Toward the development of an anisotropy-resolving subgrid-scale model for large eddy simulation

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Abstract

To improve the prediction accuracy of large eddy simulation, an anisotropy-resolving subgrid-scale (SGS) model is a promising strategy. Although an anisotropic term in this type of SGS model is known to effectively enhance unsteady motions of vortex structures particularly in the near-wall region, it has not been made clear how well this term reproduces the real SGS-stress components. Therefore, we performed a detailed investigation of the model performance by an *a priori* test using the direct numerical simulation (DNS) data of a plane channel flow. The anisotropic SGS model is constructed by combining an isotropic linear eddy-viscosity model with an extra anisotropic term that does not produce undesirable energy transfer between the grid-scale and SGS components. This modeling concept contributes to a reasonable prediction, while maintaining computational stability. Comparison of the SGS stresses evaluated by the model expressions with those obtained directly from the DNS data provided several insights useful for further development of this type of SGS models. From the present investigation, this anisotropic SGS model was found generally to produce a reasonable trend for the SGS-stress anisotropy.

Key words : Turbulence, Large eddy simulation, Subgrid-scale model, Extra anisotropic term, Scale-similarity model

1. Introduction

In a large eddy simulation (LES), the flow variables are decomposed into a directly resolved grid-scale (GS) component and an unresolved subgrid-scale (SGS) component that derives from the small-scale eddies. A canonical form of the governing equations for incompressible turbulence may be written as

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{D\bar{U}_i}{Dt} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \left\{ \nu \left(\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right) - \tau_{ij} \right\}, \quad (2)$$

where $\bar{(\quad)}$ denotes a filtered value. In Eq. (2), ρ , $\bar{P}$, $\bar{U}_i$ and ν denote the density, filtered static pressure, filtered velocity and kinematic viscosity, respectively. The SGS-stress tensor τ_{ij} is originally expressed as

$$\tau_{ij} = \bar{U_i U_j} - \bar{U}_i \bar{U}_j. \quad (3)$$

LES has long been recognized as a promising way to predict complex turbulence in engineering applications. Since the success of LES depends strongly on the accurate representation of the SGS stresses, a number of research groups have proposed several kinds of SGS models for τ_{ij} (see for example, Germano et al., 1991; Lilly, 1992; Zang et al., 1993; Vreman et al., 1994; Salvetti and Banerjee, 1995; Horiuti, 1997; Sarghini et al., 1999; Morinishi and Vasilyev, 2001). Although these models have provided encouraging results, there still remain several aspects to be further improved. Among them, an important concern may be in the reduction of the prediction accuracy, when they are applied to engineering applications using coarse grid resolution in the near-wall region.

To overcome this difficulty, Abe (2013) recently proposed a new anisotropy-resolving SGS modeling concept, where the SGS-stress expression is constructed by combining an isotropic eddy-viscosity model (EVM) with an extra anisotropic term. This SGS model successfully improved the prediction accuracy, particularly with a coarse grid resolution in the near-wall region, while maintaining computational stability. Although the application of the model to several test cases indicated the basic capability of this SGS modeling concept (Abe, 2013; Abe, 2014), it had not been made clear how the extra anisotropic term worked for improving the predictive performance. To investigate this issue, Ohtsuka and Abe (2013) compared the simulation results obtained by this anisotropic SGS model with those obtained using a linear isotropic SGS model. They found that an anisotropic term in this type of SGS model effectively enhanced unsteady motions of vortex structures particularly in the near-wall region. Nevertheless, another important issue remained, namely, assessing how correctly this term reproduces the real SGS-stress components. This being the case, understanding this fundamental performance of an SGS model is thought to be important and useful in further developing this kind of turbulence modeling.

With that in mind, the objective of the present study is to elucidate the fundamental features of the SGS stresses predicted by the SGS models used in LES. For this purpose, we perform a quantitative investigation of the model performance by an *a priori* test using the direct numerical simulation (DNS) data of a plane channel flow. We make several reduced velocity fields from the DNS data with different filter widths. We then evaluate the SGS stresses by applying some representative SGS models to these filtered velocity-field data. The results obtained are compared with the true values estimated directly from the DNS data and their predictive performance is discussed in detail.

