Application of linear hydrodynamic stability analysis to reacting swirling combustor flows

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Abstract
This paper presents recent research activities on coherent structures in combustor flows employing linear hydrodynamic stability theory. Large-scale coherent structures play an important role in swirling combustor flows. On the one hand isothermal swirling jets undergoing vortex breakdown are susceptible to self-excited flow oscillations. They manifest in a precessing vortex core and synchronized growth of large-scale helical vortical structures. On the other hand, thermoacoustic oscillations are often related to axisymmetric flow structures that are driven by the acoustic field. Despite the qualitative difference between the self-excited helical instabilities and the acoustically forced axisymmetric instabilities, the linear analysis is capable to describe both phenomena with an astonishing accuracy. The proposed theoretical framework allows for a systematic analysis of the dominant flow dynamics in the turbulent combustor flow and their interplay with the flame. This is demonstrated by considering two examples: a swirl-stabilized flame featuring a precessing vortex core that becomes suppressed when changing from steam-diluted to dry conditions, and a swirl-stabilized flame subjected to strong axial forcing mimicking thermo-acoustic oscillations.

Key words: Linear hydrodynamic stability, Reacting swirling flows

1. Nomenclature

- \( Re_D \) : Reynolds number based on \( D_h \)
- \( Re_t \) : Reynolds number with eddy viscosity
- \( S \) : swirl number
- \( K \) : normalized coherent velocity fluctuation intensity
- \( A \) : amplitude ratio
- \( \nu \) : eddy viscosity
- \( \nu \) : kinematic viscosity
- \( D_h \) : hydraulic diameter of the burner
- \( OH^* \) : OH-chemiluminescence intensity
- \( \alpha = \alpha_r + i\alpha_i \) : complex axial wave number
- \( \omega = \omega_r + i\omega_i \) : complex frequency
- \( \omega_0 = \omega_{0r} + i\omega_{0i} \) : complex local absolute frequency
- \( \omega_g = \omega_{gr} + i\omega_{gi} \) : complex global frequency
- \( m \) : azimuthal wave number
- \( t \) : time
- \( f \) : frequency
- \( \phi \) : complex eigenfunctions
- \( \mathbf{x} = (x, y, z) \) : Cartesian coordinates
- \( \mathbf{x} = (x, r, \Theta) \) : cylindrical coordinates
2. Introduction

Swirl-stabilized combustors, which are commonly used for enhanced flame stabilization, exhibit a complex flow field topology consisting of an emanating jet, an Inner Recirculation Zone (IRZ), an Outer Recirculation Zone (ORZ), and turbulent shear layers between the jet and the recirculation zones. The backflow due to the vortex breakdown and the regions of high shear enhance the flame stability but also facilitate self-excited flow oscillations that manifest in a precessing vortex core and large-scale anti-symmetric flow structures. Due to the skew symmetric shape of the precessing vortex core, no direct coupling of the helical structure to integral heat release fluctuations, which is necessary for thermoacoustic instabilities, is possible (Moeck et al., 2012), but they may play an important role due to indirect coupling (e.g. mixing) or in multiburner arrangements or annular combustion chambers. Axisymmetric coherent structures, in contrast, are generally related to thermoacoustic oscillations. The pronounced vortex-flame interactions associated with large-scale flow structures (Paschereit et al., 1999, Paschereit et al., 2000) cause heat-release fluctuations with a feedback loop between the hydrodynamic field, the flame, and the acoustic field (Keller, 1995).

Both, self-excited helical anti-symmetric (Oberleithner et al., 2013) and forced axisymmetric (Terhaar et al., 2014b) flow structures can be analyzed using linear hydrodynamic stability theory. Accordingly, the current study is structured into two parts. In the first part we employ the linear hydrodynamic stability theory to explain why combustion can suppress the occurrence of self-excited helical structures. In the second part we use this tool to illustrate the role of the saturation of the shear layer amplification for the flame response saturation, which is of crucial importance for the limit-cycle amplitude of thermoacoustic instabilities.

