Novel Algorithms of 3D Particle Tracking Velocimetry Using a Tomographic Reconstruction Technique*

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Abstract

New algorithms of 3D particle tracking velocimetry (3D PTV) based on a tomographic reconstruction approach have been developed and tested by using synthetic images of unsteady 3D flows. The new algorithms are considered not only in the tomographic reconstruction process of the fluid volume with particles but also in the subsequent process of individual particle detection and validation. In particular, the tomographic reconstruction accuracy is boosted up by using a new recursive validation scheme through which many of ghost particles can be removed effectively. The particle detection process includes the particle mask correlation operator and the dynamic threshold scheme to extract individual particle centroids from the reconstructed intensity clusters of the fluid volume. The overall reconstruction accuracy is checked by the synthetic image data sets with different particle density and different volume thickness.

Key words: 3D Particle Tracking Velocimetry, Volumetric PTV, Tomographic Reconstruction, Camera Calibration, Flow Measurement.

1. Introduction

In many applications of experimental fluid and thermal science, the particle imaging methods, typically known as Particle Image Velocimetry or PIV, have been widely used as a powerful flow diagnostic technique(1). The term PIV in the broad sense stands for various imaging techniques using small seed particles as tracers but more precisely it stands for basically two major groups of techniques for measuring image disparity. The first one is the PIV in the narrow sense (the term PIV referred to hereafter is used in this narrow sense), which tracks the movement of group of particles statistically in small areas or volumes between a short time interval and is more suitable for Eulerian flow analysis, while the second one is the PTV (particle tracking velocimetry), which tracks the movement of individual particles in a whole measurement field and is considered suitable for Lagrangian flow analysis(2).

In recent years, the trend of both techniques is more and more 3C-3D (3 components of velocity in 3D space) oriented in stead of 2C-2D or 3C-2D and many interesting and meaningful flow measurement results have been reported from these 3C-3D PIV or 3C-3D PTV experiments(3). In the case of PIV, the only possible methods (for the time being) for full 3C-3D measurement are the scanning 3D PIV(4) and the tomographic PIV (Tomo-PIV)(5), while in the case of PTV, there are more choices in methodology. The most classical method is the stereoscopic PTV(6) in which 3D location of individual particles are determined by using some matching algorithm applied to two stereoscopic particle images and then a triangulation algorithm. The key factor of this method is the matching algorithm between the stereoscopic images and many efforts have been made to improve the accuracy of matching(7). Nevertheless any refined algorithm is not free from a certain amount of matching errors. The defocus effect
of the particle images at different depth positions is another crucial factor in the determination of 3D location of particles.

This defocus effect is positively utilized in the defocusing (defocused) particle tracking velocimetry\(^{(8)}\), in which the particle image in the measurement volume is captured by a single camera with three axisymmetrically arranged eccentric apertures. The in-plane centroid of the particle is determined as the gravity center of the triplet image of a single particle and the depth of the particle is computed from the separation of the triplet points. As assumed from this principle, the defocusing approach is only applicable to relatively low density particle images. Another drawback of this method is the use of eccentric apertures in front of the camera objective lens, which means that the particle images have to be captured through a most inconvenient part of the lens from the viewpoint of optical distortion.

The holographic particle tracking velocimetry\(^{(9)}\) also uses a single camera for image capturing but there is no eccentric aperture or objective lens. This image capturing (more precisely the hologram recording) records particles as a hologram image with phase information and the depth position of each particle has to be extracted from the 3D reconstruction result of the hologram. The holographic method is a refined and promising 3D flow measurement technique but the limitation for the time being is the size and the resolution of the imaging devices in the hologram recording process, which inevitably limit the measurement volume size into the same order of the image sensor size. The particle depth extraction from the hologram reconstruction result is another issue to be solved.

The scanning technique and the tomographic technique are not only used in 3C-3D PIV but also in PTV. The basic principle of image capturing in these two methods is the same either in PIV or in PTV. In the scanning PIV or PTV, the laser light sheet is scanned at high speed perpendicular to the average viewing angle of two high-speed cameras in stereoscopic arrangement. The high-speed cameras capture stereoscopic particle images at each cross-section hit by the light sheet and the control PC stores the 3D velocity data as a time-multiplexed 3C-2D PIV or PTV result. The critical factor of this method is the time resolution of the laser light scan with respect to that of the unsteadiness of the flow to be measured. As a matter of fact the method is not applicable to high-speed and/or time-resolved flow measurement.

By contrast, the tomographic PIV or PTV has much less limitation from the viewpoint of applicability. In addition, the tomographic method seems to overcome many of the technical issues encountered in the earlier methods. It is a fully 3C-3D flow measurement method with much less limitation of volume size and flow speed and the measurement result is time-resolved up to the maximal frame rate of the imaging devices. But this method has its own specific technical issues to be solved, which are firstly related to the current capacity of the major hardware like volumetric light source intensity, synchronized high-speed image capturing and data transfer by more than three high resolution cameras and costly computation for volumetric reconstruction. The secondary but no less important issues are the lack of information regarding the detailed process of reconstruction algorithm, the optimization of computation parameters and the methodology for systematic removal of ghost particles. And these second issues apply much more in the tomographic PTV than in the tomographic PIV because most of the earlier and ongoing research works on the tomographic application are concerned only with PIV.

