The effect of upstream disturbance on the angle of sound emission in a supersonic round jet

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Abstract
The three-dimensional time-dependent compressible Navier-Stokes equations are numerically solved to study acoustic emission mechanisms in a supersonic round jet at high convective Mach numbers. A 5th-order compact upwind algorithm developed by Deng et al. (1996) is used for spatial derivatives and a 4th-order Runge-Kutta scheme for time advancement. The Navier-Stokes characteristic boundary conditions are used in the streamwise and radial directions and periodic boundary conditions in the azimuthal direction. Numerical results for the convective Mach number $M_c = 0.97$ are presented ($M_c$ is defined by Eq.(14) in Section2). Two different cases were investigated. The first case is the jet forced by the linear unstable modes. The second case is the jet flow forced randomly. The numerical results provide new physical features of the Mach wave generated in supersonic round jets, which lead to extinction of the Mach wave by introducing disturbance condition. Upstream disturbance conditions play an important role for the emission of the Mach wave in supersonic jets. The pressure fluctuations generated by the growth of the opposite helical mode are shown to be linearly superimposed into the jet near sound field. The numerical results indicate that the jet forced with a pair of first helical modes can indicate the elimination of Mach waves at restricted emission azimuthal angles due to the interference of these modes. The partial elimination of the Mach wave also appears in a turbulent jet at the frequency of the artificially forced an optimal combination of the helical modes at the inlet region. This forcing technique will be extended to the Mach wave reduction in the distinct azimuthal direction.

Key words : Supersonic jet, Flow control, Turbulent flow, Aerodynamic noise, DNS, Stability

1. Introduction

Compressible jets, which can be found in many applications such as rocket, scramjet, ramjet and turbojet engines, have been of fundamental importance in the study of compressible free shear flows. With new noise regulations, reducing of acoustic noise is the one of key technological challenges facing proposed supersonic commercial aircraft. The numerical investigation of supersonic jets is expected to guide such technological progress on aircraft where the jet exhaust velocity is supersonic. On the other hand, methods have long been sought to find an efficient means for reduction of jet noise using either active or passive turbulence control measures (see Seiner, 1992). Progress in this area has been limited by unclear understanding of the physical supersonic jet noise source mechanism. These mechanisms have been extensively studied using round jets. Non-round geometry of the jet exit has been experimentally studied for beneficial noise reduction relative to the round jets. Mach wave emission and shock noise are the dominant acoustic sources of supersonic jet noise. Improperly expanded nozzles produce shock noise that dominates acoustic emission in the jet forward quadrant. Shock noise, however, could be minimized through appropriate design of nozzle geometry. It has been observed experimentally that the acoustic radiation from jets is dominated by Mach waves (see review by Tam, 1995). Turbulent structures traveling at supersonic speed within the jet are generally thought to be responsible for Mach waves (see Papamoschou, 1997 and Kearney-Fischer et al., 2011), and they have been modelled as a combination
of linearly unstable modes. Reduction of Mach wave emission represents the most serious challenge to the successful
design of a suppressor nozzle. The understanding of jet turbulence and noise emission is very crucial for jet
investigations. DNS of supersonic round jet was performed and brought much light to these mechanisms. Freund et al.
(2000) simulated a perfectly expanded Mach 1.92 jet at ReD=2000 (based on jet diameter) and its sound field. Watanabe
et al. (2003, 2006) performed linear stability analysis and DNS of a supersonic plane jet. Their results show that the
plane jet (Mc =1.17) forced by a pair of oblique modes (the oblique mode is most unstable at high convective Mach
numbers) with the subsonic phase speed suppresses the emitted Mach wave intensity. On the other hand, in high
convective Mach number, the most unstable (first helical mode) modes have the supersonic phase speed in a round jet
(Luo et al., 1997 and Parras et al., 2010). The supersonic phase speed of the most unstable mode in a round jet makes
the suppression of Mach waves more difficult.

