Deconvolution Correction for Wandering in Wingtip Vortex Flowfield Data*

and Louis CATTAFAESTA
**Florida Center for Advanced Aero-Propulsion
Florida State University
2003 Levy Ave
Tallahassee, Florida 32310, USA
E-mail: lcattafesta@fsu.edu

Abstract
A deconvolution method is presented for correcting the smoothing effect caused by vortex wandering in velocity measurements of wingtip vortices. Flowfield velocity data are acquired using SPIV in an open-jet wind tunnel at several streamwise locations. A parametric Wiener filter that incorporates an estimate of the noise-to-signal ratio and the joint probability density function of the vortex motion is applied to correct for the wandering using a Fourier-domain deconvolution approach. In order to assess the accuracy of the correction, the instantaneous vector fields obtained from SPIV are shifted to the mean vortex center, and then averaged to yield the uncorrupted vortex flowfield. The unshifted mean flowfield data is then corrected by deconvolution and is then compared with the uncorrupted vortex flowfield. The results show that the deconvolution technique accurately corrects for the vortex wandering.

Key words: Tip-Vortex, Wandering, Deconvolution, Wiener Filter

1. Introduction

When an aircraft wing produces lift, the pressure difference between the suction and pressure sides of the wing causes a net flow to wrap around the tip of the wing, generating swirling flow patterns at each wingtip. As the aircraft advances, the swirling flow develops into stable, counter-rotating vortices that decay over time, referred to as wingtip vortices. These vortices cause many adverse effects including induced drag, increasing air traffic congestion, and aerodynamic noise.

Induced drag results from the downwash created by the vortices and contributes to the total drag experienced by a lifting body (Anderson, 2001). This is especially true during high-lift, low-speed flight typical during takeoff and approach. Additionally, wingtip vortex downwash influences flow across control surfaces and stabilizers, potentially causing aircraft instability (Yechout et al., 2003). The vortices generated by the wings of larger aircraft are strong enough to cause a large rolling moment on following aircraft. To combat this, strict regulations exist dictating the time between successive aircraft takeoffs and landings. The blades of turbines, rotors, and propellers also generate tip vortices, which manifest as performance losses and noise generation. For instance, vortex ring state and blade slap are phenomena that occur in helicopters due to the tip

Due to these adverse effects, the mitigation of these vortices is a popular research topic. Fixed-probe measurements are commonly undertaken in a grid behind the wing in a wind tunnel to obtain the velocity field of the vortex wake. However, it has been observed that the vortex core meanders within the measurement plane, corrupting the fixed-probe velocity measurements (Baker et al., 1973). This coherent motion of the vortex core is referred to as wandering. The effects of the wandering motion must be separated from the vortex flowfield to adequately quantify the velocity field.

The presence of wandering causes over prediction of the mean core radius and under prediction of the peak tangential velocity (Devenport et al., 1996). Thus, the vortex wandering causes the mean velocity field to be “smoothed.” It has been shown that if the amplitude of the wandering motion is 50% of the mean vortex core size, an overestimation of the core radius and underestimation of the peak tangential velocities may be as high as 64% (Devenport et al., 1996). However, the actual wandering amplitudes measured by Devenport et al. do not exceed 30% of the corrected core radius.

Considerable work has been done regarding the wandering of wind tunnel generated wingtip vortices. Devenport et al. describes the first attempt to correct for wandering. Devenport et al. provide a method for determining the wandering motion and estimating the corrected vortex flowfield. The mean square amplitude of wandering motion is defined as the variance of the vortex center location. Several independent investigations have provided validity to the analysis given by Devenport et al. Heyes et al. propose a method that relocates the instantaneous particle image velocimetry (PIV) images to the mean vortex location prior to averaging them, yielding the “true” vector field. Their corrected data are compared with the method proposed by Devenport et al. and are in close agreement. This method, however, requires the data to be taken with PIV, and is not usable with fixed-probe measurements. Bailey and Tavoularis (2008) describe a method for directly measuring the wandering amplitude by observing the frequency that a probe records zero velocity within the mean center of the vortex. The authors also use a two-point measurement method to reconstruct the vortex velocity profile in the wandering reference frame. Although their results closely agree with Devenport’s method, the use of multiple probes is required for their correction method.

