Validation of Integral Method for Numerical Prediction of Hybrid Rocket Internal Ballistics*

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Abstract

In order to design hybrid rocket engines, we developed a numerical prediction method which describes the internal ballistics, thus the solid fuel regression rate. In order to obtain a fast code, simple but efficient models have been implemented. The fluid dynamics and the thermal heat conduction in the solid fuel have longer characteristic times than chemical phenomena, hence time-dependency is required. The regression rate is evaluated by solving the energy-balance equation at the solid fuel surface. The code validation is made by comparing calculation results with open literature experimental data. This comparison confirmed that the numerical estimation of time- and space-averaged regression rates has the same order of magnitude as the experimental data of time- and space-averaged regression rates. However the dependency of regression rate on oxidizer mass flux differs between the calculations and the experiments. This difference could be mainly due to the use of only convective heat transfer between flame and fuel surface. Nevertheless, by considering also the radiative heat transfer, it is possible to obtain a higher accuracy in the calculation of the regression rate dependency on oxidizer mass flux.

Key words : Hybrid Rocket, Internal Ballistics, Numerical Prediction, Validation, Heat Feedback

1. Introduction

Hybrid rocket propulsion is considered a promising technology for the next generation of space transportation. A hybrid rocket typically features the fuel in the solid phase, while the oxidizer in the liquid or gas phase. If compared with solid rocket propulsion, hybrid technology is characterized by intrinsic safety and high theoretical specific impulse. Nevertheless, some technical drawbacks have to be overcome before developing hybrid rocket propulsion into practical space applications. Low fuel regression rate, low combustion efficiency and combustion instability are main disadvantages. As one can see in Fig. 1, the hybrid combustion mechanism is characterized by a diffusion flame within the turbulent boundary layer developed on the fuel surface. The gasification of the solid fuel is promoted by heat transfer from the flame, with convective and radiative mechanisms, to the fuel surface. The rate of gasification depends on many factors, such as convective and radiative heat transfer, blowing effect and heat conduction inside the solid fuel.

Many researchers developed the simulation codes for fuel regression rate prediction, such as Antoniou et al., University of New Orleans⁽¹⁾. However, these simulation analyses consider short combustion times, less than about 10 seconds in two-dimensional cases while no more than 20 seconds in one- or zero-dimensional cases. Concerning combustion instability, for example, a specific simulation code was developed by Stoia-Djeska et al., Politehnica University of Bucharest⁽²⁾. Nevertheless, this code has been applied only to few cases and a

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Fig. 1 Schematic of boundary layer combustion.

parametric study has not been conducted.

We have developed a design tool for hybrid rocket internal ballistics analysis, such as the estimation of fuel regression rates under various operating conditions, and parametric study. We did not consider a rigorous turbulent flame mathematical model (i.e. direct numerical simulation) because of its high calculation cost. Hence we have employed a faster model, which includes quasi-one-dimensional flowfield and one-dimensional thermal conduction into the solid fuel. In this model, the fluid dynamics and the thermal heat conduction into the solid fuel have longer characteristic times than chemical phenomena. On the other hand, characteristic times of combustion reactions are significantly short and quasi-steady flame can be assumed. In our previous study⁽³⁾, two combustion models were examined. One was the global single-step reaction model for 4 chemical species and the other one was the chemical equilibrium model for 9 chemical species. As a result, the former model was not appropriate to predict the state in the chamber, because the maximum temperature was enormously high. Therefore we use the latter model in this paper. In order to estimate regression rate, the energy-flux balance equation at the solid fuel surface is also solved.

The purpose of this paper is the validation of our method by comparing numerical results with experimental data available in the open literature. Furthermore, we discuss the dependency of fuel regression rate on oxidizer mass flux, considering the heat feedback from the flame to the solid surface.