The onset of self-excited global flow oscillations in isothermal swirling jets has been extensively studied within the last years, employing the concept of linear hydrodynamic stability analysis. It was shown experimentally (Liang and Maxworthy, 2005, Oberleithner et al., 2011, Oberleithner et al., 2012b), numerically (Ruith et al., 2003), and theoretically (Gallaire et al., 2006, Oberleithner et al., 2011) that these oscillations correspond to a so-called unstable global mode triggered by internal resonance. For the isothermal jet, the size of the recirculation bubble is thereby crucial for the set-in of a global instability (Oberleithner et al., 2012b). Despite numerous experimental and numerical investigations, it remains still unclear how combustion affects the self-excited oscillatory mode. Depending on the combustor design and operating conditions, the precession of the vortex core, clearly identified at isothermal conditions, may (Moeck et al., 2012, Terhaar et al., 2014a, Boxx et al., 2010, Fokaides et al., 2009, Froud et al., 1995, Galley et al., 2011) or may not (Syred, 2006, Giauque et al., 2005, Roux et al., 2005) exist in the reacting flow field. Several mechanisms for the observed damping have been suggested. These are, for instance, a reduction of the tangential velocities near the jet centerline (Syred, 2006) or a flame-induced dilatation and increased viscosity (Roux et al., 2005). However, recent experimental investigations within our group suggest that the density stratification is the main mechanism that suppresses the self-excited flow oscillations (Terhaar et al., 2014a, Oberleithner et al., 2013). The density field that is present in the reacting flow is assumed to have a strong effect on the growth of hydrodynamic instabilities, and thus, on the onset of global instability and the precessing vortex core. Considering the fact that low density (hot) jets are globally unstable (Monkewitz et al, 1990), the unburned (cold) jet entering a hot combustion chamber should be stabilized due to the density field. However, as described (Oberleithner et al., 2013), the global flow stability in a swirl-stabilized combustor depends crucially on the specific density field resulting from a specific flame location.

Consequently, the concept of global flow instability is first demonstrated on a combustor flow that exhibits a precessing vortex core. The associated coherent helical flow structures are described in detail and are compared to theoretical predictions based on a local linear stability analysis. As a next step, the analysis is applied to the reacting flow of a V-
shaped flame. This configuration does not reveal any self-excited flow oscillations. The linear stability analysis provides clear evidence that the density stratification induced by the V-flame stabilizes the global instability.

In the second part of the study, we investigate the role of shear layer amplification for the saturation of acoustically driven coherent structures and their impact on the nonlinearity of heat release fluctuations. Flame describing functions are of crucial importance for the prediction of the limit cycle amplitude of thermoacoustic instabilities (Ćosić et al., 2013). The phenomenological explanation of one of the flame response saturation mechanisms was found by Oberleithner et al. (2012a), who showed that axial velocity fluctuations introduced by loudspeakers lead to changes in the mean flow field. They performed linear hydrodynamic stability analysis on mean flow fields measured for increasing forcing amplitudes and showed that the growth rate of the vortices in the shear layers is considerably decreased for higher forcing amplitudes. With their findings, they explained an intermediate saturation of the flame response and emphasized the importance of the mean flow field. Similar trends were observed in several studies (Bellows et al., 2007, Thumuluru and Lieuwen, 2009), which reported changes in the time-averaged flame shape.

To obtain insight into the growth of vortical structures and changes of the shear layer amplification, a combined empirical and theoretical approach is used in the second part of this study. First, we employ a triple decomposition of the experimental data, as proposed by Hussain and Reynolds (2006), to derive the coherent velocity fluctuations at the forcing frequency. Next, the influence of the mean flow field on the growth of vortical structures is shown, employing linear hydrodynamic stability theory. The results are well in line with the empirically derived results. Both show the crucial importance of the shear layer saturation mechanisms related to mean flow field changes for the growth of vortices. Furthermore, the resulting amplitudes of the large-scale coherent structures, which are assumed to play an important role for the heat release fluctuations, show similar characteristics to the flame response at the same forcing frequency and amplitude.