A more direct method for correcting for vortex wandering involves deconvolution. As is shown by Baker et al. (1974) a fixed probe measures the convolution of the true vortex velocity field with the joint probability density function (PDF) of the vortex wandering distribution. The paper by Iungo et al. (2009) describes an analysis and implementation of correcting wingtip vortex velocity by multiple deconvolution methods. The correction is applied to both measured and simulated velocity fields containing wandering. Four methods of deconvolution are used. Since the wandering contributions of the simulated data are known explicitly, the simulated data corrections are directly compared for validity. The authors found that, of the methods they applied, direct deconvolution (inverse filtering) yields promising results. However, when measurement noise exists, direct deconvolution is ill-posed. The noisy contributions cause
dramatic errors in the deconvolution as demonstrated by Iungo et al. (2009). To circumvent this, the authors simply delete the spurious contributions due to noise and do not consider the problem of noise directly.

In contrast, the present analysis features a deconvolution procedure that accounts for noise within the flowfield data, enabling a more robust flowfield correction. This is achieved by using the Wiener filter to deconvolve the wandering from the vortex flowfield. Wiener filter deconvolution is a common image restoration method that provides significant improvement over direct inverse filtering by accounting for noisy contributions (Gonzalez et al., 2010). It is commonly used for image de-blurring.

In the following sections, the process of acquiring wingtip vortex data, correcting for vortex wandering by Wiener filter deconvolution, and quantifying the effectiveness of the correction are explored. The methods presented require no assumptions concerning the physical nature of the vortex wandering. To begin, the methodology behind the deconvolution wandering correction is presented. Then the setup used for acquiring vortex data using stereo particle image velocimetry (SPIV) is described. Vortex and wandering motion are then estimated. Finally, the correction method is applied to the SPIV data. Once the data are corrected, the results are compared with the “true” vortex data, which is obtained in a similar manner to Heyes et al.

2. Deconvolution Methodology

The vortex core center location can be described as a random variable, with an unknown distribution in the (y,z) plane assumed to be perpendicular to the freestream (x) direction. In the presence of wandering without any correction, the measured velocity is given by the convolution of the true mean vortex velocity with the joint PDF of the wandering motion:

\[ \bar{v}(y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(y_c, z_c) v(y - y_c, z - z_c) dy_c dz_c. \]  

(1)

where \((y_c, z_c)\) is the location of the vortex center, \(\bar{v}(y, z)\) is the mean measured velocity field, \(v(y, z)\) is the true mean velocity field, and \(p(y_c, z_c)\) is the joint PDF of the vortex center location determined from the mean velocity data as shown in §5.

The true mean flowfield of the vortex can be estimated as follows. Exploiting the fact that a convolution in the time and space domain is multiplication in the Fourier domain, Fourier transforms simplify the analysis considerably:

\[ \tilde{V}(k_y, k_z) = P(k_y, k_z) V(k_y, k_z) \]

(2)

\[ V(k_y, k_z) = \frac{\tilde{V}(k_y, k_z)}{P(k_y, k_z)} \]

Capital letters denote the corresponding Fourier transform. After determining \(V(k_y, k_z)\), \(v(y, z)\) is determined by performing an inverse Fourier transform. This method is commonly referred to as inverse filtering.
As it turns out, the method for deconvolving the true velocity field from PDF by eq. 2 is highly sensitive to noise (McNally et al., 1999). If we consider the added noise, \( n(y,z) \), the mean measured flowfield can be represented as

\[
\bar{v}(y,z) = p(y,z) * v(y,z) + n(y,z).
\] (3)

Inverse filtering is sensitive to noise because the numerator and denominator of eq. 2 both approach zero for high spatial frequencies. However, the noise present in the numerator has significant high frequency content relative to the denominator. This causes the ratio of eq. 2 to amplify the noise of the numerator, resulting in an ill-posed problem.

