Experimental study on the Schmidt number dependence of the scalar derivative statistics in a liquid axisymmetric jet

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Abstract
In this study, the Schmidt number dependence of the scalar statistics and velocity-scalar joint statistics is investigated experimentally in a liquid axisymmetric jet. In the experiments, the axial velocity, radial velocity, concentration of the diffusing dyes, and temperature are measured by an X-type hot-film, optical fiber probes, and an I-type cold-film, respectively. In addition simultaneous measurements of velocities and concentration, and velocities and temperature are also conducted. As diffusing dyes, C.I.Direct Blue 86 and Rhodamine 6G are chosen whose Schmidt numbers are 3,800 and 1,000, respectively. The Prandtl number of temperature is about 7. Experimental results show that the Schmidt number dependence of the spatial derivative statistics is observed. The skewness of concentration spatial derivative is smaller than that of temperature spatial derivative. Further, the radial velocity-concentration cospectrum and radial velocity-temperature cospectrum show differences each other. The slope of radial velocity-temperature cospectrum is $-7/3$, while that of radial velocity-concentration cospectrum is about $-2$.

Key words: Axisymmetric jet, Scalar, Schmidt number, Derivative statistics, Velocity-scalar cospectrum

1. Introduction

It is important in engineering to understand the transport mechanism of the scalar in turbulent flows in relation to the mixing of chemicals and diffusion of pollutants. So there are many reports on the turbulent scalar diffusion in the to date. Chavray et al. (1978) measured the temperature in a heated plane jet and Panchapakesan and Lumley (1993) measured the temperature in the air flow and the Schmidt number $Sc = \nu/D_m$, $\nu$ is the kinematic viscosity and $D_m$ is the molecular diffusion coefficient) or Prandtl number $Pr = \nu/D_t$, $D_t$ is the thermal diffusion coefficient is low ($Sc = 0.7$ and $Pr = 0.3$, respectively). Some researchers also investigated the Schmidt number dependence of the scalar statistics. Yeung et al. (2002) investigated the Schmidt number dependence of the scalar statistics in the various Schmidt number conditions from 0.25 to 64 by the direct numerical simulation. Antonia and Orlandi (2003) investigated the Schmidt number dependence of scalar derivative statistics which is the indication of the characteristics in the fine scale scalar field. However, these researches are only for the range of Schmidt number less than double-digits ($10^2$) and it has not been investigated for the high Schmidt number scalar whose order is more than $10^3$.

With the above backgrounds, in this study, the Schmidt number dependence of the scalar statistics and velocity-scalar joint statistics is investigated experimentally in a liquid axisymmetric jet. In the experiments, the axial velocity, radial velocity, concentration of the diffusing dyes, and temperature are measured by an X-type hot-film, optical fiber probes, and an I-type cold-film, respectively. The simultaneous measurements of velocities and concentration, and velocities and temperature are also performed by combining the X-type hot-film and the optical fiber probes for the scalar measurement. As diffusing dyes, C.I.Direct Blue 86 and Rhodamine 6G are used whose Schmidt numbers are 3,800 and 1,000, respectively. The concentration of C.I.Direct Blue 86 is measured by an optical fiber probe based on the light absorption method.
and that of Rhodamine 6G is measured by an optical fiber probe based on LIF method. Here, it should be noted that the Prandtl number of temperature is about 7.

In this paper, the experimental apparatus, measurement systems, and experimental conditions are shown in Sect. 2. The results of velocities measurement, concentration measurement, and temperature measurement are shown in Sect. 3, Sect. 4, and Sect. 5, respectively. Sect. 6 shows the result of simultaneous measurement of velocities and concentration of Rhodamine 6G. The result of simultaneous measurement of velocities and temperature are shown in Sect. 7. Finally, the conclusions are summarized in Sect. 8.

**Nomenclature**

- \( b \): Half width of mean concentration \([\text{m}]\)
- \( b_U \): Half width of axial mean velocity \([\text{m}]\)
- \( c \): Concentration fluctuation \([\text{g/L}]\)
- \( C \): Mean concentration \([\text{g/L}]\)
- \( d \): Nozzle diameter \([\text{m}]\)
- \( C_u c \): Axial velocity-concentration cospectrum
- \( C_u \): Axial velocity-temperature cospectrum
- \( C_v c \): Radial velocity-concentration cospectrum
- \( C_v \): Radial velocity-temperature cospectrum
- \( E_c (f) \): Power spectrum of concentration fluctuation
- \( E_u (f) \): Power spectrum of velocity fluctuation
- \( f \): Frequency \([\text{Hz}]\)
- \( p(c) \): Probability density function of concentration fluctuation
- \( p(u) \): Probability density function of velocity fluctuation
- \( p(\theta) \): Probability density function of temperature fluctuation
- \( Pr \): Prandtl number
- \( r \): Radial coordinate \([\text{m}]\)
- \( Re_D \): Jet Reynolds number (= \( U_0 d / v \))
- \( Sc \): Schmidt number
- \( t_3 \): Integral time scale of concentration \([\text{s}]\)
- \( t_d \): Integral time scale of axial velocity \([\text{s}]\)
- \( t_\theta \): Integral time scale of temperature \([\text{s}]\)
- \( u \): Axial velocity fluctuation \([\text{m/s}]\)
- \( U \): Mean axial velocity at jet exit \([\text{m/s}]\)
- \( (uc) \): Axial mass flux
- \( (ud) \): Axial heat flux
- \( (uc) \): Radial mass flux
- \( (\theta) \): Radial heat flux
- \( x \): Axial coordinate \([\text{m}]\)
- \( \theta \): Temperature fluctuation \([\text{K}]\)
- \( \Theta \): Mean temperature \([\text{K}]\)
- \( v \): Kinematic viscosity \([\text{m}^2 / \text{s}]\)
- \( Superscript_0 \): Root Mean Square (RMS) value
- \( Subscripts_c \): related to the nozzle exit
- \( c \): related to the axis

