Lattice Boltzmann simulation of motion of red blood cell in constricted circular pipe flow

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Abstract
The lattice Boltzmann method for two-phase flows containing a deformable body with a viscoelastic membrane is improved to simulate circular pipe flows by incorporation of the immersed boundary method. In order to examine the validity of the red blood cell (RBC) model, the method is applied to the motion of a biconcave disk-shaped body in a pressure-driven circular pipe flow. The validation is demonstrated by investigating the relation between the deformation index and terminal axial velocity of the RBC in the pipe flow. In this study, the behavior of a biconcave disk-shaped body in constricted pipe flows is simulated under various geometrical conditions. The square and circular pipes with various lengths and sizes of the constriction are considered, and the flow is induced by the pressure difference between the inlet and outlet. From the results, it is found that as the length of the constriction becomes smaller, the body is deformed larger and accelerated at the entrance and exit in the constriction, although the speed of the body is reduced while passing through the constriction. Also, it is found that as the size of the constriction becomes larger, the deformation index linearly decreases and the axial velocity exponentially increases. These results indicate that the present method has applicability to simulation of the motion of RBCs in microscale capillary blood vessels.

Key words: Lattice Boltzmann method, Immersed boundary method, Two-phase flow, Red blood cell, Viscoelastic membrane, Circular pipe, Constriction

1. Introduction
Problems of the motion of viscoelastic bodies in fluid flows can be found in not only many engineering applications but also biomedical fields such as red blood cells (RBCs) in blood plasma. Normal RBCs have viscoelastic membrane and are easily deformed in small blood vessels where the cellular size is comparable to the vessel diameter. In the past, it was difficult to examine the phenomena that were involved, particularly in microscale vessels, by experimental and numerical approaches. Nevertheless, owing to the advances in measurement devices and numerical methods, many researches of the behavior of RBCs in fluid flows have been hitherto reported.

With regard to experimental studies on the behavior of RBCs in fluid flows, Hochmuth et al. (1970) investigated the motion of RBCs passing through glass capillaries of 4-10 μm diameter, and found that the longitudinal length of RBC becomes larger as the RBC velocity increases. Fischer et al. (1978) reported tank-tread like motion of human erythrocyte membrane in shear flow. Then, Tsukada et al. (2001) measured the erythrocyte deformability in vitro by making a crystal microchannel system with a high-speed video camera. In their experiments, the deformability of hardened erythrocytes (e.g., diabetic erythrocytes) is indeed measurably lower than that of usual erythrocytes in healthy controls. As for numerical simulations, Ramanujan and Pozrikidis (1998) studied the deformation of a liquid capsule enclosed by an elastic membrane in shear flows with the boundary element method. Dzwinel et al. (2003) proposed discrete particle models for simulation of RBCs in capillary vessels by the Lagrangian coordinates technique. Imai et al. (2010) have recently used the moving particle semi-implicit (MPS) method (Koshizuka and Oka, 1996), which is one of the particle methods, to
investigate the motion of the RBC flowing into a constricted square pipe.

Over the last two decades, the lattice Boltzmann method (LBM) (Rothman and Zaleski, 1997; Chen and Doolen, 1998; Wolf-Gladrow, 2000; Succi, 2001) has been developed into an alternative and promising numerical scheme for simulating viscous fluid flows and for multicomponent and multiphase fluid flows. Sui et al. (2008) simulated the motion of a body with elastic membrane in shear flows by the LBM with the immersed boundary method (IBM) (Peskin, 1977). Dupin et al. (2008) also simulated the motion of red blood cells in fluid flows. They discretized the three-dimensional capsule membrane into flat triangular elements, and the elastic forces acting at the triangle vertices are inserted into grid points in the LBM. The authors’ group (Yoshino and Murayama, 2009; Murayama, et al., 2011a; Murayama, et al., 2011b) has also proposed a numerical method based on the LBM to simulate behavior of a deformable body with a viscoelastic membrane in fluid flows. They confirmed the validity of the method in fundamental flow problems, and demonstrated the potential for microscale two-phase flows. However, because the method is basically formulated in Cartesian coordinates, it needs to be extended to deal with rounded boundaries such as circular pipes in order to simulate the motion of real RBCs in capillary blood vessels.