To circumvent this, algorithms have been devised that sacrifice some high frequency content of the data for noise attenuation (McNally et al., 1999). The Wiener filter deconvolution is one such method for addressing the problem of noise. However, measurements must be taken such that the noise can be adequately quantified. Since the goal is to recover \( v(y,z) \), we seek a function, \( G(k_y,k_z) \), that provides an estimate of the velocity field \( \bar{v}(k_y,k_z) \):

\[
\bar{v}(k_y,k_z) = G(k_y,k_z) \bar{V}(k_y,k_z).
\] (4)

\( \bar{V}(k_y,k_z) \) is the Fourier transform of \( \bar{v} \) from eq. 3. The function \( G(k_y,k_z) \) is found by minimizing the mean square error between \( V(k_y,k_z) \) and \( \bar{V}(k_y,k_z) \), resulting in the Wiener filter

\[
G(k_y,k_z) = \frac{1}{P(k_y,k_z)} \left( \frac{|P(k_y,k_z)|^2}{|P(k_y,k_z)|^2 + NSR} \right).
\] (5)

The term NSR is the noise-to-signal ratio; defined as the ratio of the autospectra of the noise to that of the signal. Though this value is a function of wavenumber, it is simpler to approximate it by the average noise power and the average signal power. This is simply the ratio of the noise variance and the signal variance. The application of this approximation of the NSR results in the parametric Wiener filter (Gonzalez, Woods, Eddins, 2010). Now, the estimated true velocity is approximated by the inverse Fourier transform of

\[
\bar{V}(k_y,k_z) = G(k_y,k_z) \bar{V}(k_y,k_z).
\] (6)

3. Experimental Setup

The experiments are performed in the University of Florida Anechoic Flow Facility (UFAFF). The model is a NACA 0012 half-wing, with a square wingtip, a span of 38.1 cm, and a chord of 30.5 cm. Fig. 1 shows a schematic of the experimental setup. The x-direction is streamwise, and the y-direction is spanwise from root to tip, and the z-direction completes the right handed coordinate system.
The wing is mounted in the UFAFF in a vertical-cantilever orientation on an acoustic foam sidewall along its root parallel to the $xz$-plane. With the exception of the foam sidewall, the model is mounted in an open-jet test section. The nature of the open-jet test section may exacerbate the wandering problems due to the jet shear layer. To vary the angle of attack, the wing is mounted on a needle bearing assembly, and is rotated about its aerodynamic center.

![Figure 1 - Experimental setup.](image)

The flow data are acquired using SPIV for a single chord Reynolds number of 530k and an angle of attack of $5^\circ$. SPIV data are acquired at several planar sections downstream of the wing, perpendicular to the freestream direction. These sections are at 3, 4, 5, and 6 chords downstream of the wing’s leading edge.

A 200 mJ Evergreen Nd:YAG laser and optics produce a planar light sheet that is approximately 2 mm thick. The flowfield is captured using a pair of LaVision VC Imager Pro X CCD cameras mounted in backscatter. The angle between the two cameras is approximately $45^\circ$. The laser and cameras are mounted together, allowing all the equipment to simultaneously traverse parallel to the flow direction in order to obtain data at multiple streamwise locations without requiring recalibration.

The flow is seeded with olive oil particles by a TSI 9307-6 oil droplet generator. This seeder produces approximately 1-$\mu$m diameter particles of olive oil (Melling 1997). The particles are introduced into the flow just after the tunnel inlet entrance plane. For each image pair, the time interval between images is $\Delta t = 32 \mu s$. For statistical convergence and consideration of data storage limits, 500 image pairs are acquired for each experiment.

The displacement vectors between image pairs are calculated via spatial cross-correlation using LaVision’s DaVis software. A multi-pass approach is taken; reducing the interrogation window size from 64 by 64 pixels for the initial pass, to 32 by 32 pixels for the final pass, with 75% overlap. This resulted in 161 by 103 vectors for each image pair, and flowfield vector...
spacing of 1.13 mm in the y and z directions. Erroneous vectors resulting from sparse seeding are discarded.

4. SPIV Uncertainty

For SPIV, the x, y, and z pixel displacements ($S_k$) within the illuminated region must be calculated from the two projections of the region as observed by the two cameras. By taking the convention described by Hu (2012), the x and y displacements recorded by camera 1 and 2 are $\Delta x_1, \Delta y_1$ and $\Delta x_2, \Delta y_2$ respectively, and the angles between the cameras and normal to the measurement plane are $\theta_1$ and $\theta_2$ for the x-z plane, and $\phi_1$ and $\phi_2$ for the y-z plane. Fig. 2 shows this convention for the x-z plane.