So in the present work, the authors proceed on some fundamental steps for establishing the standard methodology in the algorithms of tomographic particle tracking velocimetry. A volumetric reconstruction is carried out from a rather limited number of camera views based on the algebraic tomography scheme. The major focus is directed towards the extraction of individual particles from the tomographic reconstructed voxel domain and the suppression of noise components including the ghost particles. A number of new algorithms are applied to deal with these issues which are often the specific problems of the particle tracking velocimetry. The new algorithms are tested using the VSJ (Visualization Society of Japan) PIV standard image\(^{(10)}\), often regarded as a benchmark of synthetic particle image by PIV and PTV.
2. Tomographic Reconstruction Principle

The tomographic PIV, firstly proposed by Elsinga et al.\textsuperscript{(5)}, is based on the tomographic reconstruction of instantaneous volumetric particle field intensity distributions from multiple camera 2D projections and the subsequent 3D cross correlation to determine the 3D particle displacements. 3D particle field reconstruction is performed by iterative reconstruction techniques which are highly computationally intensive and memory inefficient. So, the methods to circumvent the use of memory intensive weighting matrices for 3D particle reconstruction have been introduced. Mass et al.\textsuperscript{(11)} used a method based on a multiple projective transformation of each camera image into one object space using the transformation parameters derived from the camera orientation and calibration instead of the iterative techniques, while Atkinson and Soria\textsuperscript{(12)} limited the weighting matrices in regions with particles using the principle of multiplicative line-of-sight (MLOS) applied to tomographic PIV. However, the cost intensive iterative process still remains. Using telecentric lenses and the combination of the epipolar geometry and earlier tomographic reconstruction approach by Atkinson and Soria\textsuperscript{(12)}, Kitzhofer and Br"ucker\textsuperscript{(13)} introduced a new simplified algorithm which avoids the iterative process and eliminates the cost intensive weighting matrix. The present authors have also tested a PTV oriented tomographic reconstruction scheme with simulated particle images\textsuperscript{(14)}.

The basic idea in the tomographic reconstruction is to divide an illuminated volume into cuboids (voxels) of a pre-defined size with a resolution adapted to the camera resolution. Based on the projection of a multi-camera system with their orientation known from the prior calibration procedure, the 3D light intensity distribution of the observation volume are reconstructed. Every pixel of the images from multi-camera views are projected into the voxel space through the projection center such that every voxel, which is hit by the projected ray, gets a grey value obtained by interpolation from the originating pixel. As a result, the voxel space will contain multiplicatively accumulated image intensity information of the instantaneous particle constellation and the voxels at valid particle positions will show very high values, while all remaining voxels will show rather low values.

The integral description of pixel intensity can be discretized to a voxel model and each pixel intensity $P_i$ can be expressed as the sum of weighting matrices $W_{ij}$, storing the contribution of each voxel $j$ to a pixel $i$, multiplied by the voxel intensity $I_j$ along the line-of-sight:

$$P_i \approx \sum_j W_{ij} I_j$$

Eq. 1 represents an equation for the calculation of pixel intensity from the voxel intensity distribution of the 3D illuminated volume. In the real experimental scenario, there are a limited number of synchronized camera views and the camera sensors record the projections of common laser illuminated particles seeded in the fluid volume. So, the problem is reversed and the reconstruction of 3D particle constellation using the intensity variations on the camera image planes is necessary. This results in an underdetermined system of linear equations such that an iterative correction of the voxel intensity is required until the equations converge within a predefined error limit.

This type of underdetermined matrix problems are often solved by using algebraic reconstruction methods. In the present work, the authors have made use of the multiplicative algebraic reconstruction technique (MART)\textsuperscript{(5)} which involves multiplicative correction to voxel intensity in each iteration $k$ based on the projected pixel intensity as given by Eq. 2. In this approach, the intensity of each voxel is corrected to match the intensity of one pixel at a time:

$$I_{ij}^{k+1} = I_{ij} \left( \frac{P_i}{\sum_j W_{ij} I_j} \right)^{\mu W_{ij}}$$

where $\mu$ is a relaxation parameter.
When the intensity of each pixel from a camera view is projected to the object space along the line of sight, those voxels which fall on the line of sight get the gray values equivalent to the contribution of each voxel to each pixel. This contribution factor is defined as a weighting function and thus depends on the camera orientation and the measurement volume configuration. The line of sight is actually defined by the camera calibration mapping function in Eq. 3, which is determined by using the 3D points in the observation space $X$ and the 2D camera image points $x$.

$$x = F(X)$$  \hspace{1cm} (3)

The method described above stands for the basic steps employed in every tomographic reconstruction method. The following sections give the entire description about the current implementation of the methodology employed for the tomographic reconstruction and the particle extraction.