3. Theoretical Principle and Approach

Linear hydrodynamic stability analysis is applied to the measured time-averaged (mean) flow and density fields. The obtained instability modes represent the coherent flow structures that exist in the mean flow. In general, a linear stability analysis based on the mean flow becomes inherently nonlinear due to the nonlinear Reynolds stresses that form the mean flow. This Ansatz has been developed by (Gaster et al., 1985) and was later supported by the generalized mean field model of (Noack et al., 2004). The analysis based on the mean flow is capable to accurately predict the oscillatory mode at its limit-cycle (Pier, 2002).

In the following, we briefly describe the main principle of linear stability analysis applied to a non-isothermal, turbulent swirling jet. For additional information, the reader is referred to (Oberleithner et al. 2011, Lesshaft and Marquet, 2010, Oberleithner et al., 2012a) and the references therein.

3.1. Local Linear Stability Problem Formulation

To apply linear stability concepts to a turbulent mean flow, it is necessary to introduce the classical triple decomposition (Hussain and Reynolds, 2006). Accordingly, all flow variables are decomposed into three parts: the steady mean flow, the coherent component, and the stochastic fluctuations, yielding

$$v(x, t) = V(x) + v^c(x, t) + v_s(x, t). \quad (1)$$

By introducing this ansatz into the Navier–Stokes equations and incompressibility condition, separated equations of motion are obtained for the mean and the coherent part. The linearized equations of the coherent part for a viscous, stratified (nonuniform density), axisymmetric flow in the zero Mach number limit are given by

$$\frac{\partial v^c}{\partial t} + v^c \cdot \nabla V + V \cdot \nabla v^c = -\frac{1}{\rho} \nabla p^c + \frac{1}{\rho_0} \nabla^2 v^c \quad (2)$$

$$\nabla \cdot v^c = 0. \quad (3)$$

Within the framework of local stability analysis, the flow field is sliced into velocity and density profiles and the flow is treated as locally parallel, meaning that the analysis is applied to a fictitious parallel flow at each streamwise location. Hence, the flow becomes homogeneous in streamwise direction and the coherent part can be decomposed in the following way:
\[ \mathbf{v}^e(x, t) = \Re \left[ \hat{v}(r)e^{i(x + \omega t - \omega t)} \right], \]  
\[ \text{with } \alpha \text{ being the complex streamwise wavenumber, } \omega \text{ the complex frequency, } m \text{ the real azimuthal wavenumber, and } \hat{v} \text{ the radial amplitude function, where } \Re \text{ refers to the real part. Note that the density perturbations in the zero Mach number limit are decoupled from the velocity and pressure perturbations and do not need to be solved for (Lesshafft, 2006).} \]

By introducing equation 4 into the equations of motion, the four partial differential equations (2-3) convert into four ordinary differential equations that can be written as an eigenvalue problem

\[ A(\alpha)\phi = \omega B(\alpha)\phi \]  
\[ \text{This is solved efficiently by using a Chebyshev spectral collocation method (Khorrami et al., 1989). A detailed description of the numerical approach is given in (Oberleithner et al., 2011).} \]

The Reynolds number \( R_e \) that scales the viscous terms in equation 2 is defined as

\[ R_e = \frac{V_0 D_h}{\nu + \nu_t}, \]  
\[ \text{with } \nu = \mu/\rho \text{ representing the molecular kinematic viscosity that is, for the stratified flow, a function of } r, \text{ and } \nu_t \text{ representing an eddy viscosity that models additional damping of the coherent structures due to small-scale turbulence. The latter is derived from the measured turbulent stresses using the well known Boussinesq’s approximation } -\nu_t^2 = \nu \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right]. \]