![Figure 2 - Schematic showing the angles used to calculate the displacement vectors within the x-z plane.](image)

The displacements in the x, y and z directions are

$$S_x = \frac{\Delta x_2 \tan(\theta_1) - \Delta x_1 \tan(\theta_2)}{\tan(\theta_1) - \tan(\theta_2)}$$  \hspace{1cm} (7)

$$S_z = \frac{\Delta x_2 - \Delta x_1}{\tan(\theta_1) - \tan(\theta_2)}$$  \hspace{1cm} (8)

$$S_y = \frac{\Delta y_1 + \Delta y_2}{2} + \frac{\Delta x_2 - \Delta x_1}{2} \left( \frac{\tan(\phi_1) - \tan(\phi_2)}{\tan(\theta_1) - \tan(\theta_2)} \right).$$  \hspace{1cm} (9)

The bias uncertainty in the displacement calculations can be calculated with respect to the partial derivatives of the given expressions. If a sensitivity coefficient is defined as $\Theta_\chi = \partial S_k / \partial \chi$, then the bias limit of the displacement calculation is

$$B_{S_k} = \left( \Theta_{\Delta x_1} B_{\Delta x_1} + \Theta_{\Delta x_2} B_{\Delta x_2} + \Theta_{\Delta y_1} B_{\Delta y_1} + \Theta_{\Delta y_2} B_{\Delta y_2} + \Theta_{\theta_1} B_{\theta_1} + \Theta_{\theta_2} B_{\theta_2} + \Theta_{\phi_1} B_{\phi_1} + \Theta_{\phi_2} B_{\phi_2} \right)^{1/2}.$$  \hspace{1cm} (10)

The bias uncertainty for the pixel displacement is the same in the x and y directions, for both cameras, and is $B_{\Delta x_1} = B_{\Delta x_2} = B_{\Delta y_1} = B_{\Delta y_2} = 0.03$ px for the LaVision DaVis software being used (DaVis v8.0, 2012).
Regarding the accuracy of the angles used to calculate the displacement, DaVis provides the angular displacement of the cameras to within 0.001° (DaVis v 8.0, 2012). However, the angular bias uncertainty was taken as $B_{\theta_1} = B_{\theta_2} = B_{\phi_1} = B_{\phi_2} = 0.1^\circ$.

The mean SPIV velocity components are computed from the mean correlation displacements by

$$ V_k = \frac{S_k l_{obj}}{l_{img} \Delta t} $$

for the $k^{th}$ velocity component. The displacement length, $S_k$, has units of pixels, $l_{obj}$ is the width (measured in meters) of the illuminated field of particles determined by the field of view of the camera, and $l_{img}$ is the width of the image projected onto the photosensor by the camera, measured in pixels. If the sensitivity coefficient is defined as $\Theta = \partial V_k / \partial x$, then the bias limit of the vector calculation is:

$$ B_{V_k} = \sqrt{\Theta^2 l_{obj}^2 B_{l_{obj}}^2 + \Theta^2 l_{img}^2 B_{l_{img}}^2 + \Theta^2 \Delta t B_{\Delta t}^2 + \Theta^2 S_k B_{S_k}^2} $$

The uncertainty in the image width distance is determined by the calibration plate grid point width to be $B_{l_{obj}} = 1.5$ mm. The uncertainty in the image width in pixels is the same as the pixel displacement uncertainty: $B_{l_{img}} = 0.03$ px. Finally, the resolution of the timing system used in the experiments is $B_{\Delta t} = 1$ ns (DaVis, v 8.0, 2012).