In the present paper, the above-mentioned method with the IBM at rounded boundaries is applied to the motion of a biconcave disk-shaped body in circular pipe flows. In particular, the characteristics of the body passing through a constriction are investigated under various geometrical conditions.

2. Modeling of body with viscoelastic membrane

2.1. Membrane discretization

Three-dimensional structural membrane models are discretized into flat triangular elements. The triangulation procedure is similar to those of Ramanujan and Pozrikidis (1998). To discretize the unstressed interface, each triangular face of a regular octahedron is subdivided into $4^n$ triangular elements, where $n$ is a positive integer. These elements are then projected radially onto a sphere. The geometry of each element is described by its three vertices. The discretization of a spherical surface is shown in Fig. 1 (left). The biconcave disk-shaped model in Fig. 1 (right) is constructed by means of the mapping system. The mapping function and parameters are the same as those used in Sui et al. (2008).

2.2. Elastic force

Because the viscous effect of the body is inherent in the original LBM for two-phase flows, only an elastic force based on the Kelvin–Voigt model is introduced into the LBM. As shown in Fig. 2, a body with a viscoelastic membrane of a finite but relatively thin thickness is considered, where the membrane is composed of particles that are connected with their neighboring particles by springs. The body has an internal viscous fluid covered by the viscoelastic membrane. The
internal fluid is referred to as Fluid B, which is distinguished from the surrounding viscous fluid called Fluid A. Since these fluids have their own character, it is required to distinguish them in some way to give different values of physical properties. In the present model, as in Murayama et al. (2011b), these fluids are distinguished by an order parameter in the LBM for two-phase flows described later. However, the membrane is not autonomously determined but the region of the cubic lattices containing particles is assumed to be a membrane, so that the elastic force is applied to all vertices of the cubic lattices.

As shown in Fig. 3, the elastic membrane models are composed of small triangular elements connected by springs where Lagrangian particles are positioned at the vertices. A body with a viscoelastic membrane which encloses a viscous fluid has mechanical property of elastic resistances to the stretching, dilation and bending. For example, a normal RBC has strong resistance to the dilation of the body as well as to the stretching of the membrane. This is attributed to the fact that such bodies have an inherent characteristics of conserving surface area of the membrane and total volume of the body. Moreover, as reported in Evans (1980), the effect of the elastic resistance to the bending is negligibly smaller than that of the others for a thin viscoelastic membrane. Therefore, only the resistances to the stretching and dilation are introduced in the present model.

The elastic force acting on the particles is determined according to the principle of virtual work, which means that the behavior of the body is determined by moving the particles so that the total elastic energy $E$ can lead to a minimum value. By reference to the idea of spring model by Wada and Kobayashi (2003) and Tsubota et al. (2006), the total energy $E$ is the sum of the following quantities: the energy for the elastic resistance to the stretching of the membrane, $E^l$, the energy for the elastic resistance to the dilation of the body, $E^A + E^V$, where $E^A$ is concerned with the surface area and $E^V$ with the total volume of the body. These energies are expressed by

$$E = E^l + E^A + E^V,$$

where

$$E^l = \frac{1}{2} \sum_{m=1}^{N_l} K_m \left( \frac{l_{m} - l_{m0}}{l_{m0}} \right)^2,$$

$$E^A = \frac{1}{2} K^A \sum_{e=1}^{N_e} \left( \frac{A_e - A_{0e}}{A_{0e}} \right)^2 + \frac{1}{2} K^A \left( \frac{A - A_0}{A_0} \right)^2,$$