### 3. Estimation of weighting function

One efficient technique for the estimation of the weighting matrix has been proposed by Atkinson and Soria\(^{(12)}\). In this technique, the camera pixels are projected along the line of sight in the form of a circular cylinder. The overlapping volume between the cylinder and the intersected voxels is assigned as a weight for each voxel. The analytic method of weight estimation becomes computationally intensive since it is often calculated repetitively to cut down the static memory space.

The present authors have made use of a simpler estimation of the weighting function. At first, a camera calibration mapping function is calculated by using the linear calibration technique proposed by Hall\(^{(15)}\) which defines the line of sight. Then, a line is projected from a center of a pixel along this line of sight such that it intersects at a point in XY plane in a voxel space. The distance between the intersection point and the center of the voxel is calculated and the weight is computed as a Gaussian function of the distance as shown in Fig. 1 and Eq. 4

$$W_{ij} = \exp(-k \times D)$$  \hspace{1cm} (4)

where $k$ is a constant parameter.

For a 1024×1024 pixel camera sensor projected into a 1000×1000×200 voxel grid, the weighting matrix has a size of approximately 420TB. The space complexity can be reduced by selectively processing the voxels and pixels during the reconstruction process. Each pixel is likely to see only a small portion of the voxels such that the mapping relation between pixel and voxel can be reduced to a very limited number of voxels in each z-plane. Further, if the average size of particle is considered of about 3 pixels (or voxels), there are a very limited number of pixels and voxels with the intensity value higher than the background intensity \(^{(12),(19)}\). The exclusion of these unwanted pixels and voxels as shown in Fig. 2 boosts up the intensity update process of the reconstruction. A limited number of pixels in the camera image are projected to the limited number of voxels by applying the iterative correction schemes as

![Fig. 1 Estimation of weight function.](image-url)
in Eq. 2. In this way, the memory allocated to the weighting matrices as well as the memory allocated to the voxel intensity field including the temporary field that will be used for intensity enhancement can be significantly reduced.

The process is repeated for all the camera views and, thus, the voxel space will contain multiplicatively accumulated image intensity information of instantaneous particle constellation. Theoretically, only the voxels at valid particle positions will show high intensity values but there is generation of particle artifacts (hereafter referred as ghost particles) in the real reconstruction process. The top view layout of the generation of real and ghost particles is shown in Fig. 3. The detailed process of removal of these particle artifacts is described in the next section.

4. Particles Identification and Tracking

A by-product of every tomographic particle reconstruction method is the generation of ghost particles. The ghost particles are defined as intensity peaks that do not correspond to actual measured particle locations but rather are created by the presence of multiple particle locations that can satisfy the recorded 2D images. The algebraic reconstruction techniques (MART, SMART, etc.) spread out the voxel intensity across these locations so that the ghost particles tend to have a lower intensity than the real particles\(^5\), \(^12\). But the difference gets diminished as the number of real particles increases and this effect can be the dominant source of errors in tomographic PIV measurements when the seeding density is increased\(^5\). Some methods have been introduced to reduce the intensity and influence of these ghost particles using the information contained in the subsequent exposures of a particle intensity field during the reconstruction process\(^16\), \(^17\). But these methods perform best only when followed by the step of cross correlation based tracking. With the exception of the work of Kitzhofer and Brucker\(^13\) in which a Gaussian fit was used to extract the particles from the reconstructed volume, all the earlier and ongoing research works on the tomographic application are focused towards PIV.
Fig. 4 Three tested Gaussian particle masks composed of 3×3×3, 5×5×5 and 7×7×7 voxels with a particle of 3 voxels diameter.

The aim of PTV is the reconstruction of individual particle paths or trajectories, thus a proper treatment is necessary to detect and remove ghost particles. Therefore, many efforts have been made to enhance the reconstructed volume and extract real particles from the reconstructed voxel domain. In the present work, a two step scheme is proposed for such a combined purpose. The first step is the enhancement of the reconstructed voxel domain by maximizing the difference of some 3D spatial parameters between the real and ghost particles. Then the second step is the discrimination of real and ghost particle voxels by exploiting the calculated results of the 3D spatial parameters.

All the operations that have to be done are carried out in the voxel domain and a \( N \)-connected neighboring scheme is introduced in these operations. The \( N \)-connected neighboring scheme can use 6, 18 or 26 connected neighbor voxels in any relevant operations (labeling, opening and closing, mask filtering, etc.) and the choice is made depending upon the type of operation and the time for process completion.