Nomenclature

A : cross-sectional area, m^2	G_{ox} : oxidizer mass flux, $kg/(m^2 \cdot s)$
A_c : preexponential factor, $1/s$	L_p : port length, m
B_a : aerodynamic blowing parameter, -	N_c : number of cells, -
B_t : thermochemical blowing parameter, -	Q_c : heat of combustion per mass unit of mixture gas, J/kg
C_f/C_{f0} : ratio of skin-friction coefficient with/without surface blowing, -	R_e : residual of specific total energy, J/kg
$D_{p,0}$: diameter of initial port, m	R_p : residual of pressure, Pa
D_t : diameter of nozzle throat, m	R_u : universal gas constant, $J/(mol \cdot K)$
E_a : activation energy, J/mol	T : temperature, K
F : Helmholtz free energy of mixture gas with constraints, J/kg	T_0 : base temperature, K
G : total mass flux, $kg/(m^2 \cdot s)$	Y_i : mass fraction of chemical species i , -

a : constant parameter in Eq. (30), -	r : port radius, m
b_i : mole number of element i per mass unit of mixture gas, mol/kg	$\dot{r}$: fuel regression rate, m/s
$c_{p,i}$: specific heat of chemical species i at constant pressure, J/(kg · K)	t : time, s
e : specific internal energy, J/kg	t_b : burning period, s
$e_{reaction}$: degradation zone thickness, m	u : x -direction velocity, m/s
e_t : specific total energy, J/kg	v : y -direction velocity, m/s
f : Helmholtz free energy of mixture gas, J/kg	w_i : molecular weight of chemical species i , kg/mol
h_i : specific enthalpy of chemical species i , J/kg	x : axial location from head-end of the chamber, m
h_v : effective heat of gasification of the solid fuel, J/kg	y : location normal to the solid fuel surface, m
h_w : enthalpy of the fuel gas evaporating from the solid fuel surface, J/kg	Δh_i^0 : standard enthalpy of formation of chemical species i , J/kg
k : constant parameter in Eqs. (9) and (16), -	Δm_F : fuel mass loss, kg
l_p : perimeter of a cross-section, m	Δt : time interval, s
$\dot{m}_F$: mass addition from the solid fuel surface, kg/(m ² · s)	Λ : Lagrangian multiplier, J/mol
$\dot{m}_{ox}$: injection oxygen mass flow rate, kg/s	α : thermal diffusivity, m ² /s
n : constant parameter in Eq. (30), -	α_R : absorptivity, -
p : pressure, Pa	ε : emissivity, -
q : constant parameter in Eqs. (9) and (16), -	λ : thermal conductivity, W/(m · K)
$\dot{q}_R$: radiative energy flux, W/m ²	μ : viscosity, Pa · s
	ξ : mixture fraction, -
	π : circle ratio, -
	ρ : density, m ³
	σ : Stefan-Boltzmann constant, W/(m ² · K ⁴)

Subscripts

1 : fuel stream	$final$: final state
2 : oxidizer stream	fl : flame location
F : fuel	g : gas
P : combustion products	$initial$: initial state
S : solid fuel surface	r : reference
a : ambient	s : solid
e : boundary layer edge	

Superscripts

$\bar{}$: time- and/or space-average	n : time step
0 : constraint condition	

2. Mathematical Models and Numerical Methods

2.1. Configuration of the Hybrid Rocket Engine

A typical hybrid rocket engine is composed of the combustion chamber, which contains the solid fuel, the de Laval nozzle and the feeding system for the oxidizer (pressurization system, tank and injector). The fuel grain, considered in this paper, has a single circular central port. The oxidizer flow is axially injected into the chamber. In this simulation, the one-dimensional calculation domain corresponding to the axial direction involves only the combustion chamber and the nozzle as one can see in Fig. 2. The calculation domain in the chamber is divided into 300 cells while the domain in the nozzle is divided into 200 cells.

The solid fuel is high density polyethylene (HDPE) and the oxidizer is gaseous oxygen (GOX). It is assumed that, during gasification processes, each polyethylene chain is broken

Fig. 2 Calculation domain.

into ethylene molecules.