2. Turbulence Models

2.1. Anisotropy-resolving SGS model

In the following, we briefly describe the anisotropy-resolving SGS model proposed by Abe (2013). The SGS stress τ_{ij} in Eq. (2) was modeled as follows:

$$\tau_{ij} = \frac{2}{3} k_{SGS} \delta_{ij} - 2 \nu_{SGS} S_{ij} + 2 k_{SGS} b_{ij}^{EAT}, \quad S_{ij} = \frac{1}{2} \left(\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right), \quad (4)$$

where k_{SGS} and ν_{SGS} are the SGS turbulence energy and the SGS eddy viscosity, respectively. In Eq. (4), S_{ij} is the strain-rate tensor and the anisotropy tensor b_{ij}^{EAT} in the extra anisotropic term is modeled as

$$b_{ij}^{EAT} = \frac{\tau'_{ij} - (-2\nu' S_{ij})}{\tau'_{kk} - (-2\nu' S_{kk})} - \frac{1}{3} \delta_{ij} = \frac{R'_{ij}}{\tau'_{kk}}, \quad R'_{ij} = \tau'^a_{ij} - (-2\nu' S_{ij}), \quad (5)$$

where $\tau'^a_{ij} = \tau'_{ij} - \tau'_{kk} \delta_{ij}/3$. In Eq. (5), τ'_{ij} is given by the following representative scale-similarity model of Bardina et al. (1980):

$$\tau'_{ij} = \left(\bar{U}_i - \widehat{\bar{U}}_i \right) \left(\bar{U}_j - \widehat{\bar{U}}_j \right), \quad (6)$$

where $\widehat{(\quad)}$ denotes a test-filtered value. In Eq. (5), ν' is an equivalent eddy viscosity evaluated by an EVM-type linear approximation for Eq. (6) as

$$\tau'^a_{ij} S_{ij} = -2\nu' S_{ij} S_{ij} = -2\nu' S^2 \quad \longrightarrow \quad \nu' = -\frac{\tau'^a_{ij} S_{ij}}{2S^2}, \quad (7)$$

where $S^2 = S_{ij} S_{ij}$. In Eq. (5), R'_{ij} is evaluated by subtracting an EVM form from the original Bardina model. Considering the fact that the production term of k_{SGS} is expressed as $-\tau'_{ij} \bar{U}_{i,j}$ ($= -\tau'_{ij} S_{ij}$), Eq. (7) means that this linearized approximation produces the same amount of energy transfer between the GS and SGS components as the original scale-similarity model. Therefore, R'_{ij} yields no undesirable extra energy transfer between the GS and SGS components; the extra term in Eq. (4) is then expected to successfully predict the SGS-stress anisotropy with no serious effect to computational stability.

Concerning the linear EVM in Eq. (4), the present model adopts the one-equation SGS model proposed by Inagaki (2011). The SGS viscosity ν_{SGS} is modeled as follows:

$$\nu_{SGS} = C_{SGS} f_{SGS} \sqrt{k_{SGS}} \Delta, \quad f_{SGS} = 1 - \exp \left\{ - \left(\frac{y'_\varepsilon}{A_0} \right)^{4/3} \right\},$$

$$y'_\varepsilon = \left(\frac{u_\varepsilon y}{\nu} \right) \sqrt{C_l \frac{y}{\Delta}}, \quad u_\varepsilon = (\nu \varepsilon_{SGS})^{1/4}, \quad (8)$$

where Δ is an SGS filter width. In this study, k_{SGS} and ε_{SGS} are evaluated using the following equations:

$$\frac{Dk_{SGS}}{Dt} = \frac{\partial}{\partial x_j} \left\{ \left(\nu + C_k f_{SGS} \sqrt{k_{SGS}} \Delta \right) \frac{\partial k_{SGS}}{\partial x_j} \right\} - \tau_{ij} \frac{\partial \bar{U}_i}{\partial x_j} - \varepsilon_{SGS},$$

$$\varepsilon_{SGS} = C_\varepsilon \frac{k_{SGS}^{3/2}}{\Delta} + \frac{2\nu k_{SGS}}{y^2}. \quad (9)$$

The model constants are as follows:

$$C_{SGS} = 0.05, \quad A_0 = 30, \quad C_l = 4, \quad C_\varepsilon = 0.835, \quad C_k = 0.1. \quad (10)$$

More detailed descriptions of the present SGS model are given in Abe (2013).