2. Experimental apparatus and conditions

2.1. Water channel and nozzle

Figure 1 shows the perspective view of the water channel. The water channel has a height of 350 mm, width of 300 mm, and length of 1,940 mm. The level of water is adjusted to 300 mm. The nozzle whose diameter \( d \) is 4 mm is located at the center of the cross section of the water channel and an axisymmetric jet is issued from the nozzle. Figure 2 shows the coordinate system in this study. The coordinate system is the cylindrical one whose origin is at the center of the nozzle exit, and \( x \) and \( r \) show the axial coordinate and radial coordinate, respectively.

![Fig. 1 Perspective view of the water channel](image)

![Fig. 2 Coordinate system](image)

2.2. Velocity measurement system

The instantaneous axial velocity and radial velocity are measured by an X-type hot-film (TSI, 1243-20W) which is
connected to the constant temperature anemometer (HAYAKAWA, HC-30). The sensing part of the X-type hot-film is platinum film, and has a diameter of 51 μm and a length of 1 mm.

2.3. Concentration measurement system (for C.I.Direct Blue 86)

The instantaneous concentration of C.I.Direct Blue 86 is measured by an optical fiber probe based on the light absorption method. Figure 3 shows the schematic diagram of the light absorption in the dye solution. As shown in Fig.3, the intensity of light is attenuated in passing through the dye solution. From Lambert-Beer’s law, the rate of the attenuation of the light intensity, $\ln(I/I_0)$, is expressed as follows;

$$\ln \frac{I(\lambda)}{I_0(\lambda)} = -k(\lambda)C$$

where $\lambda$ is the wavelength of the light, $I_0(\lambda)$ is the intensity of the incident light, $I(\lambda)$ is the intensity of the light passing through the dye solution, $l$ is the distance of the light passes through the dye solution, $C$ is the concentration of the dye solution, and $k$ is a constant. As suggested by Eq.(1), the rate of the attenuation of the light intensity is proportional to the distance of the light passing through the dye solution, $l$, and the concentration of the dye $C$. If $l$ can be assumed as constant, the concentration of the dye can be evaluated from the rate of the attenuation of the light intensity $\ln(I/I_0)$.

Figure 4 shows the schematic of the optical fiber probe based on the light absorption method (Sakai, et al., 2007). The probe consists of two facing optical fibers with the outer diameter 0.25 mm and core diameter 10 μm, and has a gap of 0.7 mm between two optical fibers. The gap between two optical fibers is considered as a measurement space. The laser light is transmitted to the measurement space by an optical fiber and passes through the space between two facing optical fibers. The light passing through the space between two optical fibers is received and transmitted by another optical fiber and detected by the photomultiplier (HAMAMATSU, H6780-20). As a light source, a laser diode with the oscillating wavelength of 670 nm is used.

![Fig. 3 Schematic diagram of the absorption of light](image1)

![Fig. 4 Schematic of the optical fiber probe based on light absorption method](image2)

2.4. Concentration measurement system (Rhodamine 6G)

The instantaneous concentration of Rhodamine 6G is measured by LIF method with an optical fiber probe because maximum absorption wavelength of Rhodamine 6G (528 nm) is different from that of C. I. Direct Blue 86 (670 nm). Fluorescent dye is excited to a higher energy state by the laser light whose frequency closely matches its excitation frequency. An excited dye is then de-excited to a lower energy state and emits fluorescence. The intensity of fluorescence $S_f$ is proportional to the concentration of dye $C$.

$$S_f = KV_c I_l \Phi e_l C$$

Here, $K$ is a constant, $V_c$ is the measurement volume, $I_l$ is the laser light intensity, $\Phi$ is the quantum efficiency of the dye, and $e_l$ is the molecular extinction coefficient of the laser light. Equation (2) is applicable if the concentration of the dye is low enough. By obtaining the value of $KV_c I_l \Phi e_l$ as a calibration coefficient in the calibration test, the concentration of the fluorescent dye can be measured by measuring the intensity of fluorescence.
Figure 5 shows the schematic of tip of the optical fiber probe based on LIF method. For the high spatial resolution measurement, a condenser lens called “GRIN lens” is fused in the tip of the optical fiber and a beam spot diameter of 60 μm can be achieved for laser incidence. LIF method in this study, the laser incidence and the collecting of the fluorescence are conducted by the optical fiber. As a light source, a laser diode with the oscillating wavelength of 532 nm is used because the maximum absorption wavelength of Rhodamine 6G is 528 nm. Here, it should be noted that the peak wavelength of the fluorescence emitted from Rhodamine 6G is about 555 nm.

