3D-PTV Measuring on Bubble and Liquid Velocities of a Single Bubble Jet*

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Abstract

This experimental research concerns a simple air-bubblejet from a bottom nozzle in water. We try to conduct the measurements of the flow by applying a three-dimensional particle tracking velocimetry (3D-PTV) technique, in order to specify both bubble (air) and liquid (water) velocities. As tracer particles, we regard bubbles themselves in bubble-velocity measurements, and polyethylene particles suspended in water in liquid-velocity measurements. Then, we visualise the three-dimensional motions of the bubbles and the liquid. As we record stereo images using a pair of high-speed video cameras, we can get temporally-consecutive spatial information of both the bubbles' and liquid's motions. As a result, we quantitatively reveal the three-dimensional and instantaneous behaviour of the unsteady bubble-jet flow. And, using the obtained three-dimensional information, we show the fundamental flow structure of the bubble jet from a statistical view point.

Key words: Bubble, Jet, Plume, Three-Dimensional Flow, PTV, Environmental Engineering

1. Introduction

The bubble jet or the bubble plume is one of key phenomena in diverse engineering aspects. For example, against the water pollutions by eutrophication which has been often seen in lakes, ponds, harbors and so on, the aeration in which we send oxygen in water by air-bubble jets is one of convenient and common methods to alleviate these water pollutions (for example, see Yamada et al. (2005)⁽¹⁾). As well, the bubble jet is used in other various fields such as chemical reactors and bubble pumps. In addition, precise knowledge of the bubble jet might contribute to more fine control of boiling water reactors.

So, there have been many studies concerning two-phase bubble-jet flow. —See Miligram⁽²⁾ as a review concerning the mean-flow characteristics.—Among them, Hsiao et al.,⁽³⁾ Sano & Mori,⁽⁴⁾ Sawada & Ohashi⁽⁵⁾ and Iguchi et al.^{(6) & (7)} estimated bubbles' and liquid's rising velocities in a bottom-blowing bubble jet, experimentally and computationally. However, we have not yet obtained enough knowledge on the bubble-jet flow not only to be applied to various engineering devices but also to grasp the statistical features of a very simple case, owing to technical difficulties in two-phase-flow measurements in experiments as well in free-boundary problems in computations.

Recently, we have applied three-dimensional particle tracking velocimetry (herein after, referred to as 3D-PTV) to a single bubble-jet flow, in order to investigate the basic characteristics of the flow.⁽⁸⁾ Strictly speaking, we have quantitatively shown the three-dimensional and instantaneous motion of each bubble. Here, the 3D-PTV is a

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technology to specify three-dimensional positions of tracer particles using a couple of stereo images, and to obtain the velocity vectors of tracer particles from their positions at different instants. The 3D-PTV has the following advantages: (1) there exists no sensor which disturbs flow; (2) we get the information of many points in space at the same time; (3) we get three-dimensional information in a whole test volume (see Doh et al. (2002, 2004)⁽⁹⁾ & ⁽¹⁰⁾).

Our final aim is to get more-general and more-precise information on the various bubble-jet flows. And, the present fundamental study is in a first step where we try to quantitatively reveal the overall structure of a air-bubble-jet flow from a simple bottom-nozzle exit into stationary water, from a statistical viewpoint based on a three-dimensional and instantaneous data acquired by the 3D-PTV technique.

Concretely speaking, in addition to the bubble velocities as well as Funaki et al.,⁽⁸⁾ we measure liquid velocities; namely, the ambient-fluid velocities of bubbles. And, we quantitatively reveal both the bubbles' and the liquid velocities. In the bubble-velocity measurements, we regard bubbles themselves as tracer particles. And, in the liquid-velocity measurements, we regard polyethylene spheres suspending in the liquid as tracer particles. Then, we visualise three-dimensional and instantaneous motions of both the bubbles and the liquid. As we record stereo images using a pair of high-speed video cameras, we can get temporally-consecutive spatial information of both the bubbles' and the liquid's motions. Besides, (1) we observe the three-dimensional motion of each bubble, and (2) we conduct the measurements by a two-dimensional particle image velocimetry (hereinafter, referred to as 2D-PIV), in order to confirm the accuracy of the present 3D-PTV measurements.—A part of the present study has been published as a note.⁽¹¹⁾—

2. Experimental method

2.1 Model

Figure 1 shows the present experimental model, together with the present measurement volume for the 3D-PTV and the present coordination system. The coordinate system is a cylindrical one (r, θ, z), with the origin O at the nozzle-exit centre. This model is the same as Funaki et al.⁽⁸⁾ Air bubbles are shed from an air nozzle attached on a chamber, in a large water container. As our concern is on the basic space structure of a bubble-jetflow, we consider one of the simplest and controllable nozzles.

