Large eddy simulation of weakly compressible turbulent flows around an airfoil

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Abstract
The purpose of this study is to develop a numerical method for turbulent flows at low Mach number range. To account for the weak compressibility, we modify the time marching method of the usual incompressible scheme which is based on the elliptic equation for pressure. To deal with high Reynolds number flows appropriately, we propose a new one-equation subgrid scale model (SGS). In this model, the concept of coherent structure function model is introduced to treat the energy transfer from grid scale to SGS portion of turbulence kinetic energy. The feature of our one-equation SGS model is that the energy production rate of SGS kinetic energy is calculated without the dynamic procedure, and any parameters such as the friction velocity on the wall or distance from the wall are not necessary. The new one-equation SGS model is incorporated into the weakly compressible scheme to treat a small density variation. Computational examinations have been conducted for fully developed turbulent flows in a plane channel and turbulent flows around NACA0012 airfoil at low Mach number, and have shown satisfactory results. We show that the consideration of the density variation even in low Mach number flows is essential to reproduce the pressure fluctuation and vortical structure appropriately.

Key words: Large eddy simulation, One-equation subgrid scale model, Low mach number flows, Channel, NACA0012 airfoil

1. Introduction
We intend to develop a numerical method for turbulent flows at low Mach number range. Examples include flow fields around a large-scale wind turbine blade, a high-speed train and an aircraft at takeoff and landing. In the computational approach, these flow fields are usually treated as incompressible flows in the book written by Frank (1998) because the compressibility is thought to be insignificant when the Mach number is approximately under 0.3. However, we think even the small fluctuation of the density may affect the fluid flow around the object. When compressible flow solvers are applied to nearly incompressible flows, a particular attention is necessary to solve the large disparity between the speed of sound and the convective velocity. With the aim of solving the problem, the preconditioned method have been proposed by Turkel (1987), and it is improved by Weiss and Smith (1995), Turkel et al. (1996) and Unrau and Zingg (1997) and so on. This method works well at low Mach number flows, but, the preconditioned method needs at least one control parameter to generate the preconditioning matrix, and it is accompanied by the significant increase of computational cost. Meanwhile, to account for the weak compressibility, we modify the time marching method of the usual incompressible scheme, which is based on the elliptic equation for pressure, by reference to the concept of the cubic interpolated pseudo particle-combined unified procedure (C-CUP) method proposed by Yabe and Wang (1991). We assume that the flow field is isothermal and the density variation is very small in one time step. The modified equation of state is obtained by applying these assumptions, and it is incorporated into the pressure equation to deal with the density variation. Our method can treat the density variation without the control parameter and the computational cost is relatively low compared to the preconditioned method.
Large eddy simulation (LES) is applicable to high Reynolds number flows with relatively low computational cost compared to the direct numerical simulation (DNS). The idea of LES is to directly resolve all turbulence scales larger than grid scale (GS), while it is to model the effect of the turbulence scales smaller than GS using a subgrid scale (SGS) model. As SGS models, algebraic eddy viscosity models, one-equation eddy viscosity models and so on have been used. Algebraic eddy viscosity models as represented by the Smagorinsky model (1963) and the dynamic Smagorinsky model (DSM) proposed by Germano et al. (1991) and modified by Lilly (1992) have been used for a variety of turbulent flows. The Smagorinsky model needs modifications in the non-turbulent region and in the vicinity of the wall. The DSM needs an averaging procedure because the negative value of the SGS eddy viscosity sometimes causes a numerical instability. Recently, many SGS models have been proposed. A modified static eddy viscosity model is proposed by Inagaki et al. (2002), and it is improved by Iizuka and Kondo (2006). Vreman (2004) suggested an SGS-viscosity model based on an algebraic theory with no explicit filtering and clipping procedures. Kobayashi (2005) proposed a coherent structure function (CSF) model based on the second invariant of the velocity gradient tensor in GS flow fields. These models may solve the problems of the conventional models, and the performance of the models is almost the same as that of the DSM for various flow fields.