4.1. Reconstruction enhancement and removal of ghost particles

Due to the intensity variation of the particles in the 2D camera image sequence as well as in the reconstructed voxel space domain, all clusters of the real particles do not always have the higher intensity value. The global threshold binarization (hereafter referred as GTB) merely on the basis of a fixed threshold intensity level removes not only the ghost particles but also the low intensity real particles. So, more refined processes have to be introduced to enhance the intensity level of the real particles. In many cases, the intensity of the real particles is often expressed as a Gaussian distribution, while the ghost particles may fail to comply this criterion. By exploiting this criterion, the reconstructed voxel intensity \( I_{\text{new}} \) is match-filtered by using a Gaussian mask correlation (hereafter referred as GMC) operator as defined by Eq. 5.

\[
I_{\text{new}} = \frac{\sum \sum (G - g)(I - m)}{\sqrt{\sum \sum (G - g)^2(I - m)^2}}
\]

where \( G \) is the Gaussian mask voxel, \( g \) is the mean Gaussian, \( I \) is the voxel intensity and \( m \) is the mean voxel intensity.

The output of this Gaussian mask operation is the normalized cross correlation coefficient computed between the mask and the local voxel intensity, the distribution of which is expected to show clear and distinct intensity peaks at the locations of individual particles. And this distribution of the cross correlation coefficient may be utilized for separating the ghost particles from the real ones. The three different types of Gaussian masks with typical particle diameter of 3 voxels, shown in Fig. 4 are used in the present study. The output of this GMC operation is binarized voxel domain and a cross correlation coefficient is used to separate the real particles from the background and the ghost particles. But in fact, many ghost particles followed the Gaussian intensity profile and they could not be sufficiently eliminated regardless of the choice of the Gaussian mask size.

A new idea here is the use of the dynamic threshold binarization (hereafter referred as DTB) scheme applied to a 3D voxel space. In this scheme, an iterative binarization is carried out in the entire voxel space by incrementing the threshold level one by one, labeling the
binarized voxels and calculating the intensity peak volume at each cluster of labeled voxels. At each step of this iteration, only the clusters of labeled voxels whose peak volume exceeds a certain threshold level are picked up as individual particles and then excluded from further increment of the threshold level. Finally, all the picked up voxel clusters are binarized to yield the final binary distribution of voxel intensity. With this approach the extraction of individual particles can be done more successfully regardless of their intensity value and profile. In addition, the separation of closely located two or more particles can be less difficult by tuning the threshold parameter. In order to detect the ghost particles from among the constellations of binarized particles, this binarization scheme can be further refined by imposing another minimum threshold to the intensity peak volume and, additionally, by estimating some other geometric parameter. The minimum threshold of the intensity peak volume is effective for filtering out the very small size voxel clusters which may be regarded as intensity noise. An additional geometric parameter to be estimated is the broadness factor of the intensity peak, which has been found to vary to some extent between the real and ghost particles.

All these enhancement steps have trade-off factors associated with each other. If the 3D spatial parameters are adjusted for the removal of ghost particles, the real particles also gets vanished. Furthermore, when the grid size for the reconstruction is reduced to increase the spatial resolution, the adjustment of the spatial parameters has a smaller effect as the reconstruction quality gets deteriorated due the increase of ghost particles.

As a result, a new perception idea is necessary for detecting the remaining ghost particles in the reconstructed voxel space. The new idea in the present work makes use of the back projection of the voxel intensity of reconstructed particles onto the pixel intensity of each camera image plane. But before checking the back-projected intensity, the location of the back-projected pixel must be examined.

Based on this observation, a complete particle extraction scheme illustrated in Fig. 5 is introduced. At first, the reconstructed voxel domain is processed with enhancement schemes and thereby binarized. After that comes the process consisting of the labeling of bright (non background) voxels and the centroid calculation of labeled voxels (detected particles). Then, each centroid in the voxel space is back-projected onto the camera image planes as in Fig. 6 and the corresponding pixel in each image plane is calculated by using the camera mapping function. Ideally, the ghost particle must not be visible in the projected image space.

But, in the case of the reconstruction based on the line of sights, all the calculated pixels...
lie inside the camera image plane and it is not possible to identify all the ghost particles merely on the basis of the calculated pixel. So, the first criterion for detecting ghost particles is the back-projected intensity on image plane.

In this case, the back projected pixel intensity of the real particles is expected to be higher than that of the ghost particles. Based on this observation, the ghost particles were first detected by thresholding the back-projected intensity on each of the image planes on the "AND" basis. More specifically, a reconstructed particle was regarded as ghost particle, even if the back-projected intensity on a single image plane was less than a threshold level. As a result this strategy was found to be too crucial so that some of the real particles were also rejected. Also, we have to consider the fact that the real particles are not always visible from all the cameras. So, the next strategy is to choose the number of valid cameras $N_T$ actually viewing the particle centroids and use the product of the back-projected intensities $MI_g$ on all those valid image planes. The idea was inspired by the concept of multiplicative first guess (MFG)(19), which aimed to rapidly identify the point of interest in a volume by multiplication of the threshold intensities from each camera. This strategy worked better than the first one but the determination of the threshold level in this strategy became a more delicate factor because of the variable pixel intensity derived from the particles with different brightness levels. This strategy eventually worked best, if it used the product of the back-projected intensities $MI_g$ divided by the related voxel intensity $V_g$. In this last strategy, the key factor is the normalization of the back-projected intensity, which facilitates the determination of the threshold level $IV_T$. And this last process filtered out a large number of ghost particles and a comparatively clean voxel domain was reconstructed.