### 3.2. Temporal, Spatial, and Spatio-temporal Analysis

The above introduced eigenvalue problem can be solved for complex \( \omega \) and given real \( \alpha \), for complex \( \alpha \) and given real \( \omega \), or for complex \( \alpha \) and complex \( \omega \). The first approach is called temporal analysis, yielding temporally growing \( (\omega_\alpha > 0) \) or decaying \( (-\omega_\alpha < 0) \) modes. It is most appropriate for the analysis of bounded flows that have no free stream velocity like the Taylor–Couette flow. The second approach is called spatial analysis, yielding spatially growing \( (-\alpha > 0) \) or decaying \( (-\alpha < 0) \) modes. It is applicable to open shear flows that are convectively unstable (noise amplifiers) (Michalke, 1965). In weakly non-parallel flows, the spatial analysis describes the streamwise growth and decay of flow perturbations initiated at a certain axial location, \( x = 0 \), and frequency \( \omega_r \). The disturbance velocity field is obtained from

\[ \mathbf{v}^e(x, t) = \Re \left[ \phi(r) \exp \left[ i \left( \int_0^r \alpha(x) dx + m\theta - \omega t \right) \right] \right], \]

where the eigenvalue \( \alpha \) and the eigenfunction \( \phi \) is derived at each streamwise slice separately.

The third approach, where the eigenvalue problem is solved for complex \( \alpha \) and complex \( \omega \) is called spatio-temporal analysis. Thereby, waves are considered that may grow/decay in time and streamwise direction. It is applicable to flows that undergo self-excited oscillations at a discrete tone, as for instance wakes (Provansal et al., 1987, Monkewitz and Sohn, 1988), hot jets (Monkewitz et al., 1990), or cold swirling jets undergoing vortex breakdown (Liang and Maxworthy, 2005, Gallaire et al., 2006, Oberleithner et al., 2011). The self-induced oscillations of these so-called globally unstable flows are strongly connected to a spatial domain where the flow is absolutely unstable (Huerre and Monkewitz, 1990). In that region, disturbances grow in time in upstream and downstream direction, ultimately contaminating the entire flow (flow oscillators). In contrast, in convectively unstable regions, modes grow solely in downstream direction and are swept away from their source (flow amplifiers). In order to distinguish between absolute and convective instability, the criterion of vanishing group velocity is applied (Briggs, 1964). This implies searching for saddle points in the complex \( \alpha \)-plane by minimizing the functional \( F = (\partial \omega_\alpha/\partial \alpha_r)^2 + (\partial \omega_\alpha/\partial \alpha_i)^2 \). The frequency at this saddle point is called the absolute frequency \( \omega_0 \). It determines the long time behavior of a wavepacket initiated at \( t = 0 \) and \( x = 0 \). The corresponding parallel flow is absolutely unstable if \( \omega_0 > 0 \) and absolutely stable if \( \omega_0 < 0 \).

### 3.3. Deriving the Global Stability Properties from the Local Analysis

Globally unstable flows are typically treated by employing a global stability analysis with a two-dimensional perturbation ansatz (Sipp and Lebedev, 2007) such as

\[ \mathbf{v}^e(x, t) = \Re \left[ \hat{v}(r, x)e^{i(x + \omega_0 t)} \right], \]

This type of analysis directly provides the unstable \( (\omega_{g,i} > 0) \) and stable \( (\omega_{g,i} < 0) \) global modes without any information about the local stability properties. The high computational costs of the global ansatz and the sensitivity to the inflow and...
outflow boundary conditions render the global analysis much more demanding than the local analysis (Theofilis, 2011).