One-equation SGS models have been developed to overcome shortcomings of the local equilibrium assumption between the SGS energy production and dissipation used in algebraic eddy viscosity models. In contrast to algebraic eddy viscosity models, the calculation of the equation for the SGS kinetic energy \( k_{sgs} \) is necessary to determine the SGS eddy viscosity \( \nu_{sgs} \) in the one-equation SGS model. The SGS eddy viscosity \( \nu_{sgs} \) does not become negative anywhere, thus the numerical instability does not occur in these models. The SGS kinetic energy \( k_{sgs} \) disappears automatically in the non-turbulent region and it becomes zero on the wall. Yoshizawa and Horiuti (1985) proposed the SGS kinetic energy model which is constructed by the statistical results obtained from the two-scale direct-interaction approximation. Ghosal et al. (1995) and Kim and Menon (1995) utilized the dynamic procedure to determine the model coefficients. Davidson (1997) proposed the dynamic one-equation SGS model in which the local equilibrium for the equation of \( k_{sgs} \) to determine the coefficient is assumed.

In this study, we use the theoretically derived one-equation SGS model by Yoshizawa and Horiuti (1985), Horiuti (1985) and then improved by Okamoto and Shima (1999) as a basis. The main reason to use this model is that the dynamic procedure is not necessary to determine the model coefficients. On the basis of this model, Kajishima and Nomachi (2006) proposed the one-equation dynamic (OD) SGS model. In the OD model, the dynamic procedure is utilized for the energy transfer from the resolved turbulence to SGS portion, while the SGS stress in the equation of motion is approximated by the eddy viscosity model which is given by \( k_{sgs} \). However, the calculation of the test filter to evaluate the model parameter dynamically causes an increase of the computational cost and it may influence the numerical result. To calculate the production term of the equation of \( k_{sgs} \), we propose an SGS model in which the CSF proposed by Kobayashi (2005) is introduced. The feature of our one-equation SGS model is that the energy production rate of \( k_{sgs} \) is calculated without the dynamic procedure, and any parameters, which are difficult to be specified for the complicated geometries, such as the friction velocity on the wall or distance from the wall, are not necessary.

Many computational approaches using LES for the prediction of turbulent flows around an airfoil have been attempted, since flows around the airfoil including a separation and reattachment of the boundary layer are one of the important mechanical elements. For example, Miyazawa et al. (2002) and Kato et al. (2007) conducted the experiment and the LES of the incompressible flow around the airfoil. They show that the characteristics of the airfoil and the time-averaged distribution of the surface pressure are in agreement with their experimental results. However, the surface pressure fluctuation on the suction side which plays an important role in acoustic analysis is considerably overestimated compared to the experimental data. The SGS model and the disregard of the compressibility effect are considered as one of the reason of the overestimation.

In this paper, our method, which is composed of the new one-equation SGS model and weakly compressible scheme, is applied to a fully developed turbulent flow in a plane channel and a turbulent flow around an airfoil. Our LES results of incompressible flows in the plane channel are compared with the DNS data by Moser et al. (1999), and the influence of grid resolution is discussed. The reliability of our weakly compressible scheme is discussed through the comparison with the DNS data of compressible flows by Foisy et al. (2004). Our LES results of the incompressible and compressible flows around NACA0012 airfoil are compared with LES results of Kato et al. (2007) and the experimental data by Miyazawa et al. (2002). The importance of the consideration of the small density variation even in low Mach number flows is discussed through the comparison of the results for incompressible and compressible flows.
2. Outline of Computational Method

We modify the usual incompressible scheme, which is based on the elliptic equation for pressure, to account for the weak compressibility. It is incorporated into the LES to deal with high Reynolds number flows.

2.1. Basic Equation

The spatial filter operation \( \overline{\cdot} \) and Favre average \( \tilde{\cdot} \) are applied to the continuity and compressible Navier-Stokes equations. The flow field is assumed to be isothermal. The continuity, momentum conservation with the eddy viscosity assumption and ideal gas equations are represented by