4.2. Determination of 3D particle centroids in voxel space

A single particle in voxel space is defined by a cluster of bright neighboring voxels. When a cluster is detected, the centroid position can be determined by using some averaging method. In the present work, a particle centroid $P_g(X_g, Y_g, Z_g)$ is calculated by the weighted averaging of all $M$ voxels occupying the particle cluster $P_m(X_m, Y_m, Z_m)$ as shown in Eq.6.

$$
X_g = \frac{\sum_{m=1}^{M} I_m X_m}{\sum_{m=1}^{M} I_m}, \quad Y_g = \frac{\sum_{m=1}^{M} I_m Y_m}{\sum_{m=1}^{M} I_m}, \quad Z_g = \frac{\sum_{m=1}^{M} I_m Z_m}{\sum_{m=1}^{M} I_m}
$$

(6)
4.3. Quality of reconstruction

Since the present method is based on the detection of individual particles, the quality of reconstruction is not evaluated by the equation as given by Elsinga(5). The quality and accuracy of reconstruction is rather evaluated by comparing the results of the ghost particles removing techniques described in Section 4.1 and by the percentage of the finally validated particles with respect to all the real (theoretical) particles in the voxel space. The accuracy is measured in terms of the root-mean-square (rms) error for individual coordinates or the rms deviation. The rms deviation hereafter refers to the distance between the theoretical 3D coordinate and the reconstructed 3D coordinate.

4.4. Time differential tracking for volume PTV

Finally, the reconstructed space will contain gray value-weighted voxels representing the instantaneous particle constellations with the known 3D spatial coordinates. These spatial coordinates represent the location of the particles at one time step \((t = t_n)\). If another set of 3D spatial coordinates after a certain time interval \((t = t_{n+1})\) are considered, then the particle tracking between the two successive sets of particle coordinates \(P_1\) at \(t = t_n\) and \(P_2\) at \(t = t_{n+1}\) can be performed. For this particle tracking, a set of associations (matching) must be established between all the particles in these two time differential voxel domains. Further, if the same process is repeated for each set of 3D spatial coordinates taken after a fixed time interval, the time differential voxel space representation of the Lagrangian particle trajectories is possible and thereby, a full 3C-3D volume tracking result can be visualized. The overall summary of the steps to be followed in this time differential tracking is illustrated in Fig. 15. The task to find out a set of associations between two successive time steps can be done by using the nearest neighbor scheme(20) or some other optimization schemes(21)–(23). In the present study, the SOM (self-organizing map) neural network optimization is used to establish the associations(23).
5. Results and Discussions

For the verification of the proposed methodology, the present authors have chosen the 3-D PIV standard images data available from the Visualization Society of Japan\(^{(10)}\). These data are composed of various sets of synthetic time-series particle images generated from the direct numerical simulation results of a 3D impinging jet flow in a square cavity. One advantage of the use of these standard images is that the particle image data sets are accompanied by the theoretical time series coordinates data of the particles used in the generated images, so that one can compare the image analysis results of particle identification and tracking always with the true theoretical data.

In the case of Series 351 and 352 of the 3-D standard images, three cameras, each with pixel resolution of 256×256 and single pixel size of 0.0245 mm, are assumed to install in the direction of a common flow volume. The viewing angles of the cameras are respectively 30°, 0° and −30° with respect to the vertical line normal to the laser light sheet. The two series of images represent the same flow volume with the same viewing angles but with different particle density, that is 0.030 ppp (particle per pixel) in Series 351 versus 0.0046 ppp in Series 352. The first set of particle images of Series 352 are shown in Fig. 8. The camera calibration parameters and the relevant mapping functions are calculated by using the calibration images supplied with the tested time-series particle images. By using these mapping functions and the geometric relations described in Fig. 1 and Eq. 4, the weighting matrices \( W_{ij} \) for all the camera pixels are estimated, which are then used to perform the volumetric reconstruction of the particles. In the present study, the volumetric reconstruction was carried out in a measurement volume of 36×26×10 mm\(^3\) and the voxel grid size was varied from 180×130×50 voxels to 360×260×100 voxels, for which the single voxel size is from 0.2 mm to 0.1 mm.

In the first step of reconstruction, the voxel intensity was initialized as unity at all the voxels. The voxel grid size was 360×260×100 voxels and the single voxel size is 0.1 mm. Then the voxel intensity was iteratively updated according to the MART. With this algorithm, in most cases, the voxel intensity calculation was completed in 10 iterations for a fixed value of relaxation factor \( \mu \) at 2.0. In this reconstruction step, the total number of valid voxels i.e the voxels on the line of the sight are found to be 10 to 30% of the total number of voxels while the number of valid pixels are found to be about 10% after pre-processing of the the particle images. So in the present implementation of MART algorithm, these valid voxels and pixels are selectively used in order to increase the memory and computational efficiency. After the reconstruction, the voxel intensity was normalized in the range from 0 to 1.0.