$$E^V = \frac{1}{2} K^V \left( \frac{V - V_0}{V_0} \right)^2.$$

In the above equations, $l_{m} = |X_j - X_k|$ is the stretch (or compression) displacement between the neighboring two particles $P_j$ and $P_k$ at Lagrangian points $X_j$ and $X_k$, $A_e$ is the area of the element $e$, $A$ is the total surface area of the membrane, $V$ is the total volume of the body, $N_p$, $N_e$ and $N_c$ are the numbers of particles, springs and elements, respectively, and $K^{l_m}$, $K^{A}$, $K^{A}$ and $K^{V}$ are elastic moduli for stretching of the membrane, dilation of the local surface area, dilation of the total surface area and the dilation of the total volume, respectively. Note that the subscript 0 indicates the initial value of each property.

Consequently, the elastic force $F_j$ acting on the particle $P_j$ at the Lagrangian point $X_j$ is given by

$$F_j = -\frac{\partial E}{\partial X_j}$$

3. Numerical method

As described in Inamuro (2006) and Suzuki and Inamuro (2011), we use non-dimensional variables defined by a characteristic length $L$, a characteristic particle speed $\tilde{c}$, a characteristic time scale $\tilde{t}_0 = L/\tilde{U}$, where $\tilde{U}$ is a characteristic flow speed, a reference order parameter $\tilde{\phi}_0$ and a reference density $\tilde{\rho}_0$.

3.1. Two-phase lattice Boltzmann method

In the LBM, a modeled fluid, which is composed of identical particles whose velocities are restricted to a finite set of vectors, is considered. The three-dimensional 15-velocity model, as shown in Fig. 4, is used in the present study. This
model has the velocity vectors:
\[
\begin{bmatrix}
  c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}
\end{bmatrix}
\]
\[
= \begin{bmatrix}
  0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
  0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
  0 & 0 & 0 & 0 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 
\end{bmatrix}
\]  

We assume two-phase fluid flows with negligibly small density difference by considering the case of human RBCs in plasma. Hence, the numerical algorithm is based on the LBM for incompressible two-phase fluids with the same density (Inamuro, et al., 2003).

In the calculations, the physical domain is divided into a cubic lattice, and the evolution of particle population at each lattice point is computed. The evolution of the particle distribution functions \( f_i(x, t) \) and \( g_i(x, t) \) with velocity \( c_i \) at point \( x \) and at time \( t \) is computed by
\[
\begin{align*}
  f_i(x + c_i \Delta x, t + \Delta t) & = f_i^c(x, t), \\
  g_i(x + c_i \Delta x, t + \Delta t) & = g_i^c(x, t),
\end{align*}
\]
where \( f_i^c \) and \( g_i^c \) are functions of macroscopic variables and their derivatives given below, and \( \Delta t \) is a time-step during which the particles travel the lattice spacing \( \Delta x \). Note that \( \Delta t = St \Delta x \), where \( St = \tilde{U}/(\tilde{u} \tilde{c}) = \tilde{U} / \tilde{c} = O(\Delta x) \) is the Strouhal number.

The order parameter \( \phi \) distinguishing the two phases, the pressure \( p \) and the velocity \( u \) of the two-phase fluids are defined as follows:
\[
\begin{align*}
  \phi &= \sum_{i=1}^{15} f_i, \\
  p &= \frac{1}{3} \sum_{i=1}^{15} g_i, \\
  u &= \sum_{i=1}^{15} g_i c_i.
\end{align*}
\]