Murray and Ukeiley (2006) note that the random fluctuations in SPIV vector calculations are caused by deficiencies associated with flow seeding, image quality, illumination, and timing uncertainty. For the purpose of this analysis, it can be assumed that these deficiencies are independent of the location within the flowfield, and the noise levels are uniform throughout the flowfield. Unfortunately, turbulence levels also contribute to the random measurement fluctuations, adding to the apparent noise in the measurement. Murray and Ukeiley (2006) decompose the variance of the vectors within the freestream region to be the sum of the noise of the image ($\sigma_{PIV}^2$) and turbulent variance ($u'^2$):

$$ \sigma_{PIV}^2 = \sigma_n^2 + u'^2 $$

This assumes that the variance due to turbulence is independent of the random error in the SPIV vector calculation, and the noise in the turbulence measurements is small compared to the turbulent fluctuations. The total variance $\sigma_{PIV}^2$ is determined for the boxed region in Fig. 3 in the freestream region. With
this value and knowledge of the turbulence intensity ($u'^2$) of the wind tunnel, the variance due to the random noise in the PIV vector calculation is estimated as

$$\sigma_n^2 = \sigma_{PIV}^2 - u'^2. \quad (14)$$

In this case, the measured freestream turbulence intensity using a four-hole Cobra probe is 0.7%. If it is assumed that the noise is Gaussian, the precision limit within 95% confidence is calculated as (Coleman and Steele, 1999)

$$P_{vk} = 2\sigma_n. \quad (15)$$

Finally, the total uncertainty is:

$$U_{vk} = \sqrt{P_{vk}^2 + B_{vk}^2}. \quad (16)$$

The maximum uncertainty is shown for several streamwise locations in Table 1.

<table>
<thead>
<tr>
<th>$x/c$</th>
<th>$U_u$ ($\frac{m}{s}$)</th>
<th>$U_v$ ($\frac{m}{s}$)</th>
<th>$U_w$ ($\frac{m}{s}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.59</td>
<td>0.70</td>
<td>0.58</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
<td>0.63</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.58</td>
<td>0.46</td>
<td>0.86</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>0.47</td>
<td>0.64</td>
</tr>
</tbody>
</table>

5. Wandering Results

To properly estimate the flowfield, relevant information concerning the wandering motion must be determined. With typical fixed-probe measurement methods, the statistical parameters of the wandering distribution can be estimated from the measured velocity at the mean core location under certain conditions. On the vortex axis, it is assumed that any deviations from zero velocity are due only to vortex wandering. This yields a proper estimate as long as the wandering amplitude is sufficiently lesser than the core radius. Specifically, Devenport et al. (1996) proposed that the wandering parameters be found from the RMS value of the measured flow data by

$$\sigma_y = \frac{\sqrt{\langle w'^2 \rangle}}{\partial w/\partial y}, \quad \sigma_z = \frac{\sqrt{\langle v'^2 \rangle}}{\partial v/\partial z}, \quad \text{and} \quad e_{yz} = \langle w'v' \rangle / (\sigma_y \sigma_w)$$

evaluated at the core of the vortex. This is based on the fact that, within the core, the tangential velocity gradient is constant and therefore the variation due to wandering is directly proportional to the variation in velocity. Iungo et al. (2009) verified the estimation of the wandering amplitude if the actual wandering amplitude is less than 60% of the core radius.
According to Chow et al. (1997), the characteristic frequency of the wandering motion is on the order of $f_{\text{w}} = \frac{u_{\infty}}{100c}$. Since the time scales of SPIV are much less than the time scales of the wandering motion (Chow et al., 1997), the wandering motion can be separated from the measured flowfield, and the statistical parameters can be calculated explicitly. This is similar to the treatment by Heyes et al. (2004). The core location of each instantaneous snapshot is determined by the location of maximum vorticity for each PIV vector field. With the core locations determined for each image, the wandering levels and other core location statistics are calculated from the 500 vector fields. A PDF is constructed from the core location data series, and the PDF is found to be Gaussian within 95% confidence. Additionally, each instantaneous flowfield can be shifted to the mean vortex center prior to the calculation of the flowfield statistics. This effectively removes the wandering and provides a baseline for comparison, referred to as the “shifted” data. The shifted flowfield vortex core radius, and maximum azimuthal velocity are shown in Table 2.

The wandering amplitude and the correlation coefficient of the core coordinates are also shown in Table 2 and plotted in Fig. 4. The values shown are calculated from the core statistics directly measured from the SPIV vector fields, and by the estimation methods proposed by Devenport et al. (1996). As shown, there is a linear increase in wandering as the flow progresses downstream, agreeing with previous research (McAlister and Takahashi, 1991; Devenport et al. 1996).