Then, as a first trial, the GTB binarization scheme was applied to the reconstructed voxel intensity with a threshold level of 0.19 and, after the labeling process, the centroids of particles constellation were calculated. As a matter of fact, the lower the threshold level the higher is the density of the particle constellation. The reconstructed result of the voxel intensity derived from the first image set of Series 352 is shown in Fig. 9 (a). In this figure, the voxel intensity is not plotted by squares or cuboids but by circles of nearly the same size. To make the figure
more informative and clear, the plot excludes all the normalized voxel intensity below the binarization threshold level of 0.19. The higher intensity value in this voxel plot indicates the presence of the particles. This method of plotting the voxel intensity will be retained in all the reconstruction results hereafter.

As illustrated in Fig. 9 (a), the voxel space reconstruction by the GTB scheme is sparse due to the removal of an increased number of low intensity particles. Another issue is the excessive randomness of the reconstructed particle size, which is a specific problem of the global threshold scheme. One simple remedy for these problems would be the reduction in the voxel grid resolution. But if it is actually done, the number of noisy particles is decreased at the expense of the deterioration of accuracy in the real particle centroid. So, to enhance the reconstruction quality without reducing the voxel grid resolution, the GMC scheme was instead applied to the reconstructed volume and binarization was done on the basis of the GMC match filtered voxel intensity. The reconstructed result of this GMC scheme is shown in

![Reconstructed voxel intensity plot of the first image set of Series 352.](image)

Fig. 9  Reconstructed voxel intensity plot of the first image set of Series 352.

![X-Y and X-Z plane views of the voxel intensity plot in Fig. 9 (d).](image)

Fig. 10  X-Y and X-Z plane views of the voxel intensity plot in Fig. 9 (d).
Fig. 9 (b). From this result, it is observed that the number of the noisy particles was drastically decreased and the randomness of particle size was largely improved, though the computational time was increased to some extent.

In spite of the improvement in reconstruction quality, the result of the GMC scheme still shows a certain number of connections of practically separate particles in highly populated space areas. So, in order to enhance the reconstruction quality furthermore, the GMC match filter process was followed by the DTB binarization scheme. The results of the reconstructed voxel intensity distribution was rather noisy due to an increased number of small size (1 to 2 voxel) particles. Bearing this in mind, the DTB scheme was then directly applied to the reconstructed voxel intensity distribution which not only revived the particles of lower voxel intensity but also enhanced the separation of individual particles. This effect of the DTB scheme can be seen in the voxel intensity result in Fig. 9 (c).

Although this last figure shows a visually successful reconstruction result of particles volume, the ghost particles from the most probably origin (shown in Fig. 3) are still present. Then, the recursive filtering scheme of ghost particles by using the back projection of the reconstructed voxel intensity onto the camera pixel intensity was finally applied to the voxel intensity distribution. Specifically, in the flow chart in Fig. 5, the number of the valid cameras $N_T$ was 2 and the threshold value $IV_T$ was evaluated as the ratio of the mean product of the pixel intensity $M_{Ig}$ with respect to the $N$-connected mean voxel intensity $V_g$ of the actually reconstructed particles. Fig. 9 (d) shows the final voxel reconstruction result after the recursive filtering scheme. In this figure, all the voxel clusters, of which the centroid does not satisfy the back projection criterion are filtered out. And from the quantitative viewpoint of reconstruction, the number of the finally reconstructed particles is 67, whereas that of the synthetically produced particle images in the voxel space is 58.

Another set of reconstructed voxel intensity after the recursive filtering scheme is presented in Fig. 10, where the voxel intensity plot in Fig. 9 (d) is projected on the X-Y as well as X-Z planes. From this figure, the location and the intensity of the finally detected particles are more clearly identified and the particle distribution density in relation to the particle size is better perceived. Another aspect of this type of projected voxel intensity plot that this can be used to estimate the overall accuracy in the measured centroid positions. But before doing so, the quality of the final reconstruction must be checked by comparing them directly with the locations of the synthetically produced particle images. In the case of the final reconstruction using the Series 352 particle image data set, out of the 67 reconstructed particles, 43 were considered as real particles if the error allowance level between the measured and theoretical particle centroid is considered as 0.5 mm in the real physical scale. On the basis of this reconstruction quality check, the deviation of the measured particle centroid from the theoretical position in the X-Y as well as X-Z planes is plotted as a four-quadrant diagram in Fig. 11. The rms error of the 3 components of the 3D particle coordinates is also mentioned in the caption of the figure, according to which the quality of the z component recovery is found to be deteriorated with respect to other two components.
In the second test, to check the quality of the reconstruction furthermore with a higher number of particles, the first set of images from Series 351 image data set was also investigated with the same processes. The volumetric reconstruction parameters are the same as indicated above. One of the noticeable change of this higher particle density data set is the significant increment of the computation time for the voxel intensity reconstruction as well as for the particles extraction process. The voxel intensity results of the final two steps of the present novel reconstruction scheme are shown in Fig. 12, which indicate a good quality of reconstruction even for the higher particle density at 0.03 ppp. The deviation of the measured particle centroid from the theoretical position in the X-Y and X-Z planes is plotted as a four-quadrant diagram in Fig. 13, together with the mention about the rms error of the 3 components of the 3D particle coordinates. The maximum rms error is found in the z component recovery with an increased value of 0.172 mm.