The functions \( f_i^c \) and \( g_i^c \) in Eqs. (5) and (6) are given by
\[
\begin{align*}
  f_i^c &= H_i \phi + F_i \left( p_0(\phi) - \kappa \phi \nabla^2 \phi - \frac{\kappa^2}{6} |\nabla \phi|^2 \right) + 3E_i \phi c_ia u_a + E_i \kappa G_{ai}(\phi)c_ia \delta_{ij}, \\
  g_i^c &= E_i \left[ 3p + 3c_ia u_a - \frac{3}{2} c_ia u_a + \frac{9}{2} c_ia c_\beta u_\beta u_\beta + \frac{3}{4} \Delta x \left( \frac{\partial u_\beta}{\partial x_\alpha} + \frac{\partial u_\alpha}{\partial x_\beta} \right) c_\alpha c_\beta \right] \\
  &+ 3E_i c_ia \frac{\partial}{\partial x_\beta} \left( \frac{\partial u_\beta}{\partial x_\alpha} + \frac{\partial u_\alpha}{\partial x_\beta} \right) \Delta x + 3 \Delta x E_i c_ia \delta_{ij} M F_a,
\end{align*}
\]
with
\[
\begin{align*}
  E_1 &= \frac{2}{9}, \quad E_2 = E_3 = \ldots = E_7 = \frac{1}{9}, \quad E_8 = E_9 = \ldots = E_{15} = \frac{1}{72}, \\
  H_1 &= 1, \quad H_2 = H_3 = \ldots = H_{15} = 0, \quad F_i = \frac{7}{3}, \quad F_i = 3E_i (i = 2, 3, \ldots, 15)
\end{align*}
\]
and
\[ p_0(\phi) = \phi T_s \frac{1}{1 - \phi} - a\phi^2, \quad (13) \]
\[ G_{0\beta}(\phi) = \frac{9}{2} \frac{\partial \phi}{\partial x_\alpha} \frac{\partial \phi}{\partial x_\beta} + \frac{3}{2} \frac{\partial \phi}{\partial x_\alpha} \frac{\partial \phi}{\partial x_\gamma} \delta_{\alpha\beta}, \quad (14) \]
where \( a, b, \gamma = x, y, z \) (subscripts \( a, b \) and \( \gamma \) indicate Cartesian coordinates and the summation convention is used), and \( \Pi = FA, FB, M \) (subscripts \( FA, FB \) and \( M \) represent fluid phase \( A \), fluid phase \( B \) and membrane, respectively). In the above equations, \( \delta_{\alpha\beta} \) is the Kronecker delta, \( \kappa_f \) is a constant parameter determining the width of the interface, \( \mu \) is the viscosity, \( a, b \) and \( T_s \) are free parameters determining the maximum and minimum values of \( \phi \), and \( F_s \) is the elastic force at the grid point described below. Note that the interfacial tension is assumed to be free in the present study; the constant parameter \( \kappa \) determining the strength of the interfacial tension, which appears in Inamuro et al. (2003), is set to zero in the present calculations.

For the calculation of the gradient and the Laplacian of a variable \( \psi \) in Eqs. (10), (11) and (14), the following second-order finite-difference approximations given by the Taylor series expansion are used:
\[ \frac{\partial \psi}{\partial x_\alpha} \approx \frac{1}{10\Delta x} \sum_{i=2}^{15} c_{\alpha i} \psi(x + c_i \Delta x), \quad (15) \]
\[ \nabla^2 \psi \approx \frac{1}{5(\Delta x)^2} \left[ \sum_{i=2}^{15} \psi(x + c_i \Delta x) - 14\psi(x) \right]. \quad (16) \]

As mentioned in Inamuro (2006), applying the asymptotic theory (Sone, 1971) to Eqs. (5) and (6) with appropriate initial and boundary conditions, we can obtain the phase-field advection–diffusion equation (the Cahn–Hilliard equation plus advection) for \( \phi \), the continuity equation and the Navier–Stokes equations for incompressible two-phase fluids.

### 3.2. Immersed boundary method

As described in Suzuki and Inamuro (2011), in the IBM, it is assumed that an incompressible viscous fluid is filled in both inside and outside of the boundary. Then, the body forces are applied on lattice points near the boundary in order to enforce the no-slip condition on the boundary. In the present computations, the IBM with the multi-direct forcing method (MDFM) proposed by Wang et al. (2008) is applied to the stationary curved boundary of circular pipes.