2.2. Quasi-One-Dimensional Flowfield

For simplicity, quasi-one-dimensional flowfield is assumed. The representative values of physical quantities are area-mean values, and their changes along the axial direction are considered. The flowfield is described by the compressible Euler equations with a conservation equation for the mixture fraction. The mass addition from the solid fuel surface is introduced as a source term.

$$\frac{\partial A\mathbf{Q}}{\partial t} + \frac{\partial A\mathbf{F}}{\partial x} = \mathbf{S}_{Q1D} + \mathbf{S}_{Mass},$$

$$\mathbf{Q} = \begin{pmatrix} \rho \\ \rho u \\ \rho e_t \\ \rho \xi \end{pmatrix}, \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ (\rho e_t + p)u \\ \rho \xi u \end{pmatrix}, \mathbf{S}_{Q1D} = \begin{pmatrix} 0 \\ p \frac{dA}{dx} \\ 0 \\ 0 \end{pmatrix}, \mathbf{S}_{Mass} = \begin{pmatrix} l_p \dot{m}_F \\ 0 \\ l_p \dot{m}_F h_w \\ l_p \dot{m}_F \end{pmatrix}. \quad (1)$$

These equations are solved numerically. Boundary conditions at head-end of the chamber are subsonic inflow conditions as shown in Table 1. At the nozzle exit, supersonic outflow conditions are imposed.

Mixture fraction ξ is defined as follows,

$$\xi \equiv \frac{b_C - b_{C,2}}{b_{C,1} - b_{C,2}} \quad (2)$$

where b_C is mole number of element C per mass unit of mixture gas. This parameter presents the ratio between fuel and oxidizer. It is related to temperature and chemical composition of combustion gas with chemical equilibrium calculation (see §2.3).

Specific total energy of mixture gas e_t is expressed as follows,

$$e_t = e + \frac{u^2}{2}, \quad (3)$$

$$e = \sum_i h_i Y_i - \frac{p}{\rho}, \quad (4)$$

$$h_i = \Delta h_i^0 + \int_{T_r}^T c_{p,i} dT. \quad (5)$$

Specific heat at constant pressure $c_{p,i}$ is a function of temperature. The function is obtained by fitting the JANAF table's data⁽⁴⁾.

The equation of state for ideal gas is

$$p = \sum_i \frac{Y_i}{w_i} \rho R_u T. \quad (6)$$

When Eq. (1) is solved, mass addition $\dot{m}_F$ must be obtained in advance, by solving the equation of energy-flux balance at the solid fuel surface (see §2.4).

Table 1 Inflow conditions.

Mass flow rate	Pressure	Temperature	Mixture fraction
given	zeroth-order extrapolation	300 [K]	0 [-]

2.3. Chemical Equilibrium

Many chemical species take part in the combustion reactions and a lot of elementary processes are involved at the same time. However when a combustion calculation step is completed, it is difficult that all chemical species and all elementary processes are considered. Hence the combustion model should deal with only significant chemical species and elementary processes during the combustion. Because of this consideration, a faster code is suitable using the chemical equilibrium model for 9 chemical species. With NASA CEA code⁽⁴⁾, chemical species are selected so that the flame temperature in limitation of chemical species is nearly equal to that in no limitation of chemical species. As the result, 9 chemical species C_2H_4 , O_2 , CO_2 , H_2O , CO , H_2 , OH , O and H are considered. Particular reaction processes are not considered.

It is assumed that gasified fuel and oxidizer are mixed and reacted instantly in the control volume. Temperature and chemical composition of the combustion gas are obtained with chemical equilibrium calculation. In this calculation, free energy is minimized by Lagrange's method of undetermined multipliers under the constraint condition that mole numbers of elements are conserved⁽⁴⁾. Helmholtz free energy is used for the free energy minimization, and the values of density ρ , internal energy e and mixture fraction ξ are required. Minimized function F is given as follows,

$$F = f + \sum_i \Lambda_i (b_i - b_i^0). \quad (7)$$

2.4. Energy-Flux Balance at the Solid Fuel Surface

In order to estimate fuel regression rate $\dot{r}$, i.e. fuel mass addition $\dot{m}_F (= \rho_s \dot{r})$, energy-flux balance at the solid fuel surface is implemented, which is described as follows. In this section, radiative heat transfer is not considered. The case including radiation will be discussed later (see §3.4).

$$\lambda_g \left(\frac{\partial T}{\partial y} \right)_{y=+0} = \lambda_s \left(\frac{\partial T}{\partial y} \right)_{y=-0} + \rho_s \dot{r} h_v. \quad (8)$$

Coordinate y is normal to the solid fuel surface. The origin of this coordinate system is fixed on the solid fuel surface. Negative values of y are referred to the inside part of the fuel. It is assumed that the solid fuel is semi-infinite, i.e. $-\infty < y \leq 0$. Considering Eq. (8), the left-hand side is the heat feedback from the flame, the first term in the right-hand side is the thermal conduction into the solid fuel, while the second term is the energy required for the solid fuel to pyrolyze and to gasify.