2.2. Conventional linear eddy-viscosity models

For a more detailed investigation of the characteristics of the SGS models, two conventional linear EVMs were also tested. One was the Smagorinsky model (Smagorinsky, 1963) and the other was the dynamic Smagorinsky model (Germano et al., 1991; Lilly, 1992).

The conventional form of the linear EVM is

$$\tau_{ij}^a = -2 \nu_{SGS} S_{ij}, \quad (11)$$

where $\tau_{ij}^a = \tau_{ij} - \tau_{kk}\delta_{ij}/3$. In the Smagorinsky model, the eddy viscosity ν_{SGS} is modeled as

$$\nu_{SGS} = (C_S f_S \Delta)^2 \sqrt{2S^2}, \quad f_S = 1 - \exp\left(-\frac{y^+}{A}\right), \quad (12)$$

where $C_S = 0.1$ and $A = 25$ are the model constants. In contrast, in the dynamic Smagorinsky model, ν_{SGS} is modeled as

$$\nu_{SGS} = C\Delta^2 \sqrt{2S^2}, \quad (13)$$

where the coefficient C is locally determined by the dynamic procedure proposed by Germano et al. (1991) with the aid of the least square approximation by Lilly (1992). Since such a dynamic SGS model generally shows strong numerical instability, we introduced a well-known strategy of clipping the negative values of ν_{SGS} in an *a posteriori* test of LES.

3. Test Cases and Computational Conditions

In our detailed investigation of the SGS model, we performed an *a priori* test using the DNS data of a channel flow at $Re_\tau = 180$ that were originally obtained by Hattori and Nagano (2004). Note that the Reynolds number Re_τ is based on the friction velocity u_τ and the half channel height δ , i.e., $Re_\tau = u_\tau \delta / \nu$. The grid resolutions of the DNS were $\Delta x^+ = 9$ and $\Delta z^+ = 4.5$ in the x - (streamwise) and z - (spanwise) directions, respectively, as well as a sufficiently fine grid resolution in the wall-normal (y) direction, where the superscript ($^+$) denotes a value in wall units. For the *a priori* test, we made a filtered velocity field from the DNS data, where we used the top-hat filtering operator. Note that, according to Horiuti (1993), we applied the filtering operator in homogeneous directions (i.e., the x - and z - directions). In this study, we made three reduced velocity fields with grid resolutions $(\Delta x^+, \Delta z^+) = (18, 9)$, $(36, 18)$ and $(72, 36)$, where the grid-filter width was set equal to the grid spacing of each reduced data, i.e., $\bar{\Delta}_i = \Delta x_i$.

Furthermore, to confirm the basic performance of the SGS models, we performed *a posteriori* simulations corresponding to the aforementioned *a priori* tests. The computational conditions are summarized in Table 1. In general, the C180F grid resolution (i.e., $\Delta x^+ = 18, \Delta z^+ = 9$) is recognized as being sufficient for LES. Although the C180M grid resolution (i.e., $\Delta x^+ = 36, \Delta z^+ = 18$) is still applicable, it may be relatively coarse for LES when a simple (non-staggered) second-order central difference scheme is adopted on a finite-volume grid cell. In contrast, the C180C grid resolution (i.e., $\Delta x^+ = 72, \Delta z^+ = 36$) is thought to be approximately the grid resolution with which the prediction accuracy of the LES begins to clearly decrease. Note that for all three test cases, the grid resolution in the wall-normal (y) direction is the same as that of the original DNS data.