2.2 Experimental apparatus

Table 1 summarises main experimental parameters in the present study (for details, see Funaki et al.⁽⁸⁾), together with other researchers' ones for comparison. A nozzle with a chamber is placed on the bottom of a large water container. The water container is a cube of $1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$, and it consists of acrylic-resin plates. A compressor produces air-bubblejet from the nozzle into water (the liquid), through a flow-control valve and an air pipe. Water depth, namely, the distance between the top water surface and the nozzle exit is 0.5m. By a pressure transducer and a thermocouple, we measure the pressure and the temperature inside the chamber, respectively. The data from the sensors are analysed on a personal computer, through interfaces together with the data from high-speed video cameras.

Specifically speaking, the nozzle is a straight pipe with an inside diameter of 0.1 mm, and with a pipe length of 15 mm. The nozzle axis is fixed to be vertical. We define a pressure ratio p_0/p_∞ as a non-dimensional governing parameter, where p_0 and p_∞ denote the static pressures in the chamber and in the ambient around a nozzle exit, respectively.—Complementally speaking, p_∞ is larger than the atmospheric pressure by less than 5 %.—

The choked condition is at $p_0/p_\infty \geq 1.9$. And, the tested range of p_0/p_∞ is 1.2 – 3.8, which corresponds to the range of air flow rate of 1.13 – 1.55 cm³/s at the nozzle exit. The relation of mass flow rate m with p_0/p_∞ is given as follows.

$$m = 7.85 \times 10^{-7} \sqrt{7 p_0 \rho_0 \left\{ \left(\frac{p_\infty}{p_0} \right)^{1.43} - \left(\frac{p_\infty}{p_0} \right)^{1.71} \right\}} \quad \text{at } p_0/p_\infty < 1.9, \quad (1)$$

and

$$m = 7.85 \times 10^{-7} \sqrt{0.457 p_0 \rho_0} \quad \text{at } p_0/p_\infty \geq 1.9, \quad (2)$$

where ρ_0 denotes air density in the chamber.

In the present study, we consider the effects of two governing parameters p_0/p_∞ and z/d upon both the mean bubble velocity and the mean liquid velocity. The p_0/p_∞ effects are directly related with the increase/decrease of air flow rate. And, the value of z/d could be a primary indicator for the balance between the bubble-jet aspect and the bubble-plume aspect.

Especially, we focus upon the z component v of a velocity vector among its components. Now, by the 3D-PTV, we can obtain huge numbers of v of both the bubble and the liquid in a three-dimensional cubic space of the measurement volume, simultaneously, consecutively and quantitatively. When we see spatial distributions, we first carry out statistical processings over raw data such as velocity vectors. To get a mean velocity V , we average over a whole circle in the azimuthal direction θ , over a 4 mm distance in the radial direction r , and over a 4 mm distance in the vertical direction z , each instance. We suppose such averaging volumes with a 5 mm interval in the r direction and a 20 mm interval in the z direction for bubble-velocity measurements, and with a 10 mm interval in the r direction and a 20 mm interval in the z direction for liquid-velocity measurements. So, we obtain five distributions corresponding to five averaging volumes in a range of z between 38 mm and 122 mm. Averaging time is over 1 s (= 500 frames) for bubble-velocity measurements, or over 6 s (= 6 × 250 frames) for liquid-velocity measurements. The averaging number of the raw data is $10^2 - 4 \times 10^3$ each averaging volume.