The left-hand side of Eq. (8) is evaluated by quasi-steady flame model, such as Karabeyoglu's model⁽⁵⁾. Karabeyoglu's model is based on Marxman's turbulent boundary layer combustion model⁽⁶⁾. While thermochemical blowing parameter is equal to aerodynamic blowing parameter in Marxman's model, Karabeyoglu distinguishes these parameters.

$$\lambda_g \left(\frac{\partial T}{\partial y} \right)_{y=+0} = \left[\frac{0.03q}{\mu^{-0.2}} \right]^{\frac{1}{1-k}} \left[\frac{B_t h_v}{\rho_s^{k/(1-k)}} \right] x^{-0.2/(1-k)} G^{0.8/(1-k)} \dot{r}^{-k/(1-k)}. \quad (9)$$

This equation, Eq. (9), expresses the convective heat transfer in the turbulent boundary layer. One can see that the heat feedback behavior depends on total mass flux G .

The thermochemical blowing parameter in Eq. (9) is represented by B_t . This parameter affects the heat transfer in boundary layer. It is defined as follows,

$$B_t \equiv \frac{u_e}{u_{fl}} \frac{h_{fl} - h_s}{h_v}. \quad (10)$$

The specific enthalpy at the flame h_{fl} and the specific enthalpy at the surface h_s are defined with base temperature T_0 ⁽⁷⁾. It is assumed that the gas composition at the surface is 100%

Table 2 Parameters related to the B_f evaluation.

Parameter	Value [Unit]
Base temperature T_0	298.15 [K]
Heat of combustion per mass unit of mixture gas Q_c	5300 [kJ/kg]
Effective heat of gasification $h_v^{(8)}$	2700 [kJ/kg]
Velocity ratio $u_e/u_{fl}^{(7)}$	1.8 [-]

fuel.

$$h_{fl} = h_P(T_{fl}) - h_P(T_0), \quad (11)$$

$$h_S = h_F(T_S) - h_F(T_0). \quad (12)$$

Assuming adiabatic conditions, the specific enthalpy at the flame h_{fl} is equal to the heat of combustion per mass unit of mixture gas Q_c ,

$$h_{fl} = Q_c. \quad (13)$$

While the specific enthalpy at the surface h_S was constant in our previous study⁽³⁾, it is a function of surface temperature T_S in this paper. The function is obtained by fitting the JANAF table's data⁽⁴⁾. The parameter list is shown in Table 2.

The constant parameters q , k in Eq. (9) are related to skin-friction coefficient. The ratio of skin-friction coefficient with/without surface blowing C_f/C_{f0} means the reduction in skin-friction caused by surface blowing, which is called "blocking effect". Marxman expressed the factor C_f/C_{f0} as a function of aerodynamic blowing parameter B_a ⁽⁶⁾.

$$\frac{C_f}{C_{f0}} = \left[\frac{\ln(1 + B_a)}{B_a} \right]^{0.8} \left[\frac{1 + 1.3B_a + 0.364B_a^2}{(1 + 0.5B_a)^2(1 + B_a)} \right]^{0.2}, \quad (14)$$

$$B_a \equiv \frac{(\rho v)_S}{\rho_e u_e C_f / 2} = \frac{\rho_s \dot{r}}{G C_f / 2}. \quad (15)$$

He also suggested the simplified expression in a particular range of B_a .

$$\frac{C_f}{C_{f0}} = q B_a^{-k}. \quad (16)$$

The parameters q , k in Eq. (9) are defined by Eq (16). In this paper, the range $1 \leq B_a \leq 15$ is considered. When Eq. (14) is applied to this range of B_a , $q = 0.74$ and $k = 0.53$ are obtained.