The linear inviscid instability wave model and Lighthill acoustic analogy approach have provided invaluable
guidance to the development of the jet noise prediction (see Zorumski, 1982). It is generally accepted that the
mechanism for Mach wave radiation (Tam et al., 1984) (as well as growth of compressible mixing layers, Morris et al.,
1990) is strongly connected with the amplification rate of instability modes and that the instability wave scales with the
convective Mach number. The real physical process, however, involves the nonlinear interaction of various instability
modes in the flow field. DNS researches of the nonlinear interaction for the supersonic jets in the transition regime are
important for understanding of these mechanisms.

We investigate, by means of DNS, the spatially development of high Mach number round jet forced with the
unstable/random disturbances. Both the fluid dynamic structures and noise of round jets are studied. In the present
study, we focus on the effect of inflow disturbances on the Mach wave emission from the round jet using spatial 3-D
DNS. The round jet is of interest, as described above, due to practical importance of supersonic combustion and jet
noise generation. The sound fields related to the energy nonlinear evolution of various linear unstable modes are
examined at high convective Mach numbers. In subsection 3.1, the sound fields emanated by single linear unstable
mode are presented. In subsections 3.2 and 3.3, the effect of a pair of first helical modes on the jet sound fields and the

2. NUMERICAL METHODS

In the direct numerical simulation, the non-dimensional equations governing the conservation laws of mass,
momentum, and energy for a compressible Newtonian fluid are solved using a fifth-order compact upwind algorithm
for conservation form (Deng et al., 1996) with a time integration 4th-order Runge-Kutta algorithm. The governing
equations in cylindrical coordinates are given as follows;

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u_r)}{\partial r} + \frac{1}{r} \frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0, \tag{1}
\]

\[
\frac{\partial (\rho u_r)}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u_{r} u_r)}{\partial r} + \frac{1}{r} \frac{\partial (\rho u_{r} u_\theta)}{\partial \theta} + \frac{\partial (\rho u_{r} u_z)}{\partial z} = - \frac{\partial p}{\partial r} + \frac{1}{r} \left( \frac{\partial (r \tau_{rr})}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} \right), \tag{2}
\]

\[
\frac{\partial (\rho u_\theta)}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u_{r} u_\theta)}{\partial r} + \frac{1}{r} \frac{\partial (\rho u_{r} u_\theta)}{\partial \theta} + \frac{\partial (\rho u_{\theta} u_z)}{\partial z} = - \frac{\partial p}{\partial \theta} + \frac{1}{r} \left( \frac{\partial (r \tau_{r\theta})}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{\theta \theta}}{\partial \theta} \right), \tag{3}
\]

\[
\frac{\partial (\rho u_z)}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u_{r} u_z)}{\partial r} + \frac{1}{r} \frac{\partial (\rho u_{r} u_\theta)}{\partial \theta} + \frac{\partial (\rho u_{z} u_z)}{\partial z} = - \frac{\partial p}{\partial z} + \frac{1}{r} \left( \frac{\partial (r \tau_{rz})}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} \right), \tag{4}
\]

\[
\frac{\partial (E r)}{\partial t} + \frac{1}{r} \frac{\partial (E r u_r)}{\partial r} + \frac{1}{r} \frac{\partial ((E + p) u_\theta)}{\partial \theta} + \frac{\partial ((E + p) u_z)}{\partial z} = \frac{1}{r} \left( \frac{\partial (r \tau_{rr})}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} \right) \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial \tau_{r\theta}}{\partial \theta} \frac{\partial T}{\partial \theta} + \frac{\partial \tau_{\theta \theta}}{\partial \theta} \frac{\partial T}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} \frac{\partial T}{\partial z}, \tag{5}
\]
where

\[ E_i = \frac{p}{(\gamma - 1)} + \frac{\rho u_i u_i}{2} + \frac{\rho u_j u_j}{2} \frac{\rho u_i u_j}{2}, \quad \text{(6)} \]