2.3 Bubble's condition

Table 2 shows the present bubble's condition. The equivalent diameter D_e of a bubble is defined by

$$D_e = \left(\frac{6B_b}{\pi} \right)^{\frac{1}{3}}, \quad (3)$$

where B_b denotes the volume of a bubble. Strictly speaking, we measure the major and minor axes of an each bubble from a visualised photograph. And, to determine the bubble's volume B_b , we assume a spheroid with the revolution about the minor axis. We should note that, in the present experiments, we directly control not the bubble size D_e , but the pressure ratio p_0/p_∞ .

Bubble's Reynolds number Re_b is defined by

$$Re_b = \frac{\rho_l D_e V_b}{\mu_l}, \quad (4)$$

where ρ_l , V_b and μ_l denote the density of the liquid, the velocity of the bubble, and the

Fig. 1 Model, together with the measurement volume for 3D-PTV and the coordinate system.

Table 1 Experimental parameters, together with other researchers' ones.

Researchers	The present and Funaki et al., 2009	Hsiao et al., 1980	Sano & Mori, 1982	Sawada & Ohashi, 1987	Iguchi et al., 1988	Iguchi et al., 1992
Exp./Comp.	Exp.	Exp.	Comp.	Comp.	Exp.	Exp. & comp.
Container	Cube	Cylinder	Cylinder	Cylinder	Cylinder	Cylinder
Size of container	1 m × 1 m × 1 m	2.44 m	2.2 m	0.6 m	0.2 m	0.13 m
Depth of Liquid	0.5 m	1.35 m	1.5 m	0.4 m	0.4 m	0.23 m
Liquid	water	water	water	water	water	water
Gas	air	argon	argon	compressed air	air	compressed air
Inner diameter of a nozzle	0.1 mm	unknown	unknown	unknown	2 mm	2 mm
Flow rate	0.000011 – 0.000015 Nm ³ /min	0.025 – 0.47 Nm ³ /min	0.001 – 1 Nm ³ /min	0.040 Nm ³ /min	0.00062 – 0.0025 m ³ /min	0.000041 m ³ /min

(a) Over-all view (b) Close-up view

Fig. 2 Photograph of a bubble-jet at $p_0/p_\infty = 1.2$.

(a) Over-all view (b) Close-up view

Fig. 3 Photograph of a bubble-jet at $p_0/p_\infty = 1.9$.

(a) Over-all view (b) Close-up view

Fig. 4 Photograph of a bubble-jet at $p_0/p_\infty = 2.9$.

Table 2 Bubble's condition.

Equivalent diameter D_e [mm]	0.3 – 4.2
Bubble Reynolds number Re_b	60 – 1400
Etovos number Eu	0.01 – 2.4
Bubble type (Clift et al., 1978)	Spherical, ellipsoidal or wobbling

Fig. 5 A consecutive series of photographs around a nozzle at $p_0/p_\infty = 1.2$.

Fig. 6 A consecutive series of photographs around a nozzle at $p_0/p_\infty = 1.9$.

Fig. 7 A consecutive series of photographs around a nozzle at $p_0/p_\infty = 2.4$.

Fig. 8 Range of bubble's diameter D_e . ○, Ensemble-mean diameter: ●, Sauter's mean diameter: ◆, surface-area-mean diameter: ◇, volume-mean diameter.

viscosity of the liquid, respectively.

Etovos Number Eo is defined by

$$Eo = \frac{gD_e^2(\rho_l - \rho_b)}{\sigma}, \quad (5)$$

where ρ_b and σ denote the density inside the bubble and the surface tension between the liquid (water) and the bubbles' gas (air), respectively.

As described in Table 2, test ranges are as follows: $D_e = 0.3 - 4.2$ mm, $Re_b = 60 - 1400$ and $Eo = 0.01 - 2.4$. According to Clift et al.,⁽¹²⁾ bubble types in the present bubble's condition are "spherical," "ellipsoidal" and "wobbling." Our observations through all tests are almost consistent with such bubble types in their classification.—We should note that we cannot detect such small bubbles as $D_e < 0.3$ mm in experiments, because of our present purpose to grasp rather large-scale flow structures.—

2.4 Flow visualisation

The 3D-PTV system are calibrated in advance, and we take two synchronized stereo images in a measurement volume using a pair of high-speed video cameras A and B with frame rates of $1/500 - 1/60$ s. Each image adequately covers the measurement volume.