In the first term of the Eq. (8) right-hand side, temperature profile in the solid fuel is obtained by solving one-dimensional thermal conduction equation at any location x ,

$$\frac{\partial T}{\partial t} + \dot{r} \frac{\partial T}{\partial y} - \alpha_s \frac{\partial^2 T}{\partial y^2} = 0 \quad (17)$$

with following boundary conditions

$$T = T_S \quad \text{at } y = 0, \quad (18)$$

$$T = T_a \quad \text{at } y \rightarrow -\infty. \quad (19)$$

In a steady state, analytical solution of this equation can be obtained,

$$T(y) = T_a + (T_S - T_a) \exp\left(\frac{\dot{r}}{\alpha_s} y\right). \quad (20)$$

Hence the heat flux into the solid fuel is given as follows,

$$\lambda_s \left(\frac{\partial T}{\partial y} \right)_{y=0} = \lambda_s (T_S - T_a) \frac{\dot{r}}{\alpha_s}. \quad (21)$$

In a hybrid rocket, fuel regression is mainly caused by fuel pyrolysis and gasification. Arrhenius-type equation is empirically used as pyrolysis rate equation⁽⁸⁾.

$$\dot{r} = A_c \exp\left(-\frac{E_a}{R_u T_S}\right) e_{reaction}. \quad (22)$$

Table 3 Parameters in Arrhenius-type equation⁽⁸⁾.

Parameter	Value [Unit]
Preexponential factor A_c	2.0×10^{16} [1/s]
Activation energy E_a	1.5×10^5 [J/mol]
Degradation zone thickness $e_{reaction}$	25 [μ m]

The parameter list is shown in Table 3. Regression rate $\dot{r}$ is eliminated from Eq. (8) by using Eq. (22). Hence, Eq. (8) is expressed as the equation of temperature at the solid fuel surface T_s . By solving Eq. (8) for T_s , it is possible to evaluate $\dot{r}$ and $\dot{m}_F$.

2.5. Numerical Schemes

Equation (1) is discretized with finite volume method. Numerical flux is evaluated with AUSMDV scheme which is famous for its robustness⁽⁹⁾. In order to achieve higher order accuracy for space, second order MUSCL method is used. Time integration is two-stage Runge-Kutta method.

It is assumed that surface regression is a quasi-static change. At a certain time t , the convergent result of Eq. (1) is calculated. The convergence conditions and the definitions of residuals are described as follow,

$$R_e \leq 1.0 \times 10^{-4} \quad \text{and} \quad R_p \leq 1.0 \times 10^{-4}, \quad (23)$$

$$R_e \equiv \frac{\sqrt{\sum_{i=1}^{N_c} (e_{t,i}^{n+1} - e_{t,i}^n)^2}}{N_c}, \quad (24)$$

$$R_p \equiv \frac{\sqrt{\sum_{i=1}^{N_c} (p_i^{n+1} - p_i^n)^2}}{N_c}. \quad (25)$$

Port configuration at $t + \Delta t$ (where $\Delta t = 0.3$ [s]) is evaluated with the obtained regression rate, and the convergent result is calculated again. This procedure is iterated during the burning period t_b . Time-histories of the state in the chamber can be obtained by compiling the convergent results.

3. Results and Discussion

3.1. Comparison between Calculations and Experimental Data for Averaged Regression Rate

In order to validate our method, calculation results are compared with experimental data available in the open literature. The conditions of the calculations are conformed to Karebeyoglu's experimental conditions⁽¹⁰⁾ as shown in Table 4.

In Fig. 3 (a), one can see the pressure time-history for Test No.1 which is a typical case in calculation results. Chamber pressure decreases gradually with time. Local regression rate profiles for Test No.1 (see Fig. 3 (b)) is maximum at the head-end of the chamber, decreasing gradually with time.

In Fig. 4, time- and space-averaged numerical regression rates $\bar{r}$ are compared with data obtained by the experiments. The averaging method of numerical results is described follow,

$$\Delta m_F = \rho_s \int_0^{L_p} \pi(r_{final}^2 - r_{initial}^2) dx, \quad (26)$$

Table 4 Experimental conditions⁽¹⁰⁾.