\[ \tau_{ss} = \frac{\mu}{\text{Re}} \left( \frac{\partial^2 u_s}{\partial z^2} - 2 \left[ \frac{1}{3} \frac{\partial (ru_s)}{\partial r} + \frac{1}{r} \frac{\partial u_s}{\partial \theta} + \frac{\partial u_s}{\partial x} \right] \right), \quad \text{(7a)} \]

\[ \tau_{rr} = \frac{\mu}{\text{Re}} \left( \frac{\partial^2 u_r}{\partial r^2} - 2 \left[ \frac{1}{3} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_r}{\partial x} \right] \right), \quad \text{(7b)} \]

\[ \tau_{\theta\theta} = \frac{\mu}{\text{Re}} \left[ \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_\theta}{\partial x} \right], \quad \text{(7c)} \]

\[ \tau_{\phi\phi} = \frac{\mu}{\text{Re}} \left( \frac{\partial}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{\partial u_\phi}{\partial x} \right), \quad \text{(7d)} \]

\[ \kappa = -\frac{\mu}{(\gamma - 1) M^2 \text{RePr}}, \quad \text{(8)} \]

\[ T = \frac{\gamma M^2 p}{\rho}, \quad \text{(9)} \]

\[ \mu = T^{2/3}, \quad \text{(10)} \]

where \( M = U_j/c_\infty \) and \( c_\infty \) is the speed of sound at the ambient fluid. The all variables, in all the following discussions, are non-dimensionalized by the characteristic physical scales such as \( U_j, \rho_\infty, T_\infty \) and the jet nozzle radius of \( r_0 \) where the subscripts \( j \) and \( \infty \) indicate the jet centerline and the ambient fluid, respectively. The Reynolds number is

\[ \text{Re} = \frac{U_j r_0}{\mu_\infty}. \quad \text{(11)} \]

The flow field of a jet issuing from a circular nozzle into an ambient fluid can be divided into a potential core region, a transition region, and a fully developed region. The "top-hat" jet profile belongs to the potential core region with a thin but finite shear layer. We employed the mean velocity profile \( U(r) \) given by

\[ U(r) = \frac{U_j}{2} \left[ 1 - \tanh \left( \frac{12.5}{4} \frac{r_r - r_0}{r_0 - r} \right) \right]. \quad \text{(12)} \]
The mean temperature \( T(r) \) was calculated with the Crocco-Busemann relation for unity Prandtl number;

\[
T(r) = M^2 \gamma \left( \frac{1}{2} \left( \frac{U}{U_j} \right)^2 + \frac{T_e}{U_j} + \frac{T_j}{U_j} \right),
\]

(13)

where \( \gamma \) is the ratio of specific heats. The jet centerline Mach number \( M_j = U_j/c_j \) is obtained from the relation \( M_j = M_e/c_e \). The jet was heated with exit temperature ratio \( T_j/T_e = 1.12 \). This ratio was used for the simulation of a perfectly expanded Mach 1.92 jet (Watanabe et al., 2002). A jet convective Mach number \( M_c \) may be defined by

\[
M_c = \frac{M_j \sqrt{T_j/T_e}}{1 + \sqrt{T_j/T_e}}.
\]

(14)

By using equation (14), the convective Mach number for the present study is \( M_c = 0.97 (M_j = 1.89) \).

For 3-D spatial DNS, after a grid-convergence analysis, the computational mesh was \( N_x \times N_y \times N_z = 801 \times 150 \times 128 \). Mesh points were clustered toward the jet shear layer in the normal direction and downstream streamwise locations near the end of the potential core. NSCBC (the Navier-Stokes Characteristic Boundary Conditions, Poinsot et. al., 1992) were implemented in the treatment of the boundaries at the in/outflow and far normal regions. Periodic boundary conditions were implemented in the azimuthal direction. Outflow boundaries were located at \( x = 40r_0 \) in the streamwise direction and at \( r = 22r_0 \).