2.5. Temperature measurement system

The instantaneous temperature is measured by an I-type cold-film (DANTEC, 55R11) and temperature measuring module 90C20 of DANTEC Stream Line. The length and diameter of the cold-film are 1.25 mm and 70 μm, respectively.

2.6. Velocity-concentration simultaneous measurement system

Simultaneous measurement of instantaneous velocities and concentration of Rhodamine 6G is performed by combining the X-type hot-film and optical fiber probe closely each other. Figure 6 shows the schematic of the combined probe of X-type hot-film and optical fiber probe. As shown in Fig.6, the optical fiber probe is located in front of the X-type hot-film. The gap between two probes Δl is 1.5 mm. This spatial gap can be treated as the time lag Δt by Taylor’s frozen-flow hypothesis:

\[ \Delta t = \frac{\Delta l}{U} \]  

where, \( U \) is axial mean velocity at the measuring position. By introducing Δt in the simultaneous measurement, the gap between two probes can be neglected.

2.7. Velocity-temperature simultaneous measurement system

Simultaneous measurement of instantaneous velocities and temperature performed by combining the X-type hot-film and I-type cold-film closely each other. Figure 7 shows the schematic of the combined probe of X-type hot-film and I-type cold-film. As shown in Fig.7, the I-type cold-film is located in front of the X-type hot-film. The gap between two probes Δl is 1.5 mm. This spatial gap also can be neglected by introducing Taylor’s frozen-flow hypothesis shown in the previous section.
2.8. Experimental conditions

Table 1 shows the experimental conditions. All the experiments are performed in the same jet Reynolds number $Re_D = 20,000$ which is based on the nozzle diameter $d$ and jet exit velocity $U_0$. Measurements are performed at $x/d = 30, 50, 60,$ and $80$.

<table>
<thead>
<tr>
<th>Nozzle diameter</th>
<th>$d = 4 \times 10^{-3}$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic viscosity</td>
<td>$\nu = 1.008 \times 10^{-6}$ m$^2$/s (293 K)</td>
</tr>
<tr>
<td>Velocity at jet exit</td>
<td>$U_0 \approx 5.04$ m/s</td>
</tr>
<tr>
<td>Jet Reynolds number</td>
<td>$Re_D = 20,000$</td>
</tr>
<tr>
<td>Initial concentration of C.I.Direct Blue86</td>
<td>$C_0 = 3.0$ kg/m$^3$</td>
</tr>
<tr>
<td>Initial concentration of Rhodamine 6G</td>
<td>$C_0 = 2.0 \times 10^{-3}$ g/l</td>
</tr>
<tr>
<td>Initial temperature difference between jet and ambient fluid</td>
<td>$\Theta_0 = 7$ K</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>40 kHz</td>
</tr>
<tr>
<td>Sampling number</td>
<td>524, 288</td>
</tr>
<tr>
<td>Measuring position (on jet axis)</td>
<td>$x/d = 30, 50, 60, 80$</td>
</tr>
<tr>
<td>Measuring position (radial direction)</td>
<td>$x/d = 50, 80$</td>
</tr>
</tbody>
</table>

3. Results of velocity measurement

3.1. Axial mean velocity

Figure 8 shows the radial profiles of axial mean velocity $U$ at $x/d = 50$ and 80. The ordinate shows the axial mean velocity normalized by the value at the jet centerline, $U/U_c$, and the abscissa shows radial direction distance normalized by the half width of axial mean velocity, $r/b_U$. In Fig.8, the solid line and dashed line show the Gaussian curve and the results of Burattini et al. (2005, $Re_D = 13,000$), respectively. From Fig.8, it is found that the radial profiles of axial mean velocity at $x/d = 50$ and 80 show a similarity with each other. Further, a good agreement with the Gaussian curve and the results of Burattini et al. (2005) is also found.

![Fig. 8 Radial profile of mean axial velocity at $x/d = 50$ and 80](image)

3.2. RMS value of axial velocity fluctuation

Figure 9 shows the radial profiles of the RMS value of axial velocity fluctuation $u'$ at $x/d = 50$ and 80. The ordinate shows the RMS value of axial velocity fluctuation normalized by the value at the jet centerline, $u'/u'_c$, and the abscissa shows $r/b_U$. In Fig.9, the solid line and dashed line show the results of Chevray and Tutu (1978) and Lemoine et al. (1999), respectively. From Fig.9, it is found that the radial profiles of $u'/u'_c$ show a good agreement with the results of previous studies (Chevray and Tutu, 1978 · Lemoine, et al, 1999).

3.3. RMS value of radial velocity fluctuation

Figure 10 shows the radial profiles of the RMS value of radial velocity fluctuation $v'$ at $x/d = 50$ and 80. The ordinate shows the RMS value of radial velocity fluctuation normalized by the value at the jet centerline, $v'/v'_c$, and the abscissa shows $r/b_U$. The meaning of the lines in the figure are the same as those in Fig. 9. From Fig.10, it is also found that the radial profiles of $v'/v'_c$ show a good agreement with the results of previous studies. Therefore, the validity of the velocity measurement apparatus and base flow is confirmed from Fig. 8-10.
3.4. PDFs of axial velocity fluctuation

Figure 11 shows the radial profiles of the probability density functions (PDFs) of axial velocity fluctuation $p(u)$ at $x/d = 80$. The ordinate shows PDF multiplied by the RMS value of axial velocity fluctuation, $p(u' u')$, and the abscissa shows $u'/u'$. In Fig. 11, the solid line shows the Gaussian curve. It is found that the PDF at the jet centerline shows the symmetry with respect to $u'/u' = 0$ and follows the Gaussian curve, and the PDFs at $r/b_U = 0.49$, 0.98, and 2.20 skew positively.