In the third test, the present particle reconstruction method was examined for different numbers of theoretical (synthetically produced) particles in the reconstruction volume. The

<table>
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<th>Case</th>
<th>Theoretical particles</th>
<th>Final reconstructed particles</th>
<th>Validated particles</th>
<th>Percentage of validated particles</th>
<th>RMS deviation (mm)</th>
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<td>IV</td>
<td>359</td>
<td>957</td>
<td>264</td>
<td>73.5</td>
<td>0.158</td>
</tr>
<tr>
<td>V</td>
<td>875</td>
<td>1896</td>
<td>631</td>
<td>72.1</td>
<td>0.163</td>
</tr>
<tr>
<td>VI</td>
<td>1350</td>
<td>2682</td>
<td>942</td>
<td>69.8</td>
<td>0.166</td>
</tr>
</tbody>
</table>
control of the number of real particles was done by changing the z-direction depth of the reconstruction volume as well as by changing the particle density between Series 351 and 352. The examined results are summarized in Table 1, where the first three experiments (Case I to III) were performed on the first image set of Series 352, while the last three experiments (Case IV to VI) were from Series 351. The “final reconstructed particles” in the table indicate the particles recovered after the complete processes of Fig. 5 and the “validated particles” stand for the particles existing within the error allowance level with respect to the theoretical location. It is observed that the percentage of the validated particles in reconstruction is generally reduced, if the voxel grid size is fixed and the particle number and/or the particle density are increased. It is considered that this is due to the number of the ghost particles in the reconstructed voxel space, which is significantly increased and has a potent influence on the accuracy of the individual particle detection.

At this step, additionally, the performance of individual particles recovery was compared with the conventional stereoscopic particle tracking approach (6). This comparative test was done with all the six cases as mentioned above and the results of the number of extracted particles and the rms deviation of reconstructed coordinates are summarized in Table 2. The “final matched particles” in the table indicate the particles which were successfully matched in all the three camera views. It should be noted that the number of matched or validated particles is relatively low here probably because many visible particles are concentrated within a rather thin layer of laser sheet which is subject to a Gaussian intensity distribution in depth direction. The significant difference of the validated particles count for the low concentration and high concentration of particles is indicative of the limitation of the spatial resolution of the stereoscopic particle tracking approach. The recovery ratio of particles within the same volume for the tomographic PTV is more or less constant as indicated in the Table 1 even at the higher particle density.

![Graph showing percentage variation of validated particles](image)

**Fig. 14** Percentage variation of the number of validated particles with three different enhancement schemes and with two different voxel grid size.

Finally the present particle reconstruction method was tested for 3 different enhancement schemes (GTB, GMC and DTB) followed up with the recursive filtering process. In all these tests, the particles were extracted as described in Fig. 5 with three different enhance-
ment schemes applied separately. The results are presented in terms of the percentage of the number of the validated particle with respect to that of the theoretical particles as shown in Fig. 14. In this figure, additionally, all the percentages are estimated with two different voxel grid size (0.1 or 0.2 mm). As understood from this figure, the number of reconstructed particles is largest in the case of the DTB binarization at lower and middle numbers of theoretical particles. The number of reconstructed particles with the DTB binarization is also found to be slightly deteriorated at highest number of theoretical particles, where the GMC match filter scheme outperforms the DTB scheme to a small extent. As far as the effect of the voxel grid size is concerned, the smaller grid size (0.1 mm) is obviously advantageous over the larger grid size (0.2 mm) by a factor of 10 to 20% from the viewpoint of the number of validated particles, though the results of the smaller grid size are found to be more fluctuant according to the experimental conditions. This is probably indicative of the fact that smaller grid size is highly sensitive to the high frequency noise of the reconstructed voxel intensity distribution.

Table 3 Comparisons of the percentage of validated particles with three different enhancement schemes and with two different error allowance levels

<table>
<thead>
<tr>
<th>Case</th>
<th>GTB 0.5</th>
<th>GMC 0.2</th>
<th>DTB 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>75.9</td>
<td>75.9</td>
<td>79.3</td>
</tr>
<tr>
<td>II</td>
<td>73.7</td>
<td>73.7</td>
<td>80.3</td>
</tr>
<tr>
<td>III</td>
<td>76.6</td>
<td>76.6</td>
<td>82.4</td>
</tr>
<tr>
<td>IV</td>
<td>79.1</td>
<td>68.8</td>
<td>82.5</td>
</tr>
<tr>
<td>V</td>
<td>72.5</td>
<td>57.3</td>
<td>79.7</td>
</tr>
<tr>
<td>VI</td>
<td>64.4</td>
<td>47.9</td>
<td>73.9</td>
</tr>
</tbody>
</table>