In this study, jet is forced by two types inlet disturbance at \( x = 0 \) plane. For subsection 3.1 and 3.2, the inlet disturbance vector \( \tilde{\mathbf{d}}_m = (\tilde{p}_m, \tilde{u}_m, \tilde{v}_m, \tilde{w}_m, \tilde{\theta}_m, \tilde{\phi}_m) \) can be written as

\[
\tilde{\mathbf{d}}_m = \sum_m A_m \hat{\mathbf{d}}_m(r) \exp[i(m \theta - 2\pi St t)],
\]

(15)

where \( \hat{\mathbf{d}}_m \) is the eigenfunction vector of corresponding instability modes calculated by linear stability analyses. \( m \) and \( St \) indicate the corresponding azimuthal wave number (\( m = 1 \) is the first helical mode) and nondimensional frequency. \( A_m \) is the amplitude. In subsection 3.3, in order to control the sound radiated from a jet, the inlet disturbance which includes random disturbance and artificial forcing disturbances is given by

\[
\tilde{\mathbf{d}}_m = A_m \varphi(r) f_{\text{random}}(r, \theta, t) + \sum_m A_m \hat{\mathbf{d}}_m(r) \exp[i(m \theta - 2\pi St t + \psi_m)],
\]

(16)

where \( f_{\text{random}} \) is random velocity disturbance made by the same technique as an isotropic turbulence introduction and \( A_m \) is the magnitude of \( f_{\text{random}} \). \( \varphi(t) \) is a function which restricts the radial direction range of random disturbance magnitude as follows,

\[
\varphi(r) = -0.5 \tanh[5(r/r_0 - 1.1)] + 0.5.
\]

(17)

In the equation (16), \( \psi_m \) is the phase difference to control the suppression azimuthal angle of acoustic pressure fluctuation from the jet.

3. Results

3.1 Single mode cases

In order to understand the effects of upstream disturbance on the jet sound field generation, a first helical mode (\( m = 1 \)) and an axisymmetric mode (\( m = 0 \)) are introduced at the jet inlet plane with the Strouhal number \( St = 0.1 \). The both unstable modes have the supersonic phase speed relative to the ambient fluid (Luo and Sandham, 1997). The supersonic phase speed of the modes is responsible to form Mach waves. The amplitude of each mode is 1% of the jet
centerline velocity. Figure 1 shows the pressure fields visualized by the pressure iso-surface of the jet where \( p_\infty = p_e T_\infty / \gamma \). In the helical mode case (Fig.1a), the Mach wave is formed helically and appears from the jet inflow region. On the other hand, in the axisymmetric mode case (Fig.1b), the Mach wave formed in an axisymmetric manner appears from slightly downstream of the inlet region. The sooner appearance of the Mach wave in the helical cases is due to the higher growth rate of the helical mode.

![Fig. 1 Downstream evolution of pressure for a) first helical mode (St=0.1) and b) axisymmetric mode (St=0.1); blue: \( p/p_\infty = 1.005 \) and red: \( p/p_\infty = 0.995 \).](image)

Figure 2 indicates the pressure fields of the two cases in the \( r-\theta \) plane at \( x=20r_0 \). The characteristic shape of the each Mach wave is clearly shown. Note that the helical mode can also have an alternative result in the different rotational direction opposite to the result of figure 2 (a) with the same property. Therefore, the sound field formed by the opposite helical mode with the opposite rotational direction presents a reflected symmetric image of the shape shown in Figures 1(a) and 2(a).

![Fig. 2 Distribution of pressure \( p/p_\infty \) at \( x=20r_0 \) for a) first helical mode (St=0.1) and b) axisymmetric mode (St=0.1).](image)

### 3.2 A pair of helical mode case

The jet sound fields forced by a pair of first helical modes are shown in figures 3, 4 and 5. A pair of equal and opposite helical modes with the Strouhal number \( St=0.1 \) are superimposed in the inlet profiles in this simulation. The amplitudes of the fluctuations are chosen to be 1% of the jet centerline velocity. In figure 3, the pressure field shows
significant differences from the single helical mode case (see figure 1a). The pressure distribution in the meridian (θ=0 and 180°) plane shows an anti-symmetry pattern along the interference line between the helical modes.