A posteriori calculations were performed using an unstructured finite-volume procedure that was almost the same as that used by Muto et al. (2012), where vertex-centered type storage was used on a grid. The second-order central difference scheme was used to discretize the spatial derivatives except for the convection term of k_{SGS} (Eq. (9)), which was discretized by the second-order upwind scheme. Time marching was based on the fractional step method (Kim and

Table 1 Computational conditions for channel-flow cases.

Case	Re_τ	Grid numbers	Domain (x - z)	Δx^+	Δy^+	Δz^+	Δt
C180F ($\Delta x^+ = 18, \Delta z^+ = 9$)	180	$61 \times 98 \times 61$	$6\delta \times 3\delta$	18	0.2–9	9	1×10^{-3}
C180M ($\Delta x^+ = 36, \Delta z^+ = 18$)	180	$31 \times 98 \times 31$	$6\delta \times 3\delta$	36	0.2–9	18	1×10^{-3}
C180C ($\Delta x^+ = 72, \Delta z^+ = 36$)	180	$31 \times 98 \times 31$	$12\delta \times 6\delta$	72	0.2–9	36	1×10^{-3}

Moin, 1985), in which the second-order Crank-Nicolson scheme was used for the velocity equations. With regard to the transport equation of k_{SGS} , the first-order Euler implicit scheme was used. The coupling of the velocity and pressure fields was based on the simplified marker and cell (SMAC) method (Amsden and Harlow, 1970). The flow rate on the control-volume surface was estimated using the Rhie-Chow interpolation (Rhie and Chow, 1983). For the boundary conditions, periodic conditions were imposed in the streamwise and spanwise directions. At the wall surfaces, the no-slip conditions were specified. Note that the effects of a first-order time-marching scheme and a second-order upwind scheme adopted for the k_{SGS} transport equation were investigated in previous studies (Abe, 2013; Abe, 2014), from which it was confirmed that the time-integration and space-discretization schemes presently used did not have any crucial effect on the computational results, at least, for fundamental turbulent flows. In this study, to obtain a statistical value for the *a priori* test, a variable was averaged in homogeneous planes (i.e., the x - and z -directions) at a fixed wall distance (“plane-averaged”, hereafter). On the other hand, the statistics used for comparison in the *a posteriori* test were the values ensemble-averaged in time and over homogeneous planes that were originally provided by Hattori and Nagano (2004). A supplementary discussion on the statistics used in this study is given in the Appendix.

4. Results and Discussion

4.1. Validation of the basic performance of the SGS models

First, to confirm the basic performance of the SGS models, we investigate the simulation results of the *a posteriori* test. Figures 1 and 2 show the results obtained by the Smagorinsky model and the dynamic Smagorinsky model, respectively. These models have a standard EVM-type linear form that has been most often used. Note that the turbulence energy and turbulence intensities are evaluated only by the GS components because the SGS turbulence energy cannot be directly obtained in an LES using a conventional linear SGS model. It is readily seen that the prediction accuracy of these linear EVMs worsens as the grid resolution becomes coarser. As discussed above, the grid resolution of C180C is generally insufficient for a conventional LES. Both the linear EVMs returned considerable overpredictions of the mean velocity for C180C. This implies an underprediction of the wall-shear stress, which is also understood from the results of the total (GS+SGS) Reynolds shear stress, as seen in Figs. 1 (b) and 2 (b). Such a grid dependence on the grid resolution has been recognized as an crucial problem to be improved. Concerning the turbulence energy in Figs. 1 (c) and 2 (c), even the GS component shows a considerable overprediction as the grid resolution becomes coarse. A similar trend is seen in the distributions of the streamwise turbulence intensity. In contrast, a slight underprediction is seen in the distributions of the wall-normal turbulence intensity for C180C. Therefore, the GS stress anisotropy predicted by these conventional models is thought to be stronger than that of the DNS.