Test number	Port length, L_p [cm]	Diameter of initial port, $D_{p,0}$ [mm]	Diameter of nozzle throat, D_t [mm]	Injection oxygen mass flow rate, $\dot{m}_{ox}$ [kg/s]	Burning period, t_b [s]
1	30.48	12.8	9.78	0.0454	3.0
2	30.48	17.0	9.78	0.0453	3.0
3	30.48	22.0	9.53	0.0328	3.0
4	30.48	12.8	9.91	0.0329	3.0

Fig. 3 Typical calculation result (Test No.1).

Fig. 4 Time-averaged oxidizer mass flux .vs. Time- and space-averaged regression rate. The interpolated lines are the fitted lines to the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. The x -axis and the y -axis are log scale.

$$\bar{r}_{final} = \sqrt{r_{initial}^2 + \frac{\Delta m_F}{\pi \rho_s L_p}}, \quad (27)$$

$$\bar{r} = \frac{\bar{r}_{final} - r_{initial}}{t_b}, \quad (28)$$

$$\bar{G}_{ox} = \frac{4\dot{m}_{ox}}{\pi(r_{initial} + \bar{r}_{final})^2}. \quad (29)$$

The values of numerical $\bar{r}$ fall within $\pm 30\%$ range of the experimental data fitting. One can see that the order of magnitude of averaged numerical regression rate is the same of the experimental data.

The calculation results and the experimental data are fitted in the empirical expression as follows,

$$\bar{r} = a\bar{G}_{ox}^n. \quad (30)$$

In Eq. (30), unit of $\bar{r}$ is mm/s and unit of $\bar{G}_{ox}$ is kg/(m²·s). The constant parameters a and n are determined by least-square method, resulting in $(a, n) = (0.00821, 0.841)$ for the calculations and $(a, n) = (0.0421, 0.498)$ for the experiments. The factor n represents the regression rate dependency on oxidizer mass flux. This factor in the calculations differs from that in the experiments. In fact, in this case the n value is higher. However, from statistical point of view, only four data points are not enough for an accurate estimation of the factors a and n . For example, Karabeyoglu obtained the factor $n (= 0.62)$ in the GOX/paraffin system by using

Table 5 The factors a and n in the various values of activation energy E_a .

	$1.5E_{a,r}$	$1.1E_{a,r}$	$1.0E_{a,r}$	$0.9E_{a,r}$	$0.5E_{a,r}$
a	0.00667	0.00801	0.00821	0.00864	0.00960
n	0.815	0.835	0.841	0.842	0.858

much more than ten data points⁽¹¹⁾. The improvement of this point will be performed in the future work.

3.2. The Effect of Parameters A_c and E_a in the Arrhenius-Type Equation

The Arrhenius-type equation, Eq. (22), is directly used by evaluating local regression rate $\dot{r}$. Hence it is inferred that the parameters A_c and E_a in this equation affect the regression rate significantly. In order to confirm this inference, the calculations with various values of A_c and E_a have been performed. At first, the value of activation energy E_a is changed parametrically. The value in Table 3 sets a reference value $E_{a,r}$, and the values $1.5E_{a,r}$, $1.1E_{a,r}$, $1.0E_{a,r}$, $0.9E_{a,r}$ and $0.5E_{a,r}$ are used by calculations. These results are shown in Table 5 and Fig. 5. In this range of activation energy, the factor n increases with decreasing activation energy and does not change greatly from about 0.8. On the other hand, the factor a also increases with decreasing activation energy. Second, the value of preexponential factor A_c is changed parametrically. The value in Table 3 sets a reference value $A_{c,r}$, and the values $100A_{c,r}$, $10A_{c,r}$, $1.5A_{c,r}$, $1.1A_{c,r}$, $1.0A_{c,r}$, $0.9A_{c,r}$, $0.5A_{c,r}$, $0.1A_{c,r}$ and $0.01A_{c,r}$ are used by calculations. The calculation results are fitted into Eq. (30). The obtained factors a and n are shown in Fig. 6. In this range of preexponential factor, the factors a and n tend to increase with the increase of the preexponential factor. However, the variation ranges of a and n in the cases with changing A_c are much more narrow than those in the cases with changing E_a . The activation energy E_a affects the evaluation of the factors a and n much more than the preexponential factor A_c . In fact E_a is in the exponent function of Eq. (22), unlike A_c .