In recent years, significant effort has been made to derive connections between the absolute growth frequency $\omega_0(x)$ obtained from local analysis and the global mode shape and frequency obtained from a global analysis. A consistent criterion for the global mode frequency $\omega_g$ is given by the saddle point criterion

$$\omega_g = \omega_0(x_s) \quad \text{with} \quad \frac{d\omega_0}{dx}(x_s) = 0,$$

that is derived from an analytic continuation of $\omega_0(x)$ in the complex $x$ plane (Chomaz et al., 1991). The streamwise location $x_s$ determines the location of the global mode wavemaker, which is the 'pacemaker' of the global oscillations. It perturbs the surrounding flow field that may amplify the perturbations due to convective instabilities. The absolute frequency at the wavemaker $\omega_0(x_s)$ determines the oscillating frequency $\omega_g$, and growth rate $\omega_{0r}$ of the global mode. For $\omega_{0r} > 0$, the flow is globally unstable, oscillating at $\omega_0(x_s) = \omega_{0r}$, while the flow is globally stable for $\omega_0(x_s) < 0$. Once the global frequency is determined, the spatial structure of the global mode can be constructed from a spatial analysis using equation 7 with $\omega_r = \omega_{0r}$.

The linear criterion 9 successfully predicts the frequency of the limit-cycle oscillation of the cylinder wake (Pier, 2002) when applied to the saturated mean flow. The analysis is thereby inherently non-linear due to the nonlinearities captured in the mean flow shape. Juniper et al. (Juniper et al., 2011) provide a detailed comparison of the local and global analysis of the cylinder wake. They show that both approaches arrive at the same global mode shape and they conclude that the local analysis provides deeper insights into the frequency selection principle and associated internal feedback mechanisms, while the global analysis is more accurate for strongly nonparallel flows.

4. Experimental Approach

The experiments were carried out in an atmospheric combustor test-rig as shown in Fig. 1. The combustor consisted of a swirl generator, a silica glass combustion chamber, and an annular mixing tube. The moveable block swirl generator (Leuckel, 1967) allowed for the adjustment of the swirl number.

<table>
<thead>
<tr>
<th>Operating conditions of the tested cases.</th>
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<tr>
<td><strong>Inlet Temp.</strong></td>
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<tr>
<td>Attached V-flame without forcing</td>
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<tr>
<td>Detached annular-flame without forcing</td>
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<tr>
<td>Acoustically forced V-flames at 196 Hz with $u'/w_0 = 0 - 0.7$</td>
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Figure 1 Sketch of the combustor test rig and the experimental setup.

The natural gas fuel was either injected into the swirl generator (partially premixed) mixed with the air flow well upstream of the combustor to assure a homogeneous fuel–air mixture (perfectly premixed). Optionally, the air mixture could be preheated and diluted with superheated steam. The flow could be excited with axisymmetric forcing using four loudspeakers. The acoustic field upstream of the combustor and inside the combustor was measured using the Multi-Microphone-Method (MMM) (Paschereit et al., 2002), and global heat release fluctuation were captured at perfectly premixed conditions using a narrow bandpass-filtered (308 nm) photomultiplier that measured the chemiluminescence of the OH*-radical. Time-resolved spatial distributions of the heat release rate were obtained using a high-speed camera coupled to a high-speed capable bandpass filtered image intensifier. The two-dimensional velocity field in the streamwise plane aligned with the combustor axis was measured using high-speed Particle Image Velocimetry (PIV).

The density distribution was obtained from the same Mie-scattering data as the PIV results using a Quantitative Light Sheet (QLS) technique as proposed in (Terhaar et al., 2014a). In the QLS technique the amount of scattered light is used to derive the spatial distribution of the seeding density, which can be correlated to either mixture fractions or the fluid density. Details about the used setup can be found in recent publications (Terhaar et al., 2014a, Terhaar et al., 2014b, Oberleithner et al., 2013).

Table 1 provides a summary of the investigated operating conditions. They include two different flame configurations (attached V-flame and detached annular flame) without forcing and one configuration (V-flame) at increasing rates of acoustic forcing. The steam content $\Omega$ is defined as the ratio of the mass flow of steam to the mass flow of air.

5. Results

In the following, the experimental and theoretical findings are briefly outlined and discussed. For more detail the reader is referred to the references (Terhaar et al., 2014b, Oberleithner et al., 2013).