3.5. PDFs of radial velocity fluctuation

Figure 12 shows the radial profiles of the PDFs of radial velocity fluctuation $p(v)$ at $x/d = 80$. The ordinate shows $p(v)/v'$, and the abscissa shows $v/v'$. In Fig. 12, the solid line shows the Gaussian curve. It is found that the PDF at the jet centerline shows the almost symmetric distribution with respect to $v/v'$ and follows the Gaussian curve, and the PDFs at $r/b_U = 0.49$, 0.98, and 2.20 skew positively and they have a sharp peak at $v/v' \approx 0$.

3.6. Power spectra of axial velocity fluctuation

Figure 13 shows the power spectra of axial velocity fluctuation $E_u(f)$ on the jet centerline at $x/d = 30$, 50, and 80. In the upper graph, the ordinate shows the power spectrum of axial velocity fluctuation normalized by the RMS value of axial velocity fluctuation and the integral time scale of axial velocity, $E_u(f)/(u'^3 t_{1a})$, and the abscissa shows the frequency normalized by the integral time scale of axial velocity, $f t_{1a}$. In the lower graph of Fig. 13, the ordinate shows the compensated power spectrum, $[E_u(f)/(u'^3 t_{1a})](f t_{1a})^{5/3}$. In this graph, the power spectrum which has a slope of $-5/3$ is shown as a flat distribution. From Fig. 13, it is found that the power spectra of axial velocity fluctuation at $x/d = 30$, 50, and 80 obey $-5/3$ power law in the frequency region $2 \times 10^{-1} \leq f t_{1a} \leq 1.5$.

3.7. Power spectra of radial velocity fluctuation

Figure 14 shows the power spectra of radial velocity fluctuation $E_v(f)$ on the jet centerline at $x/d = 30$, 50, and
80. In the upper graph, the ordinate shows $E_v(f)/(v'^2 f_{1d})$, and the abscissa shows $f_{1d}$. In the lower graph of Fig.14, the ordinate shows $[E_v(f)/(v'^2 f_{1d})](f_{1d})^{5/3}$. In this graph, the power spectrum which has a slope of -5/3 is shown as a flat distribution. From Fig.14, it is found that the power spectra of radial velocity fluctuation at $x/d = 30$ obeys -5/3 power law in the frequency region $3 \times 10^{-1} \leq f_{1d} \leq 8 \times 10^{-1}$. The power spectrum at $x/d = 80$ also obeys -5/3 power law in the wider frequency region $3 \times 10^{-1} \leq f_{1d} \leq 1.2$. Further, in comparison with the power spectra of axial velocity fluctuation shown in Fig.13, it is found that the power spectra of radial velocity fluctuation obey -5/3 power law in the narrower frequency region than those of axial velocity fluctuation.

![Fig. 13 Power spectra of axial velocity fluctuation at $x/d = 30, 50,$ and 80](image1)

![Fig. 14 Power spectra of radial velocity fluctuation at $x/d = 30, 50,$ and 80](image2)

4. Results of concentration measurement

4.1. Mean concentration

Figure 15 shows the radial profiles of mean concentration $C$ of C.I.Direct Blue 86 and Rhodamine 6G at $x/d = 50$ and 80. The ordinate shows the mean concentration normalized by the value at the jet centerline, $C/C_c$, and the abscissa shows radial direction distance normalized by the half width of mean concentration, $r/b_C$. In Fig.15, the solid line and dashed line show the Gaussian curve and the measurement results of temperature field by Mi et al. (2001, $Re_p = 16,000$, $Pr = 0.7$), respectively. From Fig.15, it is found that the radial profiles of mean concentration of C.I.Direct Blue 86 and Rhodamine 6G at $x/d = 50$ and 80 have a similarity with each other. Further, a good agreement with the Gaussian curve and the results of Mi et al. (2001) is also found.

4.2. RMS value of concentration fluctuation

Figure 16 shows the radial profiles of the RMS value of concentration fluctuation $c'$ of C.I.Direct Blue 86 and Rhodamine 6G at $x/d = 50$ and 80. The ordinate shows the RMS value of concentration fluctuation normalized by the value at jet centerline, $c'/c_c$, and the abscissa shows radial direction distance normalized by the half width of mean concentration, $r/b_C$. In Fig.16, the dashed line and dot-dashed line show the measurement results of fluctuating temperature by Chevray and Tutu (1978, $Re_p = 370,000$, $Pr = 0.7$) and Mi et al. (2001), respectively. From Fig.16, it is found that the radial profiles of the RMS value of concentration fluctuation of C.I.Direct Blue 86 and Rhodamine 6G at $x/d = 50$ and 80 have a peak at $r/b_C \approx 0.8$ and they show a good agreement with the results of other researchers. Therefore, it is shown that the radial profile of the RMS value of concentration fluctuation have no dependence on $Sc/Pr$.