3.3. The Effect of Index Number of Mass Flux G in the Heat Feedback Model

In this section, we discuss how the index number of mass flux G in the heat feedback model, Eq. (9), affects on the factor n in the empirical expression, Eq. (30).

In the energy-flux balance equation at the solid fuel surface, Eq. (8), it is assumed that the heat flux of thermal conduction into the solid fuel is much smaller than the energy required

Fig. 5 Time- and space-averaged regression rate. The activation energy E_a is changed into various values. The interpolated lines are the fitted lines to the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. The x -axis and the y -axis are log scale.

Fig. 6 The values of the factors a and n of the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. The preexponential factor A_c is changed into various values. The x -axis is log scale.

for the solid fuel to pyrolyze and to gasify.

$$\lambda_s \left(\frac{\partial T}{\partial y} \right)_{y=-0} \ll \rho_s \dot{r} h_v. \quad (31)$$

Actually, in the calculations, $\rho_s \dot{r} h_v$ is 3 to 4 times higher than $\lambda_s (\partial T / \partial y)_{y=-0}$. Under this assumption, the energy-flux balance equation can be simplified as follows,

$$\lambda_g \left(\frac{\partial T}{\partial y} \right)_{y=+0} \approx \rho_s \dot{r} h_v. \quad (32)$$

The following equation, Eq. (33), is obtained with Eqs. (32) and (9),

$$\dot{r} \propto G^{0.8}. \quad (33)$$

Hence the dependency of local regression rate $\dot{r}$ on total mass flux G is roughly determined by the index number of G in Eq. (9).

It is inferred that the averaged regression rate $\bar{r}$ also depends on the index number of G in Eq. (9). This is confirmed by calculating $\bar{r}$ after changing $G^{0.8/(1-k)}$ into $G^{0.5/(1-k)}$ in Eq. (9) (This model in which the index number of G is changed into $0.5/(1-k)$ is called "changed model"). In Fig. 7, one can see this calculation results, which, fitted in Eq. (30), provide $(a, n) = (0.00871, 0.497)$. The value of n in this calculations is almost the same as 0.5 of the index number of G in the changed model. This confirms that the factor n in Eq. (30) is greatly affected by the numerator of the index number of G in Eq. (9). Next, the effect of the factor k , which can be eliminated with the assumption of Eq. (31), is examined. The value of the factor k is changed parametrically by setting the reference value k_r equal to 0.53. The values $1.5k_r$, $1.1k_r$, $1.0k_r$, $0.9k_r$ and $0.5k_r$ are used by calculations. The calculation results are shown in Table 6 and Fig. 8. In this range of the factor k , the variation of the factor n is very small.

The value 0.8, of the index number of G in the heat feedback model, means that the dominant form of the heat feedback from the flame to the fuel surface is the convective heat transfer in the turbulent boundary layer. In order to conform the estimated values to the experimental values of n and $\bar{r}$, a heat feedback model must include not only convective heat transfer but also other phenomena, such as radiative heat transfer, chamber pressure effect (i.e. finite-rate chemical reactions) and so on.

Table 6 The factors a and n in the various values of the factor k .

	$1.5k_r$	$1.1k_r$	$1.0k_r$	$0.9k_r$	$0.5k_r$
a	0.00679	0.00804	0.00821	0.00868	0.0101
n	0.837	0.837	0.841	0.838	0.842

Fig. 7 Time- and space-averaged regression rates are estimated with the heat feedback model in which the index number of G is changed into $0.5/(1-k)$. The interpolated lines are the fitted lines to the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. The x -axis and the y -axis are log scale.

Fig. 8 Time- and space-averaged regression rate. The factor k is changed into various values. The interpolated lines are the fitted lines to the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. The x -axis and the y -axis are log scale.