**Figure 3** Down Downstream evolution of pressure for a pair of helical mode's case (St=0.1); blue: \( p/p_\infty = 1.01 \) and red: \( p/p_\infty = 0.99 \).

**Figure 4** Distribution of pressure \( p/p_\infty \) at \( x=20r_0 \) for a pair of helical mode's case (St=0.1).

**Figure 5** Distribution of pressure at (a) \( \theta=0^\circ \) plane, (b) \( \theta=90^\circ \) plane (c) \( \theta=180^\circ \) plane and (d) \( \theta=270^\circ \) plane for St=0.1.

Figure 4 shows the pressure field in the \( r-\theta \) plane at \( x=20r_0 \). As shown in figure 4, views of pressure field in \( r-\theta \) planes show the generated waves are highly directional. Arc-like large fluctuations of the pressure are shown outside of the jet in the meridian (\( \theta=0^\circ \) and 180°) plane. In contrast, the fluctuation magnitude in the meridian (\( \theta=90^\circ \) and 270°) plane is very small. Figures 5(a), (b), (c) and (d) show the pressure fields in the meridian planes of \( \theta=0^\circ \), 90°, 180° and 270°, respectively. The pressure waves in the \( \theta = 0^\circ \) and 180° planes in figures 5(a) and 5(c) show an instantaneous
wave front indicating an emission angle of about 60° from the jet axis determined by the jet velocity and the convection velocity, which may propagate away from the jet at the nearly same angle downstream. The observed pressure wave radiation inclined about 60° from the jet axis corresponds Mach-wave emission. On the other hands, in the θ-planes of 90° and 270°, as shown in figures 5 (b) and 5 (d), more tranquil pressure waves are observed. The observed calm pressure fluctuation is caused by interfering between the pair of first helical modes with the same frequency propagating mutually in the opposite directions ($m=\pm 1$). This result suggests that the Mach waves radiated away from the jet can be eliminated by introducing an optimal combination of helical modes interfering each other in turbulent jets.

Fig. 6 Downstream evolution of pressure for random case; blue: $p/p_\infty=1.01$ and red: $p/p_\infty=0.99$.

Fig. 7 Distribution of pressure $p/p_\infty$ at $x=20r_0$ for random case.

Fig. 8 Downstream evolution of second invariant $Q(=0.1)$ structure (iso-surfaces) and pressure field (color contour) at $\theta=90$ and 270° plane for random case.

Fig. 9 Jet centerline velocity.
3.3 Random disturbance cases

In order to investigate the development of a pair of helical modes in turbulent jets, random disturbances and a combination of helical modes is introduced at the jet inlet plane. In this section, two cases are performed. The first case is the jet flow forced randomly (hereafter referred to as random case). The second case is the jet forced by the random disturbances and an optimal combination of helical modes (hereafter referred to as random with optimal combinations of helical modes case). For the random disturbance, the magnitude of the forced streamwise velocity disturbance is chosen to be 10% of the jet centerline velocity. Figures 6, 7 and 8 show the sound fields forced by only the random disturbance. In the near jet fields, arc-like pressure fluctuations are emitted in the distinct directions from the jet (Figures 6 and 7). In figure 8, the pressure fluctuations in the meridian ($\theta=90^\circ$ and $270^\circ$) plane are visualized with vortical structures for the random case. In the downstream regions, the developed three-dimensional structures appear inside the jet. Furthermore, the emitting Mach wave fronts have their normals between $45^\circ$ and $60^\circ$ off the jet axis. These pressure patterns are very close to the results of Freund's $M=1.92$ jet simulation (Freund, et. al. 2000). Figure 9 show the jet centerline velocity. The potential core exists up to $x=17r_0$.