Next, we calculate the three-dimensional positions of tracer particles from a pair of the stereo images. The velocity vectors of tracer particles are determined from two consecutive informations of the tracer-particle positions. All data processings have been carried out in a personal computer.

In bubble-velocity measurements, we use air bubbles themselves as the tracer particles,

which are illuminated by a halide lamp. In liquid-velocity measurements, we use chemically-bridged polyethylene-resin spheres with a mean diameter of 0.12 mm as tracer particles, which are coated with fluorescent paint, and which are illuminated by a YAG laser. —About the minimum size of the tracer particles which the present 3D-PTV system can recognise, see Funaki et al.⁽⁸⁾—

The calculated position of a tracer particle is not identical with its centre of gravity, in a strict sense. Because, the calculated position corresponds to the position where the luminance on the tracer particle indicates locally maximum. Especially in bubble-velocity measurements, we could expect such an error in the calculated position as $D_e/2$ (will be shown in Fig. 8). The error in the determined velocity vector is likely to be smaller than that in the calculated position, as the tracer particles are illuminated by a single light source and recorded with a high frame rate.

Moreover, for accurate measurements to specify the bubble's diameter D_e , we visualize bubble in a narrower range of $z = 40 - 80$ mm, using one high-speed video camera attached with a close-up lens. (Later, we will show examples in Figures 2(b), 3(b) and 4(b).)

From a theoretical point of view, it is possible to conduct the 3D-PTV for both bubble-velocity and liquid-velocity measurements at the same time. However, from a practical point of view, it is difficult to know which an obtained velocity vector represents for the bubble or the liquid. Moreover, the measurement accuracy becomes lower due to a large discrepancy between the averaged velocities of the bubble and the liquid. Thus, in the present study, we separately conduct the measurements for the bubbles and the liquid. Especially in bubble-velocity measurements, there are no polyethylene particles to reduce error vectors.

Besides, in order to observe the behaviours of individual bubbles and to confirm the accuracy of the present 3D-PTV measurements, we conduct two kinds of preliminary experiments; namely, (1) the three-dimensional motion analysis of each bubble and (2) the measurements by a 2D-PIV. The former shows about 2 % relative error in comparison between the present 3D-PTV and a commercial three-dimensional motion analyser. Concerning the latter, we will discuss later in Subsection 3.3.

3. Results and Discussion

3.1 Mean bubble's diameter

Figures 2, 3 and 4 show sample photographs of the bubble jet at $p_0/p_\infty = 1.2, 1.9$ and 2.9 , respectively. In each figure, figures (a) and (b) represent an over-all view at $z = 0 - 500$ mm and a close-up view at $z = 40 - 80$ mm, respectively. We can see that the number of bubbles increases and that the space where bubbles are distributed becomes wide, as p_0/p_∞ increases. Especially, the number of small bubbles tends to increase, as p_0/p_∞ increases. We confirm such tendencies even in the choked condition at $p_0/p_\infty \geq 1.9$, as well as the un-choked condition at $p_0/p_\infty < 1.9$ (compare Figs. 3 and 4).

Figures 5, 6 and 7 shows the consecutive series of photographs around the nozzle at $p_0/p_\infty = 1.2, 1.9$ and 2.4 , respectively. We can recognise observe the complexity in bubble's formation process, in addition to the effect of p_0/p_∞ .

Among the properties to identify the bubble-jet-flow characteristics, which is a function of p_0/p_∞ in the present study, the bubble size is one of the most important and principal properties. So, we preliminarily specify the bubble size as a characteristic length scale. As the bubble size, we now consider the equivalent bubble's diameter D_e defined by Eq. (3).

However, when we specify the bubble size by averaging D_e , there are some alternatives such as the Sauter's mean and so on in addition to a simple ensemble-mean. Figure 8 shows

the range of D_e plotted against p_0/p_∞ , together with such alternatives. Specifically speaking, error bars denote the ranges. And, open circle, solid circle, solid diamond and open diamond denote ensemble-mean, the Sauter's mean, surface-area-mean and volume-mean, respectively. For each p_0/p_∞ , total bubble's number N_{total} for averaging is 50 – 200.