<table>
<thead>
<tr>
<th>$x/c$</th>
<th>Measured from SPIV flowfield</th>
<th>Estimated from single point</th>
<th>Shifted Vortex Core Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_y/c$</td>
<td>$\sigma_z/c$</td>
<td>$e_{yz}$</td>
</tr>
<tr>
<td>3</td>
<td>0.014</td>
<td>0.012</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>0.018</td>
<td>0.017</td>
<td>0.34</td>
</tr>
<tr>
<td>5</td>
<td>0.021</td>
<td>0.021</td>
<td>0.24</td>
</tr>
<tr>
<td>6</td>
<td>0.026</td>
<td>0.028</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 4 – Measured and Estimated wandering amplitude and correlation coefficient.
In analyzing the quantities in Table 2, the maximum observed wandering amplitude is over 90% of the core radius, and as the amplitude increases, an overestimation of the wandering amplitude results (Devenport et al. 1996, Iungo et al. 2009). In estimating the wandering amplitude, it is assumed that the measured velocity deviations are linearly related to the spatial deviations due to wandering as discussed above. This is based on the fact that the velocity gradient within the core is constant. If the wandering motion spreads beyond the core region, this assumption breaks down, and the estimates exceed the actual wandering amplitude. The maximum overestimation of the wandering amplitude is 14%. Iungo et al. (2009) show similar overestimation for this level of wandering. This overestimation corresponds to an apparent increase in the wandering motion. However, regarding the wandering correction, this amount of overestimation is insignificant. Furthermore, the correlation coefficient is underestimated. The results shown below indicate that the overestimation of the wandering amplitude and the underestimation of the correlation coefficient is not detrimental to the final corrected velocity field.

Continuing with the task of correcting the measured flowfield, a bivariate Gaussian PDF is constructed from the estimated variances and correlation coefficient from the mean flowfield data. The following expression is used to construct the PDF from the estimated wandering amplitude (standard deviations) and correlation coefficient (Bendat and Piersol, 2010)

\[ p(y_c, z_c) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - e_{yz}^2}} \exp \left( -\frac{1}{2(1 - e_{yz}^2)} \left( \frac{y_c}{\sigma_y} - \frac{2e_{yz}z_c y_c}{\sigma_x \sigma_y} \right) \right). \] (17)

This PDF is used to perform the correction of the flowfield data.

6. Correction Results

The Wiener deconvolution technique requires both an estimate of the joint PDF of the core location and an estimate of the NSR. As a first approximation of the NSR, consider the variance due to random noise calculated in §4 \((\sigma_n^2)\), and the signal variance of the mean flowfield \((\sigma_s^2)\). Because wandering corrupts the measured flowfield, the true signal variance cannot be measured. As a first approximation of the signal variance, it is assumed that the measured flowfield variance is close to that of the corrected flowfield. The parametric Wiener filter is implemented for these parameters. Then, after the deconvolution is performed, the signal variance is updated from the resulting flowfield, providing a better approximation of the NSR. This process is iterated until the NSR converges. This is performed for the recorded SPIV flow data for each streamwise location.
Since it is assumed that the shifted SPIV data is the correct vortex flowfield, the error associated with the estimate of NSR can be calculated. In Fig. 5, the mean square error between the entire corrected tangential velocity field and the correct flowfield is shown for $\frac{x}{c} = 6$.

The red circle in Fig. 5 indicates the NSR associated with the minimum achievable error using a constant NSR with respect to the corrected velocity field. As shown, the estimated NSR is close to the optimal value. It should be noted that for decreasing NSR, the error increases drastically. As the NSR approaches zero, the Wiener filter simplifies to the direct inverse filter. The high error associated with zero NSR illustrates the limitation of direct inverse filtering with noisy data.