The pressure fluctuations obtained at the various locations $x$ are analyzed in Fourier spectrum of time and the azimuthal direction to investigate the modal properties of the jet near sound field ($5r_0 < r < 20r_0$). The Fourier mode amplitude of the pressure fluctuations $|\hat{p}(x,m,St)|$ is defined by,

$$|\hat{p}(x,m,St)| = \left| \int_{5r_0}^{20r_0} \hat{p}(x,r,m,St)\hat{p}^*(x,r,m,St)dr \right|, \quad (18)$$

where $m$ indicates azimuthal wave number, $m=\pm 1$ corresponds the first helical mode and plus or minus denotes the rotational directions. In equation (18), $\hat{p}^*(x,r,m,St)$ denotes the complex conjugate.

![Fig. 10 Amplitude distribution of Fourier mode of the pressure $|\hat{p}(x,m,St)|$ for random case at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.](image)

![Fig. 11 Amplitude distribution of Fourier mode of the pressure $|\hat{p}(x,m,St)|$ for random with optimal combination of helical modes case at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.](image)
Figure 10 show the amplitude distributions of Fourier mode for the random case. In this figure, the pressure amplitude of $m=1$, $St=0.11$, that corresponds to the first helical mode, can be seen in Fourier wave space distinguished from other modes in the jet near fields at $x=24r_0$ and $x=28r_0$. This observed peak Strouhal number of $St=0.11$ is in good agreement with the $M=2.0$ jet experimental value of $St_0=0.2$ by McLaughlin et al. (1977) that is based on the jet diameter. Further downstream at $x=32r_0$, the first helical mode of $St=0.09$ and $m=1$ is dominant. The observations of the most dominant modes of the pressure fluctuation in the random case suggest the choice of the forcing mode at the jet inlet. Therefore, we chose a combination of the helical modes ($m=1$, $St=0.09$) and ($m=-1$, $St=0.11$) to reduce the acoustic pressure fluctuation in the random case. In the optimized process of the interfering of the specific pressure fluctuations with a combination of the forcing modes, the relative phase difference between the observed mode and the forcing mode is essential to reduce the magnitude of acoustic pressure fluctuation at distinct azimuthal angles (90° and 270°). The magnitude of the forcing mode can be a tunable parameter for interfering between the forcing modes. For this case of forcing, the relative phase differences between the observed pressure fluctuation and the opposite helical modes ($m=1$, $St=0.09$) and ($m=-1$, $St=0.11$) are 0 and $-2\pi/3$, respectively. The magnitudes of the opposite helical modes ($m=1$, $St=0.09$) and ($m=-1$, $St=0.11$) are 0.8%, respectively. In the case of forcing with an optimal combination of the opposite helical modes ($m=1$, $St=0.09$) and ($m=-1$, $St=0.11$), as shown in figure 11, the forcing mode ($m=-1$, $St=0.11$) appears in the jet near fields at $x=24r_0$ and $x=28r_0$. On the other hand, the forcing mode ($m=1$, $St=0.09$) appears downstream at $x=32r_0$. The introduced helical mode grows up to the same level as predicted by linear stability theory. This numerical result indicates that the acoustic waves by growing of the helical modes in the jet sound field propagate almost linearly along the jet upstream region ($x<20r_0$).

Furthermore, we will see the magnitude distribution of the pressure fluctuation at the frequency corresponding the helical modes ($m=\pm1$, $St=0.09$) in figures 12 and 13. The root mean square (RMS) of pressure fluctuation $p'_{\text{rms}}$
associated with \( \pm m \) and \( St \) component is defined by

\[
p'_{\text{rms}}(x, r, \theta; m, St) = \sqrt{\frac{p'(x, r, \theta, t, m, St)}{p_x}}
\]  

(19)

where,

\[
p'(x, r, \theta, t, m, St) = \hat{p}(x, r, m, St)e^{[i(m\theta-\omega t)]} + \hat{p}(x, r, -m, St)e^{[i(-m\theta-\omega t)]}.
\]  

(20)

A comparison between the random case in Figure 12 and the optimal combination in Figure 13 indicates that the magnitude of acoustic pressure fluctuation for \( m=\pm 1 \) and \( St=0.09 \) at distinct azimuthal angles (90° and 270°) is well reduced at \( x=16, 24 \) and 32\( r_0 \).