Let \( X_k(t + \Delta t) \) \( (k = 1, 2, \ldots, N) \) be the Lagrangian points of the boundary. Note that the boundary velocities at the points are set to \( U_k(t + \Delta t) = 0 \) for the stationary boundary. Also, note that the boundary is represented by \( N \) points, and the boundary Lagrangian points \( X_k \) generally differ from the Eulerian grid points \( x \). Then, the temporal velocities \( \mathbf{u}^* (X_k, t + \Delta t) \), which are calculated by using the distribution functions evolved without a body force, are interpolated by
\[ \mathbf{u}^* (X_k, t + \Delta t) = \sum_{X} \mathbf{u}^* (x, t + \Delta t) W(x - X_k)(\Delta x)^3, \quad (17) \]
where the summation symbol represents the sum over all Eulerian grid points \( x \), and \( W \) is the weighting function proposed by Peskin (2002), which is given by
\[ W(x, y, z) = \frac{1}{\Delta x} \frac{w(x)}{\Delta x} \frac{1}{\Delta x} \frac{w(y)}{\Delta x} \frac{1}{\Delta x} \frac{w(z)}{\Delta x}, \quad (18) \]
where
\[ w(r) = \left\{ \begin{array}{ll}
\frac{1}{8} (3 - 2 |r| + \sqrt{1 + 4 |r| - 4 r^2}), & |r| \leq 1 \\
\frac{1}{8} (5 - 2 |r| - \sqrt{7 + 12 |r| - 4 r^2}), & 1 \leq |r| \leq 2 \\
0, & \text{otherwise.}
\end{array} \right. \quad (19) \]
The body force \( \mathbf{G}(x, t + \Delta t) \) is determined by the following iterative procedure (Suzuki and Inamuro, 2011).

Firstly, before iteration, we compute the initial value of the body force at the boundary Lagrangian points by
\[ G_0(X_k, t + \Delta t) = -\frac{S_l}{\Delta t} \mathbf{u}^* (X_k, t + \Delta t), \quad (20) \]
where \( S_l/\Delta t = 1/\Delta x \). Secondly, we compute the body force at the Eulerian grid points of \( t \)th iteration by
\[ G_t (x, t + \Delta t) = \sum_{k=1}^{N} G_t (X_k, t + \Delta t) W(x - X_k)(\Delta x)^3. \quad (21) \]
Thirdly, we correct the velocity at the Eulerian grid point by

\[
\mathbf{u}_t(x, t + \Delta t) = \mathbf{u}^* (x, t + \Delta t) + \frac{\Delta t}{St} G_t(x, t + \Delta t).
\]

Fourthly, we interpolate the velocity at the boundary Lagrangian point with

\[
\mathbf{u}_t(X_k, t + \Delta t) = \sum_X \mathbf{u}_t(x, t + \Delta t) \mathcal{W}(x - X_k) (\Delta x)^3.
\]

Lastly, we correct the body force with

\[
G_{t+1}(X_k, t + \Delta t) = G_t(X_k, t + \Delta t) - \frac{St}{\Delta t} \mathbf{u}_t(X_k, t + \Delta t),
\]

and repeat the steps using Eqs. (21)–(24) several times. From the preliminary computations, \(G_{t=6}(x, t + \Delta t)\) is enough to keep no-slip condition on the boundary points. Therefore, we iterate the above procedure until \(\ell = 6\) in the following computations.