5.1. Mean Flow and Density Field

Figure 2 provides an overview of the two unforced configurations. Three flow regions can be identified for both configurations. An inner recirculation zone (created by vortex breakdown), an outer recirculation zone, and an annular swirling jet in between.

Significant differences can be found between the flame shapes and density distributions of the two configurations. The V-shaped flame is attached to the combustor inlet with the main reaction zone located close to the combustor inlet in the inner recirculation zone (Fig.2c). This induces a large region of burned gas and hot temperatures shortly downstream of the combustor inlet (Fig.2e). The annular flame is detached from the inlet and shifted far downstream (Fig.2d). The hot gases do not penetrate into the region near the centerbody, which is indicated by the small radial density gradients in the inner recirculation zone (Fig.2f).

5.2. Self-excited Coherent Structures

Of the two investigated flame configurations, the V-flame did not show any periodic flow oscillations, while the detached annular flame showed self-excited flow oscillations also referred to as the precessing vortex core. These energetic flow oscillations are investigated by means of proper orthogonal decomposition (POD), a well-established technique in fluid mechanics (Berkooz et al., 1993). This system reduction strategy allows for an accurate description of turbulence data using only a limited number of modes. The POD of the flow field of the annular flame yields the first two spatial POD modes depicted in Fig. 3a,b). They represent the identical vortical structures shifted by a quarter wavelength and describe travelling vortices in the shear layers, which are synchronized to the precessing vortex core (PVC) (Oberleithner et al., 2011). Figure 3c shows the phase distribution of this oscillatory mode weighted by its magnitude. Most strikingly, it shows that the waves emanate from a single point that is located at the jet center at $x/D_0 = 1.2$. From this location, waves travel in upstream, downstream, and radial directions. This particular point coincides with the wavemaker of the theoretically derived linear global mode, as will be shown later. For the V-flame configuration, no such structures can be detected.
5.3. Stability Analysis of Self-excited Coherent Structures

In order to derive the global stability properties of the underlying mean flow, it is necessary to perform a spatio-temporal stability analysis at each streamwise location as described in (Oberleithner et al., 2013). The major outcome of this analysis is the absolute growth rate ($\omega_0$) as a function of $x$. It determines where the flow becomes absolutely unstable and allows to predict the global oscillation frequency and the streamwise location where this frequency is selected. The individual steps to arrive at $\omega_0(x)$ are provided in (Oberleithner et al., 2013).

Figure 4 shows the absolute growth rate $\omega_0$ and the absolute frequency $\omega_0 f$ computed for the single-helical mode found in the experiment. Since the main object of this work is to investigate the impact of the density field on the onset of global instability, the stability analyses were performed for a virtual uniform density (virtual isothermal flow) and for the actually measured density distribution (stratified jet). The corresponding curves of $\omega_0$ are shown in Fig 4 as black and red lines, respectively.
For the annular flame, only minor differences between the stratified and the isothermal calculations are found. Both, the stratified and the isothermal calculations show a large region of absolute instability ($\omega_{ij} > 0$). The application of a frequency selection criterion (Chomaz et al., 1991) yields the global frequency and the theoretical wavemaker location from the local frequency distribution. In the present case, the location coincides with the location of the maximum absolute frequency. It predicts the measured global frequency with an absolute error of less then 3 %. Furtermore, the predicted location of the wavemaker is in very good agreement to the experimentally determined location, as shown in Fig. 3.

For the V-flame, significant differences are evident between the isothermal and the stratified analysis. When the density variations are neglected (virtually isothermal), the flow is absolutely unstable ($\omega_{ij} > 0$) for a large streamwise region. The frequency selection criterion indicates that the flow is globally unstable and that the wavemaker is located at $x_s/D_h \approx 0.4$. Hence, the isothermal analysis provides a wrong prediction of the global instability since in the experiments no global oscillations were observed. The stratified analysis, in contrast to the annular flame case, yields significantly lower growth rates $\omega_{ij}$ near the nozzle and the flow remains absolutely stable upstream of $x/D_h = 2$ (see Fig. 4b). Consequently, the frequency selection criterion indicates a globally stable flow.