Figures 14 and 15 also show that the forcing with a combination of the helical modes (\( m=\pm 1, St=0.11 \)) is effective to reduce the pressure fluctuation at distinct azimuthal angles (90° and 270°) which is well observed at \( x=16 \) and 24\( r_0 \). However, as shown in Figure 15(c) at 32\( r_0 \), the forcing of the helical mode (\( St=0.11 \)) is not effective due to the pressure fluctuations newly generated from around the jet center (see Figure 15b). These pressure fluctuations emanates due to the strong nonlinear effect downstream of the potential core breakdown region (\( x>17r_0 \)), where various pressure fluctuations shown in Fig.8 are generated. These results show that the artificially forced disturbances can form a pair of helical modes as shown in section 3.2 on a specific frequency band in a turbulent jet. Therefore, in the jet near downstream region where the potential core still appears, as shown in Fig.9, the suppression azimuthal angles of pressure fluctuation are controllable by interference between the helical modes.
Figures 16 and 17 indicate RMS of overall fluctuating pressure $p'_{all\ rms}$ for random case at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.

Fig. 16 RMS of overall fluctuating pressure $p'_{all\ rms}$ for random case at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.

Fig. 17 Difference of overall fluctuating pressure RMS $\Delta p'_{all\ rms}$ between optimal combination of helical modes case and random case at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.

Fig. 17 Difference of overall fluctuating pressure RMS $\Delta p'_{all\ rms}$ between optimal combination of helical modes case and random case at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.

Figures 16 and 17 indicate RMS of overall fluctuating pressure $p'_{all\ rms}$ for the random case and the difference of overall fluctuating pressure RMS $\Delta p'_{all\ rms}$ between the optimal combination of helical modes case and the random case, respectively. In the directions around 90° and 230°, the over-all pressure fluctuation of the artificially forced case is slightly lower than random case (Figure 17) and the suppression angle is around 230° shifted from 270°. Probably, these results were caused by the influence of the pressure fluctuation of the other frequency modes which is not controlled. Figure 18 shows the pressure fluctuation RMS at distinct azimuthal angles at 90° and 270°. In figure 18(c), due to the suppression of pressure fluctuation in the different direction, the pressure fluctuation suppression effect was low at the angle of 270°. The numerical results suggest that it is necessary to increase the number of the artificial forcing mode at the inlet, in order to achieve the higher suppression effect.

Fig. 18 Radial distribution of overall fluctuating pressure RMS at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$.

Fig. 18 Radial distribution of overall fluctuating pressure RMS at (a) $x=24r_0$, (b) $x=28r_0$ and (c) $x=32r_0$. 
4. Conclusions

Spatial DNSs of a supersonic round jet for $M_j = 1.89$ have been performed. The numerical results provide new physical insights into Mach wave emission from the jet. Upstream disturbance conditions play an important role for the evolution of sound field. Growth of a pair of first helical modes affects the Mach wave radiation. The pressure fluctuations generated by the growth of the helical modes are superposed into the jet near sound field. The numerical results indicate that the radiated pressure fluctuations generated by the growth of a pair of equal and opposite first helical modes with the Strouhal number yields a Mach wave reduction at the distinct azimuthal angle by interference between the modes. The partial suppression of the Mach wave also appears in a turbulent jet at the frequency of the artificially forced an optimal combination of the helical modes at the inlet region. Furthermore, the suppression azimuthal angle is controllable by changing the phase of the forced helical mode.

On the other hand, a clear noise reduction was not observed in the result of overalls pressure fluctuation. The main reason seems to be the influence of the mode at other frequency and nonlinear interactions between the other uncontrolled modes. Therefore, the suppression effect may be improved by increasing the number of artificially forcing mode at the jet inlet.

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