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial (\tilde{\rho} \tilde{u}_i)}{\partial x_j} = 0, \quad i = 1, 2, 3, \quad (1)
\]

\[
\frac{\partial (\tilde{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial (\tilde{\rho} \tilde{u}_i \tilde{u}_j)}{\partial x_j} + \frac{\partial}{\partial x_i} \left( \tilde{p} + \frac{2}{3} \tilde{k}_{sgs} \right) = \frac{\partial}{\partial x_i} \left[ 2\tilde{\rho}(\nu + \nu_{sgs}) \left( \tilde{S}_{ij} - \frac{1}{3} \delta_{ij} \tilde{S}_{kk} \right) \right], \quad i = 1, 2, 3 \quad (2)
\]

\[
\tilde{p} = \tilde{\rho}RT, \quad (3)
\]

where \( \tilde{\rho} \) denotes the density, \( k_{sgs} \) the SGS kinetic energy, \( \nu \) the kinetic viscosity, \( \nu_{sgs} \) the kinetic viscosity due to SGS turbulence, \( \delta_{ij} \) the Kronecker symbol, \( R \) the ideal gas constant, and \( T \) the absolute temperature. The rate-of-strain tensor of GS flow field \( \tilde{S}_{ij} \) is represented by

\[
\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right). \quad (4)
\]

2.2. One-Equation Subgrid Scale Model

The kinetic viscosity due to SGS turbulence of Eq. (2) is represented by the dimensional analysis of Yoshizawa (1982)

\[
\nu_{sgs} = C_v \Delta_x \sqrt{k_{sgs}}. \quad (5)
\]

Here, \( C_v \) is the model constant and the characteristic length \( \Delta_x \) is given by Okamoto and Shima (1999) as following equation

\[
\Delta_x = \frac{\tilde{\Lambda}}{1 + C_k \tilde{\Lambda} |\tilde{S}|^2 / k_{sgs}} \quad (6)
\]

where \( |\tilde{S}| = \sqrt{\tilde{S}_{ij} \tilde{S}_{ji}} \) denotes the strength of the rate-of-strain tensor, \( C_k \) the model constant and \( \tilde{\Lambda} = (\tilde{\Lambda} \Delta_x \Delta_y)^{1/2} \) the width of the filter. Equation (6) is used to meet the correct asymptotic behavior to the wall. Since \( C_k \) is the empirical constant of positive value, \( \nu_{sgs} \) cannot have negative value anywhere. The kinetic viscosity due to SGS turbulence \( \nu_{sgs} \) becomes automatically zero on the wall due to the boundary condition of \( k_{sgs} \to 0 \). The SGS kinetic energy \( k_{sgs} \) is calculated by the following transport equation

\[
\frac{\partial k_{sgs}}{\partial t} + \tilde{u}_j \frac{\partial k_{sgs}}{\partial x_j} = -\tau_{ij} \tilde{S}_{ij} - C_s \frac{k_{sgs}^{3/2}}{\Lambda} - \varepsilon_{\omega} + \frac{\partial}{\partial x_j} \left( C_d \Delta_x \sqrt{k_{sgs}} + \nu \right) \frac{\partial k_{sgs}}{\partial x_j}, \quad (7)
\]

where \( C_s \) and \( C_d \) are model constants. In Eq. (7), each term of right side denotes the production, dissipation and diffusion terms. As for the dissipation term, the additional term \( \varepsilon_{\omega} \) of Jones and Launder (1972) type to account for low Reynolds number effect near the wall

\[
\varepsilon_{\omega} = 2\nu \frac{\partial \sqrt{k_{sgs}}}{\partial x_j} \frac{\partial \sqrt{k_{sgs}}}{\partial x_j}, \quad (8)
\]

is used in accordance with Okamoto and Shima (1999). The production term \( -\tau_{ij}\tilde{S}_{ij} \) in Eq. (7) is represented as

\[
-\tau_{ij}\tilde{S}_{ij} = \left[ 2\nu C \left( \tilde{S}_{ij} - \frac{1}{3} \delta_{ij} \tilde{S}_{kk} \right) - \frac{2}{3} \delta_{ij} k_{sgs} \right] \tilde{S}_{ij}. \quad (9)
\]
As a model of kinetic eddy viscosity $v_C$, we introduce the coherent structure function (CSF) proposed by Kobayashi (2005). The CSF consists of the second invariant of a velocity gradient tensor $Q$ and the magnitude of a velocity gradient tensor $E$, and it plays a role of wall damping. The kinetic eddy viscosity $v_C$ is represented by

$$v_C = C_1 |F_{CS}|^{3/2} \Delta \sqrt{k_{sgs}},$$

where $C_1$ is the model constant and $F_{CS}(= Q/E)$ denotes the CSF. Considering the compressibility, $Q$ is given by Chakraborty et al. (2005)

$$Q = \frac{1}{2} \left[ \tilde{S}_{ij} \tilde{S}_{jj} - \tilde{S}_{ij} \tilde{S}_{ij} + \tilde{W}_{ij} \tilde{W}_{ij} \right],$$

where $\tilde{W}_{ij}$ means the vorticity tensor in the GS flow field. The magnitude of the velocity gradient tensor is defined by

$$E = \frac{1}{2} \left[ \tilde{S}_{ij} \tilde{S}_{ij} + \tilde{W}_{ij} \tilde{W}_{ij} \right].$$