For comparison, Fig. 3 shows the results of the present anisotropy-resolving SGS model. The computational results of the mean velocity for all grid resolutions correspond fairly well to those of the DNS data. Such a grid-independent trend in the mean-velocity distributions is very encouraging from an engineering viewpoint. Although the balance of the GS and SGS components in the Reynolds shear stress is largely different between the various test cases, the various distributions of the total (GS+SGS) values coincide and are in good agreement with the DNS data. This fact properly accounts for the prediction accuracy of the mean velocity. Concerning the turbulence energy, as the grid resolution becomes coarser, the SGS component of the turbulence energy increases. Although some overpredictions appear in the total turbulence energy for the coarser grid cases, the predictions correspond generally well to the DNS data. The total turbulence intensities are found to be well predicted in general, although slight discrepancies still remain as the grid resolution becomes coarse. From these results, it is confirmed that the prediction accuracy was considerably improved by the present anisotropic SGS model, compared with conventional linear EVMs that have been widely used.

4.2. Evaluation of true values of the SGS stresses using the DNS data

To discuss the model performance quantitatively, knowing the correct answer for the model prediction is indispensable. For this purpose, we obtained the true values of the SGS stresses for C180F and C180C by decomposing the DNS data into the GS and SGS components through a grid-filtering process. For instance, the true values of the total (GS + SGS)

Fig. 1 Computational results of Smagorinsky model.

Fig. 2 Computational results of dynamic Smagorinsky model.

Fig. 3 Computational results of the present anisotropy-resolving SGS model.

Reynolds stresses and total turbulence energy are originally defined as follows:

$$\langle U_i U_j \rangle - \langle U_i \rangle \langle U_j \rangle, \quad k_{\text{total}} = k_{(GS+SGS)} = \frac{1}{2} (\langle U_k U_k \rangle - \langle U_k \rangle \langle U_k \rangle), \quad (14)$$

where $\langle \rangle$ denotes the aforementioned plane-averaged value. Similarly to Eq. (14), the plane-averaged values of the true GS Reynolds stresses and GS turbulence energy are defined as

$$\langle \bar{U}_i \bar{U}_j \rangle - \langle \bar{U}_i \rangle \langle \bar{U}_j \rangle, \quad k_{GS} = \frac{1}{2} (\langle \bar{U}_k \bar{U}_k \rangle - \langle \bar{U}_k \rangle \langle \bar{U}_k \rangle). \quad (15)$$

On the other hand, based on Eq. (3), the plane-averaged values of the true SGS Reynolds stresses and SGS turbulence energy are defined as

$$\langle \tau_{ij} \rangle = \langle \bar{U}_i \bar{U}_j - \bar{U}_i \bar{U}_j \rangle, \quad \langle k_{SGS} \rangle = \frac{1}{2} \langle \tau_{kk} \rangle, \quad k_{SGS} = \frac{1}{2} \tau_{kk} = \frac{1}{2} (\bar{U}_k \bar{U}_k - \bar{U}_k \bar{U}_k). \quad (16)$$

Consequently, the anisotropy tensor for each of these stresses is defined as follows:

$$b_{ij}^{(GS+SGS)} = \frac{\langle U_i U_j \rangle - \langle U_i \rangle \langle U_j \rangle}{2 k_{(GS+SGS)}} - \frac{1}{3} \delta_{ij}, \quad b_{ij}^{GS} = \frac{\langle \bar{U}_i \bar{U}_j \rangle - \langle \bar{U}_i \rangle \langle \bar{U}_j \rangle}{2 k_{GS}} - \frac{1}{3} \delta_{ij}, \quad b_{ij}^{SGS} = \frac{\langle \tau_{ij} \rangle}{2 \langle k_{SGS} \rangle} - \frac{1}{3} \delta_{ij}. \quad (17)$$

Figure 4 shows the decomposed parts of the turbulence energy and the distributions of the SGS stresses. As clearly seen in the figure, the SGS components become larger as the grid resolution becomes coarser. In particular, for the coarse-grid resolution case of C180C, the SGS turbulence energy covers more than 25% of the total turbulence energy in the near-wall region. It is also noted that a strong anisotropy trend is seen in the distributions of the SGS stresses in the near-wall region for both grid-resolution cases.