The comparison of the (virtual) isothermal and the stratified analysis clearly shows that for the V-flame configuration, the density field causes the damping of the global mode. More precisely, the strong reduction of the global mode growth rate is caused by the radial density gradient in the inner recirculation zone. Particularly in the region of the global mode wavemaker, a negative radial density gradient coinciding with a negative radial gradient of the axial velocity component induces a stabilization of streamwise traveling perturbations and the internal resonance dies out.
5.4. Forced Coherent Structures

In contrast to the self-excited helical structures investigated above, in the following the growth of acoustically forced axisymmetric structures is assessed for three different swirl numbers. Figure 5 shows streamlines of the coherent velocity fluctuations superimposed on the coherent through-plane vorticity fluctuations of the intermediate swirl number ($S = 0.8$) at an intermediate forcing amplitude of $u'/u_0 = 0.3$ for four equidistant phases. The bottom row shows the phase-averaged flow fields superimposed on the phase-averaged OH*-chemiluminescence images for the same phases. Strong vortices are formed near the combustor inlet due to the forcing. These vortices are convected downstream by the jet and are first further amplified in the shear layers and then gradually damped. The phase-averaged OH*-chemiluminescence images show the corresponding large-scale fluctuations of the heat release.

In order to quantitatively assess the amplification of the vortices, the coherent velocity fluctuation intensity for every point is integrated along radial profiles, yielding the integral coherent velocity fluctuation intensity $K(x)$ (for details see (Terhaar et al., 2014b)). The streamwise development of the coherent velocity fluctuation intensity $K$ is shown for exemplary chosen forcing amplitudes in Fig. 6. Up to a certain streamwise location, $K$ is considerably amplified and subsequently decays. It is evident that for higher forcing amplitudes, the maximum coherent velocity fluctuation intensity $K_{\text{max}}$ is reached further upstream and the maximum amplification ($K_{\text{max}}/K_{x=0}$) is considerably reduced, thus indicating a saturation of the amplification of the vortices in the shear layers. This saturation becomes evident when the maximum coherent velocity fluctuation intensity $K_{\text{max}}$ is plotted over the forcing amplitude, as shown in Fig. 7(a) for the three measured swirl numbers. After an initial linear increase, all curves show a considerably flattening that corresponds to the saturation.

A direct comparison (Fig. 7) of the maximum coherent velocity fluctuation intensity to the measured integral heat release fluctuation shows that both quantities show very similar trends. A comparison of the different swirl numbers yields the highest linear amplification of the coherent velocity fluctuation intensity for the lowest swirl number and the strongest saturation at higher forcing amplitudes. This trend holds for the coherent velocity fluctuation intensity and for the heat release fluctuations. The good similarity between both quantities evidences that the velocity fluctuations are the driving mechanism for heat release fluctuations in perfectly premixed flames.

5.5. Stability Analysis of Forced Coherent Structures

In a next step, the influence of the mean flow field on the growth rate of shear flow instabilities is investigated theoretically by the application of spatial stability analysis to the time-averaged forced flow fields. It yields the axial distributions of the growth rates, which can be integrated to obtain the amplitude ratio $A$. It is defined as the ratio of the amplitude of the instability at an axial location $x$ to the amplitude at the combustor inlet ($x = 0$) and shown in the first row of Fig. 8 for the three measured cases. The damping related to the changes in the mean flow field can be observed very well for all three cases. At very low forcing amplitudes, the amplitude ratio increases up to an axial position of $x/D_h \approx 1.5$. For the lowest swirl number ($S = 0.6$), a maximum amplitude ratio of $A \approx 2.4$ is reached. For the higher swirl numbers, the maximum ratio is lower at $A \approx 1.8$. At higher forcing amplitudes, the maximum amplitude ratios are considerably reduced but remain slightly larger for the higher swirl numbers.
Figure 5  Phase-averaged flow fields and flame positions at four equidistant phase angles ($S = 0.8, u'/u_0 = 0.3$).