The negative value of the production of $k_{sgs}$ is possible as the compressibility is taken into account in Eq. (9), and it results in the decrease of $k_{sgs}$. In this process, any numerical instability is not caused. Only the clipping for negative $k_{sgs}$ is required, but the fraction of clipped area is negligible and this does not affect the numerical result. In addition, any parameters, which are difficult to be specified for the complicated geometries, such as the friction velocity on the wall or distance from the wall, are not required. Considering the stability and universality, our one-equation SGS model has a probability of wide applications in the complicated geometries and high Reynolds number flows.

Nondimensional constants used in our SGS model are as follows: $C_e = 0.05$, $C_x = 0.835$, $C_d = 0.10$, $C_l = 0.08$ and $C_1 = 0.05$. The values of $C_e$, $C_x$ and $C_d$ are the same as recommended values for flows in plane channel by Okamoto and Shima (1999), and $C_1$ is close to the theoretically derived value (0.0784). $C_1$ are quoted from Kobayashi (2005).

### 2.3. Time Marching Method for Weakly Compressible Flows

For the time marching of the compressible continuity and Navier-Stokes equations, we introduce the concept of C-CUP method developed by Yabe and Wang (1991) into the usual incompressible scheme. The different point against the C-CUP method is that our method does not divide the advection term and the non-advection term during the time marching of equations.

The time marching of Eq. (2) is divided by the fractional step method:

$$\frac{(\tilde{\rho} \tilde{u})^n - (\tilde{\rho} \tilde{u})^p}{\Delta t} = \nabla \cdot \left( - (\tilde{\rho} \tilde{u}) + \tau \right)^n,$$

$$\frac{(\tilde{\rho} \tilde{u})^{n+1} - (\tilde{\rho} \tilde{u})^F}{\Delta t} = -\nabla \tilde{p}^{n+1},$$

where $n$ denotes time step count, $F$ the fraction step partially marched without the pressure gradient, $\Delta t$ the time increment and $\tau$ the viscous stress. Coupling the mass conservation equation $(\tilde{p}^{n+1} - \tilde{p}^n)/\Delta t + \nabla \cdot (\tilde{\rho} \tilde{u})^{n+1} = 0$ with Eq. (14) derives the elliptic equation for $\tilde{p}^{n+1}$

$$\frac{1}{\Delta t} \left[ \frac{-\tilde{p}^{n+1} - \tilde{p}^n}{\Delta t} - \nabla \cdot (\tilde{\rho} \tilde{u})^F \right] = -\nabla^2 \tilde{p}^{n+1}.$$

To obtain a pressure equation in which the compressibility is considered, we make the following assumptions: the flow field is isothermal and the change of $\Delta \tilde{p}$ and $\Delta \tilde{\rho}$ in one time step is small. The equation of state Eq. (3) by applying these assumptions is expressed as

$$\tilde{p}^{n+1} - \tilde{p}^n = (\tilde{\rho}^{n+1} - \tilde{\rho}^n)RT.$$

Thus, the pressure equation is derived by substituting Eq. (16) to Eq. (15)

$$\nabla^2 \tilde{p}^{n+1} - \tilde{p}^{n+1}/(\Delta t)^2 RT = \frac{\nabla \cdot (\tilde{\rho} \tilde{u})^F}{\Delta t} - \frac{\tilde{p}^n}{(\Delta t)^2 RT}.$$

The compressibility effect is represented by the second term of each side. The pressure equation is solved by the SOR method. The relative error of the inhomogeneous Helmholtz equation based on the norm of the right hand side of Eq. (17) is $O(10^{-10})$, and we checked that the large difference is not observed in results even if the convergence criterion of
Flow
Non-slip
Periodic

Fig. 1 Computational domain and boundary conditions.