3.4. The Effect of Radiative Heat Transfer

As a first step, radiative heat transfer, which is one of the unconsidered phenomena in above sections, is now considered. For simplicity, it is assumed that all amount of energy transferred by radiation is absorbed at the solid fuel surface. The term of radiative heat transfer $\dot{q}_R$ is added into the energy-flux balance equation at the solid fuel surface, Eq. (8).

$$\lambda_g \left(\frac{\partial T}{\partial y} \right)_{y=+0} + \dot{q}_R = \lambda_s \left(\frac{\partial T}{\partial y} \right)_{y=-0} + \rho_s \dot{r} h_v, \quad (34)$$

$$\dot{q}_R = \alpha_{R,s} \varepsilon_g \sigma T_{fl}^4 - \varepsilon_s \sigma T_s^4. \quad (35)$$

The term $\dot{q}_R$ includes not only the energy transferred from the flame to the surface but also the energy transferred from the surface to the gas-phase region. The parametric calculations with radiation effects have been performed. The parameters for radiation ε_g , ε_s and $\alpha_{R,s}$ are arbitrarily changed into various values as shown in Table 7. Here, Kirchhoff's law $\varepsilon_s = \alpha_{R,s}$ is assumed. The calculation results are shown in Figs. 9 and 10. The parameters a and n obtained by the results fitting in Eq. (30) are also listed in Table 7. In this range of emissivity, the factor n increases with decreasing gas emissivity. On the other hand, the factor n also

Table 7 The parameters for radiation, the factors a and n .

Case number	Gas emissivity ε_g	Solid fuel emissivity ε_s	Factor a	Factor n
1	0	0	0.00821	0.841
2	0.5	0.95	0.537	0.218
3	0.3	0.95	0.273	0.284
4	0.1	0.95	0.0327	0.613
5	0.1	0.85	0.0284	0.636
6	0.1	0.75	0.0245	0.660

increases with the decreasing of solid fuel emissivity. The variation range of the factor n in this calculation includes the value, 0.50, obtained by the experimental data fitting. Hence the model with radiation allows to improve the prediction accuracy of the regression rate dependency on oxidizer mass flux, i.e. the factor n .

The next calculation should employ an estimation model of the parameters ε_g , ε_s and $\alpha_{R,s}$ in the GOX/HDPE combustion system. Then the importance of the radiation effect should be

Fig. 9 Time- and space-averaged regression rates are estimated with the model which includes radiative heat transfer. The gas emissivity ε_g is changed into various values. The interpolated lines are the fitted lines to the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. However, Test No.3 (lowest mass-flux point) in Case 2 is not used by fitting the expression owing to poor convergence of the numerical solution. The x -axis and the y -axis are log scale.

Fig. 10 Time- and space-averaged regression rates are estimated with the model which includes radiative heat transfer. The solid fuel emissivity ε_s is changed into various values. The interpolated lines are the fitted lines to the empirical expression $\bar{r} = a\bar{G}_{ox}^n$. The x -axis and the y -axis are log scale.

understood in more detail. This kind of study is planned for next works.

3.5. Calculation Cost

In order to evaluate calculation cost, computational time is measured. The computer is DELL Precision™ T7400. The CPU is Intel® Xeon® CPU X5482, 3.20 GHz, 4 cores. The compiler is Intel® Fortran Compiler XE 12.0 Update 5 for Linux. The automatic-optimization option (-fast) and the automatic-parallelization option (-parallel) are used when the source codes are compiled. Each test in numerical simulations involves 11 convergent results corresponding to time t . Typical test has been selected as the representative case for the calculation time measurement. In this simulation, the initial condition at $t = 0$ [s] is the uniform oxidizer flowfield. As a result, for calculating one convergent result, the wall clock time is about 9 minutes, while the CPU time is about 31 minutes.

4. Concluding Remarks

We developed an estimation method for hybrid rocket internal ballistics. In order to validate the developed method, the calculation results were compared with the experimental data. While the numerical estimation of time- and space-averaged regression rates is of the same order of magnitude as the experimental data, the factor n for the calculations differs from that obtained by experiments. The main reason is that only convective heat transfer is considered as the form of heat feedback from flame. It has been demonstrated that the model with radiation allows to improve the prediction accuracy of the regression rate dependency on oxidizer mass flux.

As future works, in order to evaluate and to validate the method with a higher accuracy, a comparison using more experimental data is required. And, in order to obtain a better comprehension of the radiation effects, an estimation model for the radiation parameters has to be implemented.

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