4.3. PDFs of concentration fluctuation

Figures 17 and 18 show the radial profiles of the PDFs of concentration fluctuation $p(c)$ of C.I.Direct Blue 86 and Rhodamine 6G at $x/d = 80$. The ordinate shows PDF multiplied by the RMS value of concentration fluctuation, $p(c)c'$, and the abscissa shows normalized concentration fluctuation, $c/c'$. In Figs.17 and 18, the solid line and dashed line show
In the Gaussian curve and the results of Venkataramani et al. (1975, Re0 = 300,000, Pr = 0.7), respectively. From Fig.17, it is found that the PDF of concentration fluctuation of C. I. Direct Blue 86 at the jet centerline (r/bc = 0) skews negatively as same as the results of Venkataramani et al. (1975). However, the PDFs at r/bc = 0.51, 0.91, and 1.57 skew positively. In the case of Rhodamine 6G shown in Fig. 18, the same tendency can be seen as the case of C. I. Direct Blue 86.

concentration fluctuation of C.I.Direct Blue 86 at the jet centerline (r/bc = 0) skews negatively, and the PDFs at r/bc = 0.51, 0.91, and 1.57 skew positively. In the case of Rhodamine 6G shown in Fig.18, the same tendency can be seen as the case of C.I.Direct Blue 86.

4.4. Power spectra of concentration fluctuation

Figures 19 and 20 show the power spectra of concentration fluctuation $E_c(f)$ of C.I.Direct Blue 86 and Rhodamine 6G on the jet centerline at $x/d = 30$, 50, and 80. In the upper graph, the ordinate shows the power spectra of concentration fluctuation normalized by the RMS value of concentration fluctuation and the integral time scale of concentration, $E_c(f)/(c^2 t_I)$, and the abscissa shows the frequency normalized by the integral time scale of concentration, $f t_I$. In the lower graph of Figs.19 and 20, the ordinate shows the compensated power spectrum, $[E_c(f)/(c^2 t_I)]/(f t_I)^{5/3}$. In this graph, the power spectrum which has a slope of -5/3 is shown as a flat distribution. From Fig.19, it is found that the power spectra of concentration fluctuation of C.I.Direct Blue 86 obey -5/3 power law in the frequency region $2 \times 10^{-1} \leq f t_I \leq 5$. From Fig.20, it is found that power spectrum of concentration fluctuation of Rhodamine 6G at $x/d = 30$ obeys -5/3 power law in the frequency region $3 \times 10^{-1} \leq f t_I \leq 1.5$, and the power spectrum at $x/d = 80$ obeys -5/3 power law in the wider frequency region $3 \times 10^{-1} \leq f t_I \leq 2$.

4.5. Skewness and flatness of concentration spatial derivative

The scalar spatial derivative statistics is very important to understand the fine scale scalar field. For example, the skewness and flatness of scalar spatial derivative are indications of local isotropy and internal intermittency of the fine
scale scalar field, respectively. In this section, the local isotropy and intermittency of the fine scale concentration field of Rhodamine 6G and C. I. Direct Blue 86 are estimated by measuring the skewness and flatness of the concentration spatial derivative.

The spatial derivative of instantaneous concentration \( \partial c_i/\partial x \) is obtained as follows. Firstly, the time derivative of instantaneous concentration \( \partial c_i/\partial t \) is calculated by the following equation.

\[
\frac{\partial f_i}{\partial t} = \frac{-c_{i+2} + 8c_{i+1} - 8c_{i-1} + c_{i-2}}{12\Delta t}
\]

(4)

Here, \( \Delta t \) is the time interval in the concentration measurement (\( \Delta t = 1/40000 \) s). Secondly, the spatial derivative of instantaneous concentration \( \partial c_i/\partial x \) is obtained by Taylor’s frozen-flow hypothesis;

\[
\frac{\partial c_i}{\partial x} = \frac{1}{U} \frac{\partial f_i}{\partial t}
\]

(5)

where \( U \) is the axial mean velocity at the measuring position.

Figure 21 shows the streamwise profiles of the skewness of the concentration spatial derivative \( S_{\partial c_i/\partial x} \) of C.I.Direct Blue 86 and Rhodamine 6G on the jet centerline. The ordinate shows the absolute value of the skewness of concentration spatial derivative for the streamwise direction, \( |S_{\partial c_i/\partial x}| \), and the abscissa shows the streamwise position normalized by the nozzle diameter, \( x/d \). From Fig.21, it is found that the skewness of concentration spatial derivative decreases for downstream direction.

Figure 22 shows the streamwise profiles of the flatness of the concentration spatial derivative \( F_{\partial c_i/\partial x} \) of C.I.Direct Blue 86 and Rhodamine 6G on the jet centerline. The ordinate shows the flatness of concentration spatial derivative for the streamwise direction, \( |F_{\partial c_i/\partial x}| \), and the abscissa shows the streamwise position normalized by the nozzle diameter, \( x/d \). From Fig.22, it is found that the flatness of concentration spatial derivative increases for downstream direction.