Table 1 Computational parameters.

<table>
<thead>
<tr>
<th>Reτ</th>
<th>Lx × Ly × Lz</th>
<th>Nx × Ny × Nz</th>
<th>Δx*</th>
<th>Δy*</th>
<th>Δz*</th>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>180</td>
<td>Case1</td>
<td>32 × 64 × 32</td>
<td>73</td>
<td>0.5–11.0</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Case2</td>
<td>12.8δ × 2δ × 4.3δ</td>
<td>48 × 64 × 48</td>
<td>49</td>
<td>0.5–11.0</td>
</tr>
<tr>
<td></td>
<td>36 × 64 × 64</td>
<td>73</td>
<td>0.5–11.0</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>395</td>
<td>Case3</td>
<td>32 × 64 × 32</td>
<td>79</td>
<td>1.2–23.8</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Case4</td>
<td>6.4δ × 2δ × 3.2δ</td>
<td>48 × 64 × 48</td>
<td>53</td>
<td>1.2–23.8</td>
</tr>
<tr>
<td></td>
<td>64 × 64 × 64</td>
<td>79</td>
<td>1.2–23.8</td>
<td>19.4</td>
<td></td>
</tr>
</tbody>
</table>

pressure equation is smaller than $O(10^{-10})$. As the procedure of calculation, $p^{n+1}$ is obtained by the Eq. (17) and then $(\rho u)^{n+1}$ is calculated by the Eq. (13). At the end, $\rho^{n+1}$ is obtained by the Eq. (1).

Because the assumption of small density variation is employed as mentioned above, our method cannot treat appropriately a high Mach number flow. However, our method is able to deal with the density variation directly in low Mach number flows without any artificial compressibility parameters used in the preconditioned method and it is applicable to incompressible flows as well as very low Mach number flows.

3. Turbulent Flow in a Plane Channel with low Mach numbers

A fully developed flow in a plane channel is simulated at various Mach numbers up to 0.3. The flow is maintained by the constant pressure gradient. We show the reliability of our method and the grid spacing for the reasonable reproduction of anisotropy of near-wall turbulences.

3.1. Outline of Computation

Figure 1 illustrates the computational domain and boundary conditions. Here, x, y and z represent the mainstream, wall-normal and spanwise directions, respectively. Table 1 shows the Reynolds number defined by the channel half width $\delta$ and the mean friction velocity $u_\tau$ at the wall, the domain sizes $L_x \times L_y \times L_z$, the grid points $N_x \times N_y \times N_z$ and the grid width in wall unit $\Delta x^*$, $\Delta y^*$ and $\Delta z^*$. The grid width in wall unit is defined by

$$\Delta y^* = \frac{u_\tau \Delta y}{\nu}, \quad u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$  \hspace{1cm} (18)

for the wall-normal direction for instance. The periodic boundary condition is used in the mainstream and spanwise directions. The non-slip boundary condition is given at the wall surface. For the spatial difference, the 2nd order central finite-difference is used. For the time marching, the Adams-Bashforth method of the 2nd order accuracy is applied to the viscous and convective terms.

LES of the incompressible flows at $Re_\tau = 180$ and 395, corresponding to the DNS by Moser et al. (1999) is performed. Our LES results are compared with the DNS data by Moser et al. (1999), and the influence of grid resolution is discussed. We also conduct LES of compressible flows at $Re_\tau = 180$ with $M_\tau = 2 \times 10^{-2}$, corresponding to the bulk Mach number $M = 0.3$. The CFL number based on the effective sound speed is 0.25. Our LES results are compared with the available DNS data of compressible flows which are assumed to be the isothermal walls by Foysi et al. (2004). We represent that the small density variation is captured by our method and turbulence intensities are in good agreement with the DNS data of the compressible flows.
3.2. Results and Discussion

Our LES results of incompressible turbulent flows in the plane channel at $Re_{\tau} = 180$ and 395 are compared with the DNS data by Moser et al. (1999). In order to discuss the influence of SGS models, computational results by using the coherent structure model (CSM) proposed by Kobayashi (2005) are shown.
Profiles of the mean streamwise velocity of both Reynolds numbers are shown in Fig. 2. In Cases 1 and 4, the mean velocity of the present model and the CSM by coarse grids are overestimated in comparison with