Figure 5 compares the distributions of the anisotropy tensor between the GS and SGS stresses. Abe (2014) previously reported that the trend in the calculated SGS stresses was generally similar to that of the GS components. The present results estimated directly from the DNS data obviously support this conclusion. The stress anisotropy of the SGS component is very similar to that of the GS component particularly in the near-wall region, although the SGS stresses

Fig. 4 Distributions of decomposed (GS and SGS) components for turbulence energy and SGS stresses with various grid-filter widths corresponding to Table 1.

tend to be more isotropic in the region far from the wall. Furthermore, it is also understood that the SGS stresses show a similar trend in the stress anisotropy between the fine (C180F) and coarse (C180C) grid resolutions.

Although it has been said that the small-scale eddies tend to be more homogeneous and isotropic than the large-scale eddies, this is not always true in the near-wall region of a wall-shear turbulent flow. Therefore, for an accurate prediction of the SGS stresses, it is necessary to correctly reproduce this strong SGS-stress anisotropy in the region close to the wall surface.

4.3. *A priori* test of the SGS models using the filtered DNS data

To investigate the basic potential of the SGS models in more detail, we evaluated the SGS stresses by an *a priori* test using the aforementioned filtered DNS data. In this study, the test-filter width of $\widehat{\Delta}_i = 2\Delta x_i$ was adopted in the test-filtering process for the present anisotropic SGS model and the dynamic Smagorinsky model.

First, the results of the Smagorinsky model and the dynamic Smagorinsky model are shown in Figs. 6 and 7, respectively. Note that, for evaluating k_{SGS} in the *a priori* test for all the SGS models presently tested, we used the true value of k_{SGS} that was originally defined in Eq. (16). Apparently, both models failed to reproduce the SGS normal stresses, obtaining instead an almost isotropic prediction, where each of the SGS normal stresses becomes close to $2k_{SGS}/3$. It is also found that the Smagorinsky model returns considerable underprediction of the SGS shear stress. Furthermore, as seen in Fig. 6 (b) and (d), b_{12} of the Smagorinsky model for each case does not approach zero but a constant value toward the wall surface. This means that the Smagorinsky model yields an incorrect wall-limiting behavior for the SGS shear stress that is proportional to y^2 , whereas its correct behavior is proportional to y^3 . In contrast, the dynamic Smagorinsky model exhibits the correct wall-limiting behavior of the SGS shear stress with no additional model function. This is regarded as a strong point for the dynamic Smagorinsky model compared with the conventional Smagorinsky model. However, a closer inspection indicates that the prediction accuracy of the SGS shear stress gradually worsens as the grid resolution becomes coarser. Although a linear EVM has been widely used, it is generally said that this type of SGS model cannot reproduce the SGS-stress tensor correctly, in particular for its normal components.

The results of the present anisotropy-resolving one-equation SGS model for C180F and C180C are shown in Figs. 8

Fig. 5 Comparison of GS and SGS anisotropy tensors evaluated from the DNS data with various grid-filter widths corresponding to Table 1.

and 9, respectively. Note that the results partially estimated using only the first two terms on the right-hand side of Eq. (4) are also included for comparison. In fact, this form is a conventional linear EVM by Inagaki (2011). Clearly, Figs. 8 (a) and 9 (a) show that the SGS stresses are well predicted in general for both grid resolutions, including the SGS normal components. Comparison of the anisotropy tensor (i.e., Figs. 8 (c) and 9 (c)) indicates that the present anisotropic SGS model returns a reasonable stress anisotropy, although it is a little weaker than the DNS data. This also indicates that the present anisotropic SGS model returns the correct wall-limiting behavior for all the SGS-stress components. Concerning the shear-stress component shown in Figs. 8 (d) and 9 (d), the prediction accuracy seems to have improved even for the coarse grid-resolution case of C180C.

That being the case, the anisotropy-resolving SGS model that incorporates the effect of a scale-similarity model is found to reproduce the SGS-stress components including the normal stresses more properly. As described above, the true k_{SGS} defined in Eq. (16) was used in the present *a priori* test for all the SGS models. Since the same value was used for k_{SGS} , it is a natural consequence that all *a priori* tests using a linear isotropic EVM form showed a very similar trend in the distributions of the SGS normal stresses. Considering the fact that the linear part of the present SGS model (i.e., Figs. 8 (b) and 9 (b)) shows a trend similar to the other linear EVMs, it is concluded that a reasonable prediction of the SGS-stress anisotropy is achieved via the extra anisotropic term introduced in the present anisotropic SGS model.