5. Results of temperature measurement

5.1. Mean temperature

Figure 23 shows the radial profiles of mean temperature \( \Theta \) at \( x/d = 50 \) and 80. The ordinate shows the mean temperature normalized by the value at the jet centerline, \( \Theta/\Theta_c \), and the abscissa shows radial direction distance normalized by the half width of mean temperature, \( r/\theta_h \). In Fig.23, the solid line and dashed line show the Gaussian curve and the results of Mi et al. (2001), respectively. From Fig.23, it is found that the radial profiles of mean temperature at \( x/d = 50 \) and 80 show a similarity with each other. Further, a good agreement with the Gaussian curve and the results of Mi et al. (2001) is also found.
5.2. RMS value of temperature fluctuation

Figure 24 shows the radial profiles of the RMS value of temperature fluctuation $\theta'$ at $x/d = 50$ and 80. The ordinate shows the RMS value of temperature fluctuation normalized by the value at the jet centerline, $\theta'/\theta_c'$, and the abscissa shows radial direction distance normalized by the half width of mean temperature, $r/b_\theta$. In Fig.24, the dashed line and dot-dashed line show the measurement results of Chevray and Tutu (1978) and Mi et al. (2001), respectively. From Fig.24, it is found that the radial profiles of the RMS value of temperature fluctuation at $x/d = 50$ and 80 have a peak at $r/b_\theta \approx 0.8$ and they show a good agreement with the results of other researchers.

5.3. PDFs of temperature fluctuation

Figure 25 shows the radial profiles of the PDFs of temperature fluctuation $p(\theta)$ at $x/d = 80$. The ordinate shows the PDF multiplied by the RMS value of temperature fluctuation, $p(\theta)/\theta'$, and the abscissa shows normalized temperature fluctuation, $\theta_c/\theta'$. In Fig.25, the solid line and dashed line show the Gaussian curve and the results of Venkataramani et al. (1975), respectively. It is found that the PDF at the jet centerline ($r/b_\theta = 0$) skews negatively, and the PDFs at $r/b_\theta = 0.49$, $0.97$, and $1.46$ skew positively.

5.4. Power spectra of temperature fluctuation

Figure 26 shows the power spectra of temperature fluctuation $E_\theta(f)$ on the jet centerline at $x/d = 30$, 50, and 80. In the upper graph, the ordinate shows the power spectrum of temperature fluctuation normalized by the RMS value of temperature fluctuation and the integral time scale of temperature, $E_\theta(f)/(\theta'^2 t_\theta)$, and the abscissa shows the frequency normalized by the integral time scale of temperature, $f t_\theta$. In the lower graph of Fig.26, the ordinate shows the compensated power spectrum, $[E_\theta(f)/(\theta'^2 t_\theta)] (f t_\theta)^{5/3}$. In this graph, the power spectrum which has a slope of $-5/3$ is shown as a flat distribution. From Fig.26, it is found that the power spectrum of temperature fluctuation at $x/d = 30$ does not have a slope of $-5/3$ and the power spectrum at $x/d = 80$ obeys $-5/3$ power law in the frequency region $2 \times 10^{-1} \leq f t_\theta \leq 2 \times 10^{-3}$.
$f_{th} \leq 7 \times 10^{-1}$.

### 5.5. Skewness and flatness of temperature spatial derivative

Figure 27 shows the streamwise profiles of the skewness of the temperature spatial derivative $S_{\partial \theta / \partial x}$ on the jet centerline. The ordinate shows the absolute value of skewness of temperature spatial derivative for the streamwise direction, $|S_{\partial \theta / \partial x}|$, and the abscissa shows the streamwise position normalized by the nozzle diameter, $x/d$. For comparison, the results of concentration are also shown in Fig. 27. From Fig. 27, it is found that the skewness of temperature spatial derivative decreases for downstream direction. Further, Fig. 27 shows that the skewness of scalar spatial derivative becomes smaller as $Sc/Pr$ becomes higher.

Figure 28 shows the streamwise profiles of the flatness of the temperature spatial derivative $F_{\partial \theta / \partial x}$ on the jet centerline. The ordinate shows the flatness of temperature spatial derivative for the streamwise direction, $F_{\partial \theta / \partial x}$, and the abscissa shows the streamwise position normalized by the nozzle diameter, $x/d$. For comparison, the results of concentration are also shown in Fig. 28. From Fig. 28, it is found that the flatness of temperature spatial derivative increases for downstream direction. Further, Fig. 28 shows that the flatness of scalar spatial derivative becomes smaller as Schmidt number becomes higher.

![Fig. 25 Radial profile of the PDFs of temperature fluctuation at x/d = 80](image)

![Fig. 26 Power spectra of temperature fluctuation at x/d = 30, 50, and 80](image)

![Fig. 27 Streamwise profile of skewness of temperature spatial derivative](image)

![Fig. 28 Streamwise profile of flatness of temperature spatial derivative](image)
6. Results of velocity-concentration (Rhodamine 6G) simultaneous measurement

6.1. Mass flux

Figure 29 shows the radial profiles of the axial mass flux $\langle uc \rangle$ at $x/d = 50$ and 80. The ordinate shows the axial mass flux normalized by axial mean velocity and mean concentration at the jet centerline, $(uc)/U_cC_c$, and the abscissa shows radial direction distance normalized by the half width of axial mean velocity, $r/b_U$. In Fig.29, the solid line, dashed line, and dot-dashed line show the results of Chevray and Tutu (1978), Papanicolaou and List (1988), and Panchapakesan and Lumley (1993), respectively. From Fig.29, it is found that the radial profiles of the axial mass flux at $x/d = 50$ and 80 have a peak at $r/b_U \approx 0.9$ and they show the symmetry with respect to $r/b_U = 0$.