### 3.3. Motion of particle and coupling with LBM

The first-order Euler method is used for the calculations of the motion of particles. During the time-step \(\Delta t\), a particle \(P_j (j = 1, 2, \ldots, N_p)\) at Lagrangian points \(X_j(t)\) moves to the following point:

\[
X_j(t + \Delta t) = X_j(t) + \frac{\Delta t}{St} \mathbf{U}_j(t),
\]

where \(\mathbf{U}_j(t)\) is the velocity of the particle. The Lagrangian point \(X_j(t)\) generally differs from the Eulerian grid points. Then, the velocity \(\mathbf{U}_j(t)\) is obtained by the weighted linear interpolation of the fluid velocities \(\mathbf{u}(t)\) at the surrounding eight points in the cubic lattice.

At the same time, the displacement of the springs, surface area and total volume are changed, and the particle is subject to the elastic force \(\mathbf{F}_j(t)\), as given by Eq. (3). In addition, because the Lagrangian point does not coincide with Eulerian grid points, \(\mathbf{F}_j(t)\) is redistributed to the surrounding Eulerian grid points by the linear interpolation. The obtained elastic force \(\mathbf{F}(t)\) at the Eulerian grid point \(x\) is substituted into the last term in the right-hand side of Eq. (11).

### 4. Results and discussion

#### 4.1. Validation of RBC model

In order to confirm the validity of the RBC model, we calculate the motion of the body in a circular pipe flow. As shown in Fig. 5, the length and diameter of the pipe are \(L_x = 100\Delta x\) and \(D = 37.2\Delta x\), respectively, and a biconcave disk-shaped body whose initial longitudinal axis is \(D_0 = 32\Delta x\) is placed on the centerline of the pipe. The membrane of the body consists of \(N_m = 258\) particles. The periodic boundary condition with pressure difference \(\Delta p\) is used at the inlet and outlet, and the no-slip boundary condition is used at walls of the pipe. The viscosities of the fluid phase A, the fluid phase B and the membrane are \(\mu_A = 3.93 \times 10^{-2}\Delta x\), \(\mu_B = \mu_m = 9.42 \times 10^{-3}\Delta x\). The elastic moduli are determined so that \(K^A = K^B = K^V = K^m\), where \(K^m\) is the arithmetic mean of \(K^m\) and set to \(2.28 \times 10^{-5}(\Delta x)^3\). Note that the fluid properties such as the viscosity correspond to a specific case of human blood flow (Shiga et al., 1990; Zhang et al., 2009). The number of Lagrangian points at the boundary of the pipe is \(N = 100\). The other parameters are the same as those in Murayama et al. (2011b). We perform the simulations for \(0.04 \leq Re \leq 0.13\), where \(Re\) is the Reynolds number defined as \(Re = \rho u_0 D_0 / \mu_B\), with \(u_0\) being the terminal axial velocity of the body. Here, the velocity of the body means the velocity at the centroid of the membrane in the body.

![Fig. 5 Computational domain of pipe flow problem.](image-url)
Figure 6 shows time evolution of the cross-sectional shape of the body at $Re = 0.10$. It is seen that the body becomes deformed into axisymmetric parachute-like shape, and goes downstream around the centerline of the pipe. This shape is qualitatively similar to that observed in the experiment by Tsukada et al. (2001). Using the variables defined in Fig. 7, the deformation index $\Gamma$ can be calculated as $\Gamma = 2D_x/(D_y + D_z)$, where $D_x$, $D_y$, and $D_z$ are the lengths of the body in the $x$-, $y$- and $z$-directions, respectively. Figure 8 shows the relation between the deformation index and the terminal axial velocity of the body at steady state. It should be noted that $U_t$ is transformed into the dimensional velocity $\tilde{U}_t$ to compare with available experimental data and calculated values. It is seen that the present results are in good agreement with experimental data by Tsukada et al. (2001) and simulation results by Reasor Jr et al. (2012). Therefore, it is verified that the present RBC model can be applied to simulation of the motion of RBCs in microscale capillary blood vessels.