4.4. Further estimations of the present anisotropic SGS model

From the above discussion, the basic potential of the present anisotropic SGS model was quite encouraging from an engineering viewpoint. Thus, for further developments, we investigated the characteristics of the present SGS model in more detail.

First, to confirm the effect of the test-filter width on the prediction accuracy, Fig. 10 shows the results of the *a priori* test of the present SGS model for C180F, with a different test-filter width of $\widehat{\Delta}_i = 3\Delta x_i$ being used. Note that as is clear from the model definition in Eq. (4), Fig. 10 (b) is the same as Fig. 8 (b). As seen in the figure, the present anisotropic SGS model shows a quite small dependence on the test-filter width, at least, in the range from $\widehat{\Delta}_i = 2\Delta_i$ to $3\Delta_i$ for the present test case. This notable feature derives mainly from the model formulation in Eq. (4), where the scale-similarity

Fig. 6 *A priori* test of the Smagorinsky model evaluated from filtered DNS data.

Fig. 7 *A priori* test of the dynamic Smagorinsky model evaluated from filtered DNS data.

Fig. 8 *A priori* test of the present anisotropy-resolving SGS model evaluated from filtered DNS data (C180F).

Fig. 9 *A priori* test of the present anisotropy-resolving SGS model evaluated from filtered DNS data (C180C).

Fig. 10 A priori test of the present anisotropy-resolving SGS model with different test-filter width of $\widehat{\Delta}_i = 3\Delta x_i$ (C180F).

model using the test-filtering operation contributes only to the evaluation of the SGS-stress anisotropy, whereas its level is determined by the transport equation of k_{SGS} . Considering the fundamental concept of a scale-similarity model, the trend for stress anisotropy evaluated with different test-filter widths over a small range is expected to be similar, and result in a solution almost independent of the test-filter width. This fact may be useful in applying the present SGS model to practical engineering applications because a CFD code sometimes encounters restrictions in performing the test-filtering process because of the grid topology and/or discretization schemes adopted.

Next, to confirm the current level of the present SGS model in a real simulation, the SGS stresses obtained by the *a posteriori* tests for C180F and C180C are compared with the true values of the DNS data in Figs. 11 and 12, respectively. As was already seen in Fig. 3, the present anisotropic SGS model considerably improved the prediction of the statistics, i.e., the mean velocity, the total (GS+SGS) Reynolds shear stress and the total turbulence intensities. However, from Figs. 11 and 12, the predictions of the SGS stresses are still unsatisfactory. They are considerably overestimated compared with those of the filtered DNS data using the same grid resolution. As is clear from Figs. 11 (b) and 12 (b), this discrepancy is thought to derive mainly from the overprediction of the SGS turbulence energy. As it is obtained from its transport equation, some areas in the present SGS model still remain to be improved. In contrast, it should be noted that the SGS-stress anisotropy is generally reproduced well even in the *a posteriori* simulations. This fundamental feature is similar to what is seen in Figs. 8 and 9, although the anisotropy of the *a posteriori* tests looks a little stronger in the region far from the wall. This fact may become a strong point of the present SGS model because all components of the SGS stresses will be improved simultaneously once the estimate of the SGS turbulence energy is improved. In this sense, further effort to improve the prediction of the SGS turbulence energy is a key factor that leads to more accurate reproduction of the SGS stresses.

Fig. 11 Profiles of the SGS stresses obtained by *a posteriori* test of the present anisotropy-resolving SGS model (C180F).

5. Concluding Remarks

Toward the development of SGS models for LES, a detailed investigation of the model performance was performed through an *a priori* test using the DNS data of a fully-developed plane channel flow. For this purpose, we produced three reduced velocity fields from the DNS data with different filter widths. We also performed *a posteriori* simulations with several SGS models using the same grid resolutions as the reduced DNS data.