Figure 30 shows the radial profiles of the radial mass flux $\langle rc \rangle$ at $x/d = 50$ and 80. The ordinate shows the radial mass flux normalized by axial mean velocity and mean concentration at the jet centerline, $(rc)/U_cC_c$, and the abscissa shows radial direction distance normalized by the half width of axial mean velocity, $r/b_U$. In Fig.30, the solid line, dashed line, and dot-dashed line show the results of Chevray and Tutu (1978), Panchapakesan and Lumley (1993), and Law and Wang (1998), respectively. From Fig.30, it is found that the radial profiles of the radial mass flux at $x/d = 50$ and 80 have a peak at $r/b_U \approx 0.9$. Further, a good agreement with the results of other researchers is also found.

6.2. PDFs of mass flux

Figure 31 shows the radial profiles of the PDFs of axial mass flux $p(uc)$ at $x/d = 80$. The ordinate shows PDF of axial mass flux multiplied by the RMS value of $uc$, $p(uc)(uc')$, and the abscissa shows the normalized correlation of axial velocity fluctuation and concentration fluctuation, $uc/(uc')$. In Fig.31, the solid line shows the Gaussian curve. From Fig.31, it is found that the PDFs axial mass flux at $r/b_U = 0$, 0.97, and 1.47 have the approximately exponential tails and its slope of negative side is steeper than that of positive side.

Figure 32 shows the radial profiles of the PDFs of radial mass flux $p(rc)$ at $x/d = 80$. The ordinate shows PDF of radial mass flux multiplied by the RMS value of $rc$, $p(rc)(rc')$, and the abscissa shows the normalized correlation of radial velocity fluctuation and concentration fluctuation, $rc/(rc')$. In Fig.32, the solid line shows the Gaussian curve. From Fig.32, it is found that the PDF of radial mass flux at $r/b_U = 0$ has the approximately exponential tails and shows the symmetry with respect to $r/b_U = 0$. Further, from the results at $r/b_U = 0.97$ and 1.47, the slope of negative side is steeper than that of positive side.

6.3. Velocity-concentration cospectra

Figure 33 shows the axial velocity-concentration cospectrum $C_{uc}(f)$ on the jet centerline at $x/d = 30, 50$, and 80. The ordinate shows axial velocity-concentration cospectrum normalized by the axial mass flux and the integral time scale of axial velocity, $C_{uc}(f)/(uc\tau_{dA})$, and the abscissa shows the frequency normalized by the integral time scale of axial velocity, $f/\tau_d$. From Fig.33, it is found that the axial velocity-concentration cospectra at $x/d = 30, 50$, and 80 obey $-7/3$ power law in the frequency region $2 \times 10^{-1} \leq f/\tau_d \leq 1$.

Figure 34 shows the radial velocity-concentration cospectrum $C_{rc}(f)$ on the jet centerline at $x/d = 30, 50$, and 80. The ordinate shows radial velocity-concentration cospectrum normalized by the radial mass flux and the integral time scale of axial velocity, $C_{rc}(f)/(rc\tau_{dA})$, and the abscissa shows the frequency normalized by the integral time scale of axial velocity, $f/\tau_d$. From Fig.34, it is found that the radial velocity-concentration cospectra at $x/d = 30, 50$, and 80 obey $-3$ power law in the frequency region $2 \times 10^{-1} \leq f/\tau_d \leq 1$. 

velocity, $f_{tol}$. From Fig.34, it is found that the radial velocity-concentration cospectra at $x/d = 30$, 50, and 80 do not obey $-7/3$ power law and they have a slope of -2 in the frequency region $2 \times 10^{-1} \leq f_{tol} \leq 1$. It shows that in the scalar field of the scale corresponding to the frequency region $2 \times 10^{-1} \leq f_{tol} \leq 1$, the concentration field of Rhodamine 6G is anisotropic. These results are consistent with the measurement results for the C. I. Direct Blue 86 in the previous study (Sakai, et al., 2007).

7. Results of velocity-temperature simultaneous measurement

7.1. Heat flux

Figure 35 shows the radial profiles of the axial heat flux $\langle u \theta \rangle$ at $x/d = 50$ and 80. The ordinate shows the axial heat flux normalized by axial mean velocity and mean temperature at the jet centerline, $\langle u \theta \rangle / U_i \Theta_i$, and the abscissa shows radial direction distance normalized by the half width of axial mean velocity, $r/b_U$. In Fig.35, the solid line, dashed line, and dot-dashed line show the results of Chevray and Tutu (1978), Papanicolaou and List (1988), and Panchapakesan and Lumley (1993), respectively. From Fig.35, it is found that the radial profiles of the axial heat flux at $x/d = 50$ and 80 have a peak at $r/b_U \approx 0.9$ and they show the symmetry with respect to $r/b_U = 0$.