4.2. RBC motion in constricted pipe flow

As shown in Fig. 9, we consider a circular pipe and a square pipe, each of which has a constriction at the center of the domain. The entrance of the constriction is positioned at $x = L_p$. For the circular pipe, the diameters of the inlet and outlet are $D_{\text{max}}$, the total length is $L_s$, the length and the minimum diameter of the constriction are $2L_s$ and $D_{\text{min}}$, respectively,
and the diameter $D(x)$ of the constriction is determined by the following equation (Kozuma and Inamuro, 2011):

$$D(x) = \begin{cases} \frac{D_{\text{max}}}{2} \left[ 1 + \cos \left( \frac{\pi}{L_{y}} (x - L_p) \right) \right] + \frac{D_{\text{min}}}{2} \left[ 1 - \cos \left( \frac{\pi}{L_{y}} (x - L_p) \right) \right], & L_p \leq x \leq L_p + 2L_s \\ \text{otherwise}. \end{cases}$$

(26)

For the square pipe, the cross-sectional square of the inlet and outlet is $L_y \times L_y$ ($L_z = L_y$), the total length is $L_z = 2.39L_y$, and the length and the minimum side of the constriction are $2L_s$ and $L_d$, respectively. Also, the constriction side is determined using the same function as mentioned above by substituting $L_y$ and $L_d$ for $D_{\text{max}}$ and $D_{\text{min}}$, respectively.

The periodic boundary condition with pressure difference $\Delta p$ is used at the inlet and outlet. The no-slip boundary condition is used at walls of the pipes. In the calculations, a biconcave disk-shaped body with initial longitudinal axis $D_0$ is placed on the centerline at $x = 0.1L_z$, and the motion of the body in pressure-driven flow is simulated. In particular, the effects of the pipe shape, of the constriction length and of the constriction size (the diameter of the cross-sectional circle) on the behavior of the body.

Computational parameters are as follows: $\rho_{\text{PA}} = 1$, $\rho_{\text{FB}} = 1$, $\mu_{\text{PA}} = 3.93 \times 10^{-2} \Delta x$, $\mu_{\text{FB}} = \mu_{M} = 9.42 \times 10^{-3} \Delta x$, $K^a = K^b = K^c = 2.28 \times 10^{-5} (\Delta x)^3$, $L_x = 105 \Delta x$, $L_y = 49.6 \Delta x$, $L_p = 40 \Delta x$, $L_s = 10 \Delta x$, $L_d = 19.2 \Delta x$, $D_{\text{max}} = 49.6 \Delta x$, $D_{\text{min}} = 19.2 \Delta x$, $D_0 = 22 \Delta x$, $N_p = 258$, $N = 100$, and $\Delta p = 5.5 \times 10^{-5}$. The other parameters are the same as those in Murayama et al. (2011b). Note that the hydraulic equivalent diameters are used for $L_y$ and $L_d$.

Grid convergence is checked for both Eulerian and Lagrangian grids. For Eulerian grid, the lattice resolutions and the number of particles in the membrane are determined by reference to the grid convergence test carried out in the previous paper (Murayama, et al., 2011b). As an example, when we carried out one case of the simulations for the constricted circular pipe flow with $1.25$ times finer grid, we found that the calculated results of the deformation index and the velocity were almost the same as the results with the standard grid. For Lagrangian grid in the IBM, on the other hand, preliminary computations were performed for several values of $N$ (the number of Lagrangian points) in fundamental problems. The value of $N$ used in the present simulations is determined from these results.