To discuss further, Fig. 9 shows the distribution of the bubble's number N in five ranges of D_e for each p_0/p_∞ . That is, the figure shows the bubble-number ratio N/N_{total} in each range of D_e plotted against p_0/p_∞ . We can see that, at $p_0/p_\infty \leq 1.9$, N/N_{total} with small values of D_e rapidly increases with increasing p_0/p_∞ , and that N/N_{total} with large values of D_e slightly increases with increasing p_0/p_∞ . In summary, this well corresponds with the fact that the range of D_e becomes rapidly wide with increasing p_0/p_∞ at $p_0/p_\infty \leq 1.9$, as shown in Fig. 8. On the other hand, at $p_0/p_\infty \geq 1.9$, the influence of p_0/p_∞ upon N/N_{total} is much smaller than that at $p_0/p_\infty \leq 1.9$. And, there exist many minute-scale bubbles with $D_e < 1.0$ mm and some grand-scale bubbles with $D_e > 3.0$ mm, at $p_0/p_\infty \geq 1.9$.

Again, we see Fig. 8. The ensemble-mean of D_e decreases, as p_0/p_∞ increases from unity to 1.9. And, at $p_0/p_\infty \geq 1.9$, the ensemble-mean tends to keep a small constant value being independent of p_0/p_∞ , due to the prevailing influence of minute bubbles with a large value of N/N_{total} (see Fig. 9). On the other hand, the Sauter's mean of D_e is always in a range of 2.2 – 2.6 mm, being independent of p_0/p_∞ . Both the surface-area-mean and the volume-mean of D_e are always between the ensemble-mean and the Sauter's mean, being functions of p_0/p_∞ with a similar manner to the ensemble-mean.

3.2 Path of bubble

The main aim of the present study is to reveal statistical characteristics, as a first step of the fundamental approach for a simple bubble-jet flow, while the detailed investigations of various instantaneous features will remain as a next step in future. However, the results obtained by the present 3D-PTV are, of course, applicable to observe such instantaneous features. Then, we briefly describe some typical bubble's motions, here.

Figure 10 shows several bubble's paths of a bubble-jet flow at $p_0/p_\infty = 1.2$. And, Figs. 11 and 12 show two examples of single-bubble motions in stationary liquid without bubble jets and in a bubble-jet flow, respectively. Specifically speaking, each figure shows time histories of the bubble-path components of a single bubble with $D_e \approx 2$ mm (in figures (a)-(c)), together with the bubble's trajectory projected on the x - y plane (in figure (d)). We can see that the bubble's motion in Fig. 12 is more complicated and more random, than that in Fig. 11 which shows a near spiral path with an initial zigzag path on a plane as well as Saffman (1956).⁽¹³⁾ Besides, when we observe many other bubbles in addition to Figs. 11 and 12, we can confirm that the fluctuating amplitudes of the paths in the bubble-jet flow tend to be much larger than those of the paths in the stationary liquid.

3.3 Radial distribution of mean bubble velocity

In this subsection, we consider the radial distribution of mean bubble velocity V_b . Figure 13 shows the distribution of V_b at $p_0/p_\infty = 2.4$, for five ranges of z/d . The lateral axis r and the longitudinal axis V_b are normalised by d and $(V_{b0})_{\text{max}}$, respectively. Here, $(V_{b0})_{\text{max}}$ represents the maximum value of the terminal rising velocity of a single bubble in stationary liquid, which is obtained by experiments and is in good agreement with other researchers' results.⁽⁸⁾

All the data indicate that $V_b/(V_{b0})_{\text{max}}$ keeps about 0.7, while there exist no data at small z/d far from the bubble-jet centre axis due to the absence of bubbles. Concerning such a fact as $V_b/(V_{b0})_{\text{max}}$ is less than unity, we should consider that the bubble's size in the bubble-jet

Fig.9 Ratio of the bubble's number N in each range of D_e against pressure ratio p_0/p_∞ .

Fig.10 Some bubbles' paths of a bubble-jet flow at $p_0/p_\infty = 1.2$.

Fig. 11 Time histories of bubble-path components of a single bubble with $D_e = 2.15$ mm, in stationary liquid without bubble jets, together with the bubble's trajectory projected on the x - y plane.