Figure 36 shows the radial profiles of the radial heat flux $\langle \theta \theta \rangle$ at $x/d = 50$ and 80. The ordinate shows the radial heat flux normalized by axial mean velocity and mean temperature at the jet centerline, $\langle \theta \theta \rangle / U_i \Theta_i$, and the abscissa shows radial direction distance normalized by the half width of axial mean velocity, $r/b_U$. In Fig.36, the solid line, dashed line, and dot-dashed line show the results of Chevray and Tutu (1978), Panchapakesan and Lumley (1993), and Law and Wang (1998), respectively. From Fig.36, it is found that the radial profiles of the radial heat flux at $x/d = 50$ and 80 have a peak at $r/b_U \approx 0.9$. Further, a good agreement with the results of other researchers is also found.
7.2. PDFs of heat flux

Figure 37 shows the radial profiles of the PDFs of axial heat flux $p(u\theta)$ at $x/d = 80$. The ordinate shows the PDF of axial heat flux multiplied by the RMS value of $u\theta$, $p(u\theta)(u\theta)'$, and the abscissa shows the normalized correlation of axial velocity fluctuation and temperature fluctuation, $u\theta/(u\theta)'$. In Fig.37, the solid line shows the Gaussian curve. From Fig.37, it is found that the PDFs of axial heat flux at $r/b_U = 0$, 0.97, and 1.47 have the approximately exponential tails and its slope of negative side is steeper than that of positive side.

Figure 38 shows the radial profiles of the PDFs of radial heat flux $p(\theta)$ at $x/d = 80$. The ordinate shows the PDF of radial heat flux multiplied by the RMS value of $\theta$, $p(\theta)(\theta)'$, and the abscissa shows the normalized correlation of radial velocity fluctuation and temperature fluctuation, $\theta/(\theta)'$. In Fig.38, the solid line shows the Gaussian curve. From Fig.38, it is found that the PDF of radial mass flux at $r/b_U = 0$ has the approximately exponential tails and shows the symmetry with respect to $r/b_U = 0$. Further, from the results at $r/b_U = 0.97$ and 1.47, the slope of negative side is steeper than that of positive side.

![Fig. 35 Radial profile of axial heat flux at x/d = 50 and 80](image1)

![Fig. 36 Radial profile of radial heat flux at x/d = 50 and 80](image2)

![Fig. 37 Radial profile of the PDFs of axial heat flux at x/d = 80](image3)

![Fig. 38 Radial profile of the PDFs of radial heat flux at x/d = 80](image4)

7.3. Velocity-temperature cospectra

Figure 39 shows the axial velocity-temperature cospectra $C_{uv}(f)$ on the jet centerline at $x/d = 30$, 50, and 80. The ordinate shows the cospectral strength normalized by the axial heat flux and the integral time scale of axial velocity, $C_{uv}(f)/(u\theta)t_{int}$, and the abscissa shows the frequency normalized by the integral time scale of axial velocity, $f_{int}$. From Fig.39, it is found that the axial velocity-temperature cospectra at $x/d = 30$, 50, and 80 obey $-7/3$ power law in the frequency region $2 \times 10^{-1} \leq f_{int} \leq 1$.

Figure 40 shows the radial velocity-temperature cospectra $C_{v\theta}(f)$ on the jet centerline at $x/d = 30$, 50, and 80. The ordinate shows the cospectral strength normalized by the radial heat flux and the integral time scale of radial velocity, $C_{v\theta}(f)/(v\theta)t_{int}$, and the abscissa shows the frequency normalized by the integral time scale of radial velocity, $f_{int}$. From Fig.40, it is found that the radial velocity-temperature cospectra at $x/d = 30$, 50, and 80 obey $-7/3$ power law in the frequency region $2 \times 10^{-1} \leq f_{int} \leq 1$.

Further, compared with the radial velocity-concentration cospectrum $C_{vc}$ shown in Fig.34, $-7/3$ power law is observed.
for temperature and it is not observed in the case of higher Sc scalar (Rhodamine 6G and C. I. Direct Blue 86, (Sakai, et al., 2007)).

8. Conclusions

In this study, Sc and Pr dependence of the scalar/temperature statistics and velocity-scalar/velocity-temperature joint statistics is investigated experimentally in a liquid axisymmetric jet.

(1) The Sc/Pr dependence is not observed in the RMS values of the fluctuating scalar, PDFs of the fluctuating scalar, mass/heat flux, and PDFs of mass/heat flux.

(2) The Sc/Pr dependence is observed in the power spectrum of the fluctuating scalar. In the power spectrum of the high Sc scalar (C. I. Direct Blue 86), -5/3 slope and -1 slope are clearly observed, while only -5/3 slope is observed for the lower Sc/Pr scalar (Rhodamine 6G and Temperature).

(3) The skewness of concentration spatial derivative for the streamwise direction is smaller than that of temperature spatial derivative and it shows that the skewness of the scalar spatial derivative becomes smaller as Schmidt number becomes higher. From this results, it is considered that the small scale scalar field is more locally isotropic as Schmidt number of the scalar becomes higher.

(4) The flatness of concentration spatial derivative is smaller than that of temperature spatial derivative and it shows that the flatness of the scalar spatial derivative becomes smaller as Schmidt number becomes higher. From this results, it is considered that the scalar field has the more internal intermittent structure as Schmidt number of the scalar becomes higher.

(5) Sc/Pr dependence of the concentration/temperature fluctuating field is observed in the slope of radial velocity-scalar cospectrum. The radial velocity-temperature cospectrum obey $-7/3$ power law in the normalized frequency region $2 \times 10^{-1} \leq f t u I \leq 1$, while $-7/3$ power law is not observed in the radial velocity-concentration cospectrum.

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References