Figure 6  Streamwise development of the coherent velocity fluctuation intensity $K(x)$ at $S = 0.8$ for increasing forcing amplitudes.

For the comparison of the calculated amplitude ratio $A$ to the empirically obtained growth of the coherent velocity fluctuations, a modified amplitude ratio $A'(x) = K'(x)/K'(x = 0)$ is introduced. The modified coherent velocity fluctuation intensity $K'$ is corrected for the spreading of the jet, which is not accounted for by the stability analysis. (For details see (Terhaar et al., 2014b).) It is shown in the second row of Fig. 8 and shows essentially the same trends as the theoretically obtained amplitude ratio $A$. The axial location of the maximum amplification and the maximal amplitude ratio are very similar. Only the decay at higher axial positions is slightly overpredicted by the linear stability analysis.

Overall, a good agreement between the theoretically obtained amplitude ratio $A$ and the empirically obtained amplitude ratio $A'$ can be observed. This shows that the stability analysis carried out on the time-averaged forced flow fields is able to capture the important saturation mechanisms. The deviations of the experimental results to the results from the stability analysis can be mainly attributed to the simplifications made for the linear stability analysis (e.g., parallel flow).

6. Summary and Conclusions

This work focuses on coherent structures in reacting swirling combustor flows. It is clearly distinguished between the helical self-excited structures that are purely driven by hydrodynamic instabilities and acoustically driven axisymmetric flow structures. In the first part, the formation of helical self-excited flow oscillations is investigated. These intrinsic dynamics are characterized by a single oscillation frequency and mode. In the swirling jet undergoing vortex breakdown,
this mode manifests in a precessing vortex core and large-scale helical flow structures that rotate in the same direction as the base low. In this study, the particular question is investigated, whether the density field induced by the flame may be the reason for the frequently observed damping of this global flow instability.

The problem is approached by considering two different combustion configurations: The first, featuring a detached annular flame, is subjected to self-excited oscillations. The second, featuring a V-shaped flame that is attached to the burner outlet, does not reveal any global oscillations. The stability properties of these two flows are derived theoretically by means of local linear hydrodynamic stability analysis and compared with experimental results. This results provide evidence for the following statements:

1. The vortex core precession and associated coherent structures are caused by a global hydrodynamic instability.
2. Combustion dampens the oscillations by creating a density stratification.
3. The density stratification at the wavemaker causes the global damping.
4. Density stratifications in the outer shear layer have only weak effect.

A detailed discussion of these statements is given in Oberleithner et al. (2013).

In the second part of this study, the saturation of forced coherent structures is investigated. The distribution of the coherent velocity fluctuation intensity for increasing forcing amplitudes was obtained from phase-averaged measurement.
data and compared to the flame response. A good qualitative agreement could be observed. In particular, the different saturation types of the three reacting cases were captured very well in the amplitude dependent coherent velocity fluctuation intensity distribution. The results clearly demonstrate that the saturation mechanism of the hydrodynamic field is the dominant driver for the saturation of the flame response.

The saturation of the coherent velocity fluctuation intensity is strongly connected to changes in the time-averaged flow field, as demonstrated employing hydrodynamic linear stability analysis to the time-averaged flow fields at the different forcing amplitudes. The results of the stability analysis successfully predict the saturation of the shear layers at intermediate and high forcing amplitudes. Also, the different characteristics of the reacting flow fields and the isothermal flow field could be reconstructed using solely the time-averaged flow fields.

The results emphasize the role of the shear layer amplification and saturation for the flame response at low, intermediate, and high forcing amplitudes. With the application of the linear stability theory on the mean flow fields, it was shown that the changes in the mean flow field are of utmost importance for the saturation of the shear layers, and thus, for the saturation of the flame describing function.

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