Fig. 12 Time histories of bubble-path components of a single bubble with $D_e = 2$ mm in a bubble-jet flow at $p_0/p_\infty = 1.2$, together with the bubble's trajectory projected on the x - y plane.

Fig. 13 Radial distribution of mean bubble velocity V_b at $p_0/p_\infty = 2.4$.

Fig. 14 Radial distribution of mean liquid velocity V_l at $p_0/p_\infty = 1.2$ by 3D-PTV, together with 2D-PIV.

flow varies widely being independent of p_0/p_∞ , as shown in Subsection 3.1, and that the terminal rising velocity of a single bubble is strongly affected by its bubble's size. Then, as well as Funaki et al.,⁽⁸⁾ we cannot clearly confirm the z/d effect upon $V_b/(V_{b0})_{max}$ near the bubble-jet centre axis at $r/d \leq 150$ (For more precise discussion on the z/d effect, see Fig. 17).

Figure 13 also shows the experiments of Funaki et al.⁽⁸⁾ and Iguchi et al.^{(1988), (6)} for comparison. We should note that the datum given by Iguchi et al. is averaged over $r/d = 0 - 100$. We can confirm good agreement between the present and Funaki et al., both qualitatively and quantitatively. This assures a high accuracy of the present measurements. On the other hand, all the present data are much smaller than Iguchi et al., due to (1) a one-order larger nozzle size, (2) a one-order larger flow rate and (3) a one-order smaller container size in their experiments than us (see Table 1).

3.4 Radial distribution of mean liquid velocity

In this Subsection, we consider the radial distributions of mean liquid velocity V_l . Figure 14 shows V_l at $p_0/p_\infty = 1.2$, for three ranges of z/d by the 3D-PTV, together with those by the 2D-PIV. We should note that the lateral axis r/d is much wider than Fig. 13, because fluid could flow even in the space without bubbles. We can see that the 3D-PTV results well agree with the 2D-PIV ones. Again, this assures the present 3D-PTV accuracy. Then, we will see the effects of z/d and p_0/p_∞ upon the distribution, next.

Figures 15 and 16 show the radial distribution of V_l at $p_0/p_\infty = 1.4$ and 2.4, for five ranges of z/d , respectively. The figures are typical examples in the un-choked and choked conditions, respectively. Each figure also shows the region where bubbles exist. Strictly speaking, the bubble region depends upon z/d . Then, a shaded zone in each figure represents the corresponding transient region.

First, we consider Fig. 15. We can see that $V_l/(V_{b0})_{max}$ in the bubble region is always much larger than that outside the bubble region. And, $V_l/(V_{b0})_{max}$ decreases monotonically and rapidly, as r/d increases from zero to about 300. In addition, it seems difficult to confirm any clear effects of z/d both in and outside the bubble region. Just in the bubble region, we can observe such a z/d effect as $V_l/(V_{b0})_{max}$ tends to increase with increasing z/d , but this tendency is complicated.

Supplementarily speaking, Fig. 15 also shows the laser-Doppler-velocimetry measurements by Iguchi et al. (1988&1992)^{(6)&(7)}, for reference. $V_l/(V_{b0})_{max}$ by them are much larger than us, due to the same reasons indicated in Fig. 13.

Second, we consider Fig. 16. The bubble region becomes wider than that in Fig. 15, due to the larger value of p_0/p_∞ and the corresponding larger air flow rate. As well as Fig. 15, $V_l/(V_{b0})_{\max}$ in the bubble region is always much larger than that outside the bubble region. However, especially in the bubble region, $V_l/(V_{b0})_{\max}$ decreases with increasing r/d , not always monotonically but complicatedly. Besides, in contrast with Fig. 15, we cannot ignore the effect of z/d both in and outside the bubble region. More specifically, outside the bubble region, we can observe such a monotonical z/d effect as $V_l/(V_{b0})_{\max}$ increases with increasing z/d . On the other hand, in the bubble region, we usually observe a more-complicated z/d effect, as will be shown in Fig. 17.