### 4.2.1. Effect of circular and square pipes

We first compare the results in the circular and square pipe flows at $Re = 6.2 \times 10^{-3}$. Time evolutions of the body shape in the circular and square pipes with constriction are shown in Fig. 10. In both cases, it is seen that the body is elongated at the constriction and after passing through the constriction, the body becomes deformed into concave shape on the upstream and convex shape on the downstream like a parachute. Figure 11 shows time variations of the calculated deformation index $\Gamma$ and the axial velocity $u$ of the body. It should be noted that the velocity of the body represents the velocity at the centroid of the membrane in the body, and this definition is applied in the following figures. The dimensionless time is given by $t^* = mU_{\text{max}} \Delta x / D_0$, where $m$ is the number of time-step, and $U_{\text{max}}$ is the maximum velocity in Poiseuille flow without a body. In both cases, the body becomes elongated in
the $x$-direction when it passes through the constriction, and there are small and large peaks in the axial velocity profiles. Nevertheless, by comparison with these two results, it is found that the body in the circular pipe flow undergoes a little larger deformation and passes more slowly than in the case of square pipe flow. Also, time variation of the axial velocity in the circular pipe flow is larger than that in the square pipe flow. Therefore, from these results, it is considered that the body in the circular pipe flow is more affected by the constriction and that it takes more time to return to its equilibrium shape after the deformation than in the case of the square pipe flow.

4.2.2. Effect of length of constriction We next investigate the effect of the constricted length in the circular pipe on the behavior of the body. In the calculations, the constricted length is changed in the range of $0.455 \leq L_s/D_0 \leq 1.36$. The other parameters are the same as those in the previous section except for $D_{\text{min}} = 22\Delta x$ in this case.

Figure 12 shows time variations of the deformation index and the axial velocity for various constricted lengths. In all cases, the body is deformed at the constriction by flow resistance and changed to an equilibrium shape. Also, the body is accelerated at the entrance and exit in the constriction, but reduces its speed while going through the constriction. In comparison with these cases, as the length $L_s$ becomes smaller, the body is deformed larger and the axial velocity

Fig. 10 Time evolutions of body shape: circular pipe (upper); square pipe (lower).

Fig. 11 Time variations of deformation index (upper) and axial velocity (lower) of the body in circular and square pipe flows ($t^* = mU_{\text{max}}\Delta x/D_0$).
becomes smaller owing to abrupt decrease in the channel width. Moreover, reduction in the velocity indicates the loss of the kinetic energy of the body, and the drag exerted on the body is considered to be significant especially at steep constrictions. Calculation of the drag by summation of the force exerted on the triangular elements in the membrane (Omori, et al., 2012) will be reported in future work.
4.2.3. Effect of size of constriction

We finally investigate the effect of the constricted size in the circular pipe on the behavior of the body. In the calculations, the minimum diameter of the constriction is changed in the range of $0.625 \leq D_{\min}/D_0 \leq 1.13$. The other parameters are the same as those in the previous section.

Time variations of the deformation index and the axial velocity for various minimum diameters of the constriction are shown in Fig. 13. Also, the deformation index and the maximum axial velocity $u_{\max}$ against the minimum diameter are shown in Fig. 14. It is found from these figures that as the minimum diameter becomes larger, the deformation index almost linearly reduces and the maximum axial velocity exponentially increases. On the other hand, as the minimum diameter becomes smaller, it is expected that the body is hard to pass through the pipe, and finally it may plug at the constriction, i.e., blocking the flow, as seen in the case of a droplet (Inamuro, 2012). Investigation into the critical conditions for the onset of the plugging is also an interesting issue.

5. Concluding remarks

The LBM for two-phase flows containing a deformable body with a viscoelastic membrane has been applied to the motion of a biconcave disk-shaped body in pressure-driven pipe flows. In particular, the characteristics of the body passing through a constriction have been investigated under various geometrical conditions. From these results, it is found that the present method has applicability to simulation of the motion of RBCs in microscale capillary blood vessels.

Finally, although the behavior of a single body has been investigated, the authors are now studying the motion of multi-bodies in pipe flows by dealing with the interaction between the bodies.

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References


Tsukada, K., Sekizuka, E., Oshio, C. and Minamitani, H., Direct measurement of erythrocyte deformability in diabetes mellitus with a transparent microchannel capillary model and high-speed video camera system, Microvascular Re-