Table 4 Comparisons of the particle extraction time and the rms deviation with three different enhancement schemes

<table>
<thead>
<tr>
<th>Case</th>
<th>Tomographic reconstruction time (s)</th>
<th>GTB 0.5</th>
<th>GMC 0.2</th>
<th>DTB 0.5</th>
<th>GTB 0.5</th>
<th>GMC 0.2</th>
<th>DTB 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3.94</td>
<td>18.87</td>
<td>65.1</td>
<td>37.6</td>
<td>0.049</td>
<td>0.105</td>
<td>0.048</td>
</tr>
<tr>
<td>II</td>
<td>8.86</td>
<td>39.0</td>
<td>169</td>
<td>123</td>
<td>0.087</td>
<td>0.113</td>
<td>0.096</td>
</tr>
<tr>
<td>III</td>
<td>14.4</td>
<td>117</td>
<td>385</td>
<td>325</td>
<td>0.117</td>
<td>0.128</td>
<td>0.117</td>
</tr>
<tr>
<td>IV</td>
<td>14.5</td>
<td>69.8</td>
<td>172</td>
<td>135</td>
<td>0.152</td>
<td>0.175</td>
<td>0.160</td>
</tr>
<tr>
<td>V</td>
<td>36.5</td>
<td>475</td>
<td>1080</td>
<td>780</td>
<td>0.169</td>
<td>0.192</td>
<td>0.1702</td>
</tr>
<tr>
<td>VI</td>
<td>55.7</td>
<td>773</td>
<td>1800</td>
<td>1410</td>
<td>0.228</td>
<td>0.236</td>
<td>0.218</td>
</tr>
</tbody>
</table>

It has been seen from Fig. 14 that the extraction efficiency changes as the voxel grid size is varied in the reconstruction process itself. So, the result with the smaller grid size is tabulated for another comparison of three different enhancement schemes in Table 3. In this table, the percentage of validated particles and the rms deviation of coordinates are estimated with one fixed grid size (0.1 mm) but with two different error allowance levels of 0.5 mm and 0.2 mm. Here, the general trend is the decrement of the number of valid particles as the image particle density is increased and the GMC and DTB schemes show almost 10% higher number of validated particles as compared to the GTB scheme. The superiority of the DTB scheme is also recognized when the error allowance level is decreased from 0.5 to 0.2 mm. The comparison of these enhancement schemes is also made in terms of the computational time and rms deviation of reconstructed coordinates as in Table 4. The computation time for each enhancement scheme shown in the table is the summation of the calculation time for that enhancement scheme followed by labeling and weighted averaging processes and the final particle extraction by recursive validation scheme. The higher computation time indicates that the large number of particle centroids are detected from the intensity constellation by the enhancement scheme. The GTB scheme is observed as the most computationally efficient one while the GMC scheme is the computationally intensive one. The rms deviation is more or less similar in all the enhancement schemes with the maximum rms deviation being 0.236 mm at the highest particle density.

In the last test, the first two time differential image sets of the PIV standard image Series
were processed with the same reconstruction process as described above and the particle tracking between these two time steps was performed by using a SOM neural network tracking algorithm\(^{(23)}\). The resultant full 3C-3D tracking result in the voxel space is depicted in Fig. 15 in the form of particle trajectories. Since the 3D coordinates of particles for this tracking were calculated from the voxel reconstruction results, the accuracy of the matching is largely dependent on the accuracy of the calculated 3D coordinates of the particles as well as on the robustness of the tracking algorithm in the presence of loss-of-pair particles. In this regard, the SOM neural network tracking algorithm\(^{(23)}\), as demonstrated by two of the present authors, is highly adaptive to time differential tracking with loss-of-pair particles. Hence, the better the voxel intensity reconstruction and the subsequent particle extraction schemes, the better is the final result of tracking as shown by Fig. 15.

**6. Conclusion**

A new tomographic reconstruction scheme of particle seeded flow volumes using the algebraic reconstruction technique was developed and tested for the use in 3C-3D particle tracking velocimetry. In the tomographic reconstruction process, we proposed an effective method to calculate weighting matrix for valid voxels and pixels. In the process of individual particle extraction and validation, different types of voxel intensity enhancement schemes are successfully applied and a new recursive validation scheme using a concept of voxel intensity back projection is also successfully applied with the reduction of the number of ghost particles in the reconstructed domain. Although the tomographic reconstruction attempted so far was limited to the synthetic image tests, the results of the particle centroid recovery and the subsequent time differential tracking of individual particles showed promising recovery ratio and accuracy which may be retained also in the case of practical particle tracking with experimental images. Further improvement and refinement of the methodology are required to increase the reconstructed voxel space resolution and to boost up the overall accuracy but this is a trade-off issue with the CPU computational power and the memory capacity of the current PC systems.

**References**

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(16) Novara, M., Batenburg, K.J. and Scarano, F.: Motion Tracking Enhanced MART for Tomographic PIV, Measurement Science and Technology, 21-3 (2010), 035401


