y^+$ increases in $20 < y^+ < 100$, and takes negative values in $40 \leq y^+$. In the near wall region in the bubbly flow ($y^+ \leq 20$), the gradients of axial velocity and void fraction are positive, and they make $\bar{u}\bar{v}$ positive. To the contrary, the gradients take negative values in the core region ($y^+ \geq 40$), and therefore, the Reynolds shear stress takes negative values in this region. This corresponds to the fact that the sign of gradient of $U'$ changes from positive to negative at $y^+ \sim 40$.

TKE budgets in the single-phase and bubbly flows are shown in Fig. 19. Although the difference in $\varepsilon'$ is small, $P'$ and $D'$ are modulated due to the presence of bubbles. This results in a non-zero residual in the TKE budget ($P'+D'+\varepsilon'$) in the bubbly flow, whereas the residual is almost zero in the single-phase flow. This non-zero residual occurs even in $y^+ < 20$ in which the modulation of $U'$ is very small. Hence this is not due to the modulation of $U'$. Since the flow was confirmed to be quasi-steady and developed in the preliminary experiment, this result indicates that the TKE budget in the bubbly flow is not described only by Eqs. (7) – (9) and modeling bubble-induced pseudo turbulence is indispensable for the bubbly flow. The small change in $\varepsilon'$ due to the presence of bubbles indicates that the
The effect of bubbles on the dissipation of TKE in a small scale is small in the present experimental range.

Kataoka and Serizawa\textsuperscript{(14)} deduced basic equations of turbulence in gas-liquid two-phase flows and pointed out that turbulence is generated by momentum transfer between bubbles and the liquid phase. Troshko and Hassan\textsuperscript{(3)} carried out a numerical simulation of bubbly flows using the following equation for the production rate $P_B$ of TKE due to momentum transfer between bubbles and the liquid phase:

$$P_B = \frac{3}{4} \frac{C_d}{d} \alpha \left| U_d - \bar{u} \right|$$  \hspace{1cm} (10)

where $U_d$ is the bubble velocity and $C_d$ the drag coefficient. Troshko and Hassan\textsuperscript{(3)} used the following equation for $C_d$:

$$C_d = \frac{24}{Re_B} \left( 1 + 0.1 Re_B^{0.75} \right)$$  \hspace{1cm} (11)

Here, $Re_B$ is the bubble Reynolds number. The applicable range of Eq. (11) is reported to be $Re_B < 500 - 1000$ in the literature. Although they reported that the numerical simulation using Eq. (10) gives good predictions for mean velocity and void fraction in bubbly flows...
with $\alpha = 4 - 38\%$, they did not validate Eq. (10) directly by comparing with experimental data.

The production rate of bubble-induced pseudo turbulence $P_B$ was calculated by substituting the measured liquid velocity, bubble velocity, the Sauter mean diameter and the void fraction into Eqs. (10) and (11), and the sum of $P^+$ and $P_B^+$ is plotted in Fig. 20 together with $D^+$ and $\varepsilon^+$. The total production rate $P^+ + P_B^+$ of shear- and bubble-induced turbulences reasonably balances with the sum of $D^+$ and $\varepsilon^+$, whereas the budget without $P_B^+$ ($P^+ + D^+ + \varepsilon^+$) is not zero as noted above. This result implies that (i) the production rate in bubbly flows can be reasonably evaluated by the sum of Eqs. (7) and (10), (ii) increase in the production rate in bubbly flows is caused by momentum transfer between bubbles and the liquid phase, and (iii) the bubble-induced pseudo turbulence dissipates through the cascade process of the turbulence eddies simultaneously with the shear-induced turbulence. We reported that the sum of $P^+$, $D^+$ and $\varepsilon^+$, i.e. without $P_B^+$ balances the TKE budget in a bubbly flow involving small bubbles ($d_B = 200 - 1000\ \mu m$, $d_{32} = 590\ \mu m$)$^{(13)}$, the size of which is smaller than the present study ($d_B = 300 - 1500\ \mu m$, $d_{32} = 690\ \mu m$). This implies that increase in the production rate depends on bubble diameter, and large bubbles, especially bubbles larger than 1 mm, contribute to the increase in the production rate. Note that the pressure diffusion neglected in the measured $D^+$ is much smaller than $\varepsilon p'\mu'/\varepsilon x = \alpha p'U_k^2/(2d)$, judging from the estimation that the maximum pressure and velocity fluctuations can be evaluated by $p' \sim pU_k^2/2$ and $u' \sim U_k$, respectively. The pressure
diffusion due to bubbles is, therefore, smaller than the production rate due to bubbles given by Eq. (10), and we cannot attribute the reason why the sum of \( P^+ \), \( D^+ \) and \( \varepsilon^+ \) does not vanish in the bubbly flow to the pressure diffusion due to bubbles. Furthermore, the pressure diffusion due to bubbles is estimated to take negative values around a void peak position \((y^+ \sim 20)\). Since \( P^++D^+ + \varepsilon^+ \) takes negative values at \( y^+ \sim 20 \), the absence of the pressure diffusion is not a reason of the unbalance in TKE budget. Hence, the production of TKE due to momentum transfer between bubbles and the liquid phase does exist in the bubbly flow.

These results have demonstrated that simultaneous measurement of velocity and velocity gradient using PB-MTV makes it possible to carry out direct evaluations of production, diffusion and dissipation rates of TKE and detailed diagnostics of TKE budget in a flow involving shear- and bubble-induced turbulences.

4. Conclusions

The photobleaching molecular tagging velocimetry (PB-MTV) was applied to a turbulent water flow in a square duct to examine its accuracy in TKE budget measurements through the comparison with DNS data provided by Horiuti. We utilized a general purpose graphic processing unit (GPGPU) for the image processing of the PB-MTV to reduce the processing time and to improve the measurement accuracy in the statistically averaged quantities by increasing the sample number. The PB-MTV was also applied to a bubbly flow in the duct to understand effects of bubbles on the turbulence properties and TKE budget. As a result, the following conclusions were obtained:

(1) The TKE budget evaluated by using PB-MTV agrees well with the DNS data, and therefore, PB-MTV is confirmed to be a useful method for evaluating TKE budget in turbulent flows.

(2) The production and diffusion rates of TKE in the bubbly flow are modulated due to the presence of bubbles, whereas the effect of bubbles on the dissipation rate is weak.

(3) Increase in the production rate due to bubbles is reasonably evaluated by Troshko and Hassan’s model, and the sum of production rates due to shear and bubbles, diffusion rate and dissipation rate is estimated to be zero. This implies that the increase in the production rate due to bubbles is attributed to momentum transfer between the phases and the bubble-induced pseudo turbulence dissipates through the cascade process of the turbulence eddies simultaneously with the shear-induced turbulence.

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