First, from *a posteriori* simulations, the predictive performance of an anisotropy-resolving SGS model was found to be much superior to the other linear EVMs examined. The anisotropic SGS model returned almost grid-independent solutions for the mean velocity as well as the total (GS+SGS) Reynolds shear stress. In contrast, the linear EVMs showed considerable grid dependency. Their prediction accuracy clearly worsened as grid resolution became coarser. This trend was not improved even if the dynamic Smagorinsky model was adopted.

Next, to elucidate fundamental features of the SGS stresses, their true values for each reduced grid resolution were evaluated directly from the DNS data. Although the SGS stresses became larger as grid resolution became coarser, it was a little surprising that the SGS-stress anisotropy did not change so much between the GS and SGS components particularly in the near-wall region. The SGS stresses also showed a similar trend in the stress anisotropy between different grid resolutions. Furthermore, to investigate how reasonably an SGS model reproduces the SGS stresses, we compared the results of *a priori* tests using the aforementioned reduced DNS data. The linear EVMs failed entirely to reproduce the SGS normal stresses at all. In contrast, the anisotropy-resolving SGS model that incorporated the effects of a scale-similarity model was found to reproduce the SGS-stress components including the normal stresses more properly. Since the linear part of the anisotropic SGS model showed a trend similar to the other linear EVMs, the reasonable prediction of the SGS stress anisotropy was achieved by the effect of the extra anisotropic term introduced. That being the case, it is strongly expected that such an anisotropy-resolving term has the capability of predicting the SGS stresses more correctly.

Finally, we briefly note the following as areas of future study. It was found from the *a posteriori* test that the present anisotropic SGS model considerably improved the predictions of the statistics such as the mean velocity. However, a closer inspection revealed that the performance in reproducing the SGS stresses was still far from satisfactory. Although

Fig. 12 Profiles of the SGS stresses obtained by *a posteriori* test of the present anisotropy-resolving SGS model (C180C).

the SGS-stress anisotropy was reproduced generally well even in the *a posteriori* simulation, the predicted SGS stresses were considerably overestimated compared with those of the filtered DNS data. It is thought that this discrepancy derived mainly from an overprediction of the SGS turbulence energy that was obtained from its transport equation. Therefore, with regard to this issue, there may remain several points to be discussed and improved. Although the predictive performance of the present SGS model is generally acceptable, more detailed discussion will contribute to its further development.

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Appendix

In the *a priori* tests of the present study, we used the values that were averaged over homogeneous planes (i.e., the x - and z -directions) at a fixed wall distance because the instantaneous DNS data were available only for a single (i.e., the final) time step. On the other hand, concerning the mean velocity and the second moments (only for the total value), the statistics ensemble-averaged over long-time periods are generally provided by the researchers who performed the DNS. Since the former “plane-averaged” procedure has often been used for *a priori* tests in many previous studies (see for example, Germano et al., 1991; Salvetti and Banerjee, 1995; Horiuti, 1997), this type of analysis is considered to be reliable. However, it is also true that the number of sample data for a plane-averaged value is much smaller than that for an ensemble-averaged value.

Fig. 13 Comparison of the plane-averaged statistics using an instantaneous dataset with those ensemble-averaged over a long period.

Therefore, to confirm how the number of sample data affects the analysis, we compared the distributions of the statistics obtained by these two procedures. Figure 13 shows the comparison of the plane-averaged statistics with those ensemble-averaged for a long period. As seen in Fig. 13 (a), good agreement is obtained for the mean-velocity distribution. Concerning the turbulence energy, the Reynolds shear stress and the turbulence intensities shown in Figs. 13 (b)–(d), the plane-averaged values generally correspond to the ensemble-averaged ones, although statistical convergence is not achieved yet and some wavy profiles still remain. As for the anisotropy tensor, it is found from Fig. 13 (e) and (f) that the plane-averaged values provide the Reynolds-stress anisotropy properly and their distributions show no crucial conflict with the ensemble-averaged ones. This being the case, it is considered that the knowledge obtained from the present *a priori* tests is valuable enough to understand the fundamental features of the model performance.

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