3.5 Pressure and distance effects upon mean velocities

In Subsection 3.4, we have confirmed the p_0/p_∞ effects upon the radial distributions, if we compare Figs. 15 and 16. As well, we have confirmed the z/d effects upon the radial distributions. However, these results are merely at two values of p_0/p_∞ , representing the typical un-choked and choked conditions. So, in this subsection, we summarise the influences of both p_0/p_∞ and z/d upon $V_b/(V_{b0})_{\max}$ and $V_l/(V_{b0})_{\max}$, referring the measurements at six values of p_0/p_∞ . That is to say, Figure 17 shows mean velocities V_b and V_l in the bubble region (at $r/d = 0$) plotted against p_0/p_∞ for several ranges of z/d . And, Figs. 18 and 19 show V_l outside the bubble region (at $r/d = 480 - 520$ and at $r/d = 980 - 1020$, respectively) plotted against p_0/p_∞ for several ranges of z/d . Both the figures show no data

Fig. 15 Radial distribution of mean liquid velocity V_l in the un-choked condition (at $p_0/p_\infty = 1.4$).

Fig. 16 Radial distribution of mean liquid velocity V_l in the choked condition (at $p_0/p_\infty = 2.4$).

Fig. 17 Mean bubble and liquid velocities V_b and V_l in the bubble region (at $r/d = 0$) against pressure ratio p_0/p_∞ .

Fig. 18 Mean liquid velocity V_l outside the bubble region (at $r/d = 480 - 520$) against pressure ratio p_0/p_∞ .

Fig. 19 Mean liquid velocity V_l outside the bubble region (at $r/d = 980 - 1020$) against pressure ratio p_0/p_{∞} .

of V_b , because there exist no bubbles at $r/d \geq 400$. In all the three figures, the longitudinal axes are normalised by $(V_{b0})_{\max}$, as well.

First, we consider Fig. 17. When we see $V_b/(V_{b0})_{\max}$, we can confirm the same features as Funaki et al.⁽⁸⁾ Specifically speaking, as p_0/p_{∞} increases from unity to 1.9, $V_b/(V_{b0})_{\max}$ monotonically decreases from unity to about 0.6. On the other hand, at $p_0/p_{\infty} \geq 1.9$ (choked), $V_b/(V_{b0})_{\max}$ is scattered in a range of $V_b/(V_{b0})_{\max} = 0.5 - 0.8$, being independent of p_0/p_{∞} . These features are consistent with the bubble-size features as discussed in Figs. 8 and 9. That is to say, we can consider that the decreasing trend of $V_b/(V_{b0})_{\max}$ with increasing p_0/p_{∞} at $p_0/p_{\infty} \leq 1.9$ well corresponds to the increasing number of minute-scale bubbles with increasing p_0/p_{∞} . And, at $p_0/p_{\infty} \geq 1.9$, we see less influence of p_0/p_{∞} upon D_e as well as that upon $V_b/(V_{b0})_{\max}$. We can also consider that the wide-range scattering of $V_b/(V_{b0})_{\max}$ at $p_0/p_{\infty} \geq 1.9$ is related to the such a complexity as a wide range of D_e at $p_0/p_{\infty} \geq 1.9$ in Fig. 8. Next, we see the z/d effects. At $p_0/p_{\infty} \leq 1.9$ (un-choked), it is difficult to find out any clear influence of z/d . On the other hand, at $p_0/p_{\infty} \geq 1.9$, we can see the influence of z/d , which is in a complicated manner. —However, especially at $p_0/p_{\infty} = 2.4$, the influence of z/d is occasionally small, as shown in Fig. 13.—

When we see $V_l/(V_{b0})_{\max}$, we can find that $V_l/(V_{b0})_{\max}$ is always less than 0.1, being almost independent of p_0/p_{∞} . Thus, $V_l/(V_{b0})_{\max}$ is smaller than $V_b/(V_{b0})_{\max}$ by about one order, in any time. Next, we see the z/d effect. We can see complicated influence of z/d at all the tested values of p_0/p_{∞} . —Strictly speaking, the z/d effect at $p_0/p_{\infty} \leq 1.9$ seems rather smaller than that of $p_0/p_{\infty} \geq 1.9$.—

Supplementarily speaking, Fig. 17 also shows $V_l/(V_{b0})_{\max}$ in the experiments and computations by other researchers, for reference. The experiment by Hsiao et al. (1980) is close to us, in spite of their much larger flow rate. However, those by Sano & Mori (1982) and Iguchi et al. (1988) are larger than us, and those by Sawada & Ohashi (1987) is much larger than us, due to various discrepancies in experimental and computational conditions (see Table 1).