Before applying this method to measured data, the correction procedure is initially verified by simulations. For this verification, a Q-vortex flowfield is simulated, and wandering is induced synthetically. The synthetic wandering is described by an isotropic, bivariate, Gaussian PDF, with wandering amplitude of $\sigma = 0.028$ (91% of the core radius). The wandering is applied by shifting the simulated measurement field according to the simulated wandering amplitude, recording each shift, and taking the mean value of the shifted velocity fields. Additionally, Gaussian noise was added to the flowfield to simulate measurement noise. The results of the simulation and correction are shown in Fig. 6.

![Figure 5 - Mean Square Error between shifted flowfield and corrected flowfield vs. NSR (blue). The red circle indicates the minimum-error NSR, and the green line indicates the estimated NSR. $x/c = 6$](image)

![Figure 6 - Velocity profiles of a simulated Q-vortex with synthetic wandering, a Q-vortex without wandering, and the profile resulting in the deconvolution correction method. The synthetic wandering amplitude is $\sigma/c = 0.028$ and the standard deviation of the added noise is $\sigma_n = 0.01 \text{m/s}$](image)
With the method computationally verified, the $NSR$ approximated, and the PDF estimated from the mean flowfield data, the Wiener filter deconvolution is performed. Comparing the resulting tangential velocity profile most drastically elucidates the effect of the deconvolution. This is shown in Fig. 7. The original mean profile, shifted profile, and corrected profile are shown for several streamwise locations, and it is apparent that this method corrects the corrupted data to within the estimated confidence intervals. The smoothing effect of the wandering is increasingly evident for the larger streamwise distances.

Considering the high level of corruption due to wandering, this correction method demonstrates robust performance for the entire range of observed wandering amplitudes. This method benefits from the fact that the entire correction is performed without the need for SPIV measurements. The parameters used to construct the PDF are estimated from the mean flowfield point measurements. Also, in contrast to the deconvolution approaches used by Iungo, et al. (2009), the method described here directly mitigates the detrimental effects of measurement noise.

However, the velocity gradient near the core is slightly under estimated. This may be attributed to the constant $NSR$ used in the parametric Wiener filter. A variable $NSR$ may possibly rectify this deficiency.

7. Conclusions

The wandering of wingtip vortices is evaluated, and a correction to mean velocity measurements via deconvolution is explored. Because of the
adverse effects wingtip vortices have in applications, the structure, dynamics, and mitigation of these vortices is a popular research topic. In order to provide a means for acquiring accurate data, the corruption due to wandering must be addressed.

Regarding the analysis of the motion of the vortex, the vortex center location for several streamwise planes is isolated, and several observations of the center location are recorded. The velocity fields are then shifted to coincide with the mean vortex center in a manner similar to the treatment described by Heyes et al. (2004). The resulting shifted flowfield is taken to be the “true” flowfield, devoid of wandering motion.

The joint PDF of the vortex axis location is calculated from the vortex center observations in the SPIV results. The PDF is found to be Gaussian and anisotropic. Additionally, it is observed that the wandering amplitude increases linearly as the flow progresses downstream, which is consistent with the findings in McAlister and Takahashi (1991) and Devenport et al. (1996).

In addition to a direct measurement of wandering parameters, the wandering amplitude and correlation coefficient are approximated by methods using point measurements proposed by Devenport et al. (1996). The estimations of the wandering amplitude proved to be accurate, while the estimates for the correlation coefficient are under predicted. Fortunately, this proved to be inconsequential with respect to the flowfield correction procedure. In addition, the joint PDF estimation method benefits from the fact that the entire correction is performed without the need for SPIV measurements.

Direct deconvolution of the wandering motion from the measured velocity field proves to be inaccurate due to the contribution of noise in the measured data. However, using an estimate of the noise, the parametric Wiener filter is adapted to allow for noise mitigation.

By comparing the corrected results with the “true” flowfield (shifted SPIV data), the error incurred by inaccurate NSR approximations was analyzed. This analysis shows that the method used to approximate NSR provides nearly optimal performance of the parametric Wiener filter. Unfortunately, there is a large increase in the error if the NSR is under-predicted.

Finally, the deconvolution correction method is applied for data acquired for streamwise locations of 3, 4, 5, and 6 chord-lengths, with a maximum observed wandering amplitude of over 90% of the core radius. The deconvolution correction method provides adequate flowfield correction for all cases presented, which is significant considering the wide range of wandering observed.

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