Second, we consider Fig. 18. As p_0/p_{∞} increases from unity to 1.9, $V_l/(V_{b0})_{\max}$ increases monotonically. However, at $p_0/p_{\infty} > 1.9$ (choked), $V_l/(V_{b0})_{\max}$ tends to have a constant value, being independent of p_0/p_{∞} . On the other hand, if we focus our attention on the z/d effect upon $V_l/(V_{b0})_{\max}$, we can confirm that z/d effect is negligibly small at $p_0/p_{\infty} < 1.9$, and that $V_l/(V_{b0})_{\max}$ increases with increasing z/d at $p_0/p_{\infty} > 1.9$. This is, of course, consistent with Figs. 15 and 16.

Finally, we consider Fig. 19. From a qualitative point of view, Fig. 19 shows the same features as Fig. 18. From a qualitative point of view, $V_l/(V_{b0})_{\max}$ tends to have a slightly-smaller value than Fig. 18, at each p_0/p_{∞} and z/d .

4. Conclusions

In order to reveal statistical characteristics as a first step of the fundamental approach for bubble-jet flows, we have consecutively measured the three-dimensional and instantaneous velocities of both the bubbles and the liquid of a simple bubble jet from a bottom nozzle, by applying a 3D-PTV technique. In the present study, we suppose two control parameters for both the mean bubble and the mean liquid velocities; namely, p_0/p_∞ and z/d .

First, to precisely specify the present experimental conditions, we have shown principal parameters such as mean bubble's diameters. And, we have also shown each bubble's path in comparison with that in stationary liquid. Then, we have revealed that the mean liquid velocity in the bubble region on its radial distribution is much larger than that outside the bubble region, while even the mean liquid velocity in the bubble region is smaller than the mean bubble velocity by about one order. So, at the transient border of the bubble region, the mean liquid velocity rapidly decreases in the radial direction.

In the bubble region on radial distribution, we have shown the mean bubble velocity first, and the mean liquid velocity second. Upon each the velocity, we have shown the p_0/p_∞ effect first, and the z/d effect second. The mean bubble velocity is affected by p_0/p_∞ in the un-choked condition; namely, the mean bubble velocity decreases with increasing p_0/p_∞ . On the other hand, it is less-affected by p_0/p_∞ in the choked condition, where the mean bubble velocity tends to scatter in a rather wide range in a complicated manner depending upon z/d . This well corresponds to the range of the bubble's diameter D_e , which becomes wide in the choked condition. The z/d effect upon the mean bubble velocity is negligible in the un-choked condition. On the other hand, the z/d effect in the choked condition can be recognised, but is complicated. The mean liquid velocity is almost independent of p_0/p_∞ in both the choked and un-choked conditions. The z/d effect upon the mean liquid velocity can be observed, but is complicated. Specifically speaking, the z/d effect in the un-choked condition seems weaker than that in the choked condition, in accordance with the z/d effect upon the mean bubble velocity.

Outside the bubble region on radial distribution, upon the liquid velocity, we have shown the p_0/p_∞ effect first, and the z/d effect second, as well. The mean liquid velocity is affected by p_0/p_∞ in the un-choked condition; namely, the mean liquid velocity increases with increasing p_0/p_∞ . On the other hand, the mean liquid velocity is less affected by p_0/p_∞ in the choked condition; namely, the mean liquid velocity tends to keep a constant value which depends not upon p_0/p_∞ but upon z/d . This p_0/p_∞ effect is similar with the p_0/p_∞ effects upon the range of D_e and upon the mean bubble velocity in the bubble region, as well. The z/d effect upon the liquid velocity is negligible in the un-choked condition. On the other hand, the z/d effect in the choked condition is rather clear; namely, the mean liquid velocity increases with increasing z/d . In summary, in the un-choked condition, we can ignore not the p_0/p_∞ effect, but the z/d effect. On the other hand, in the choked condition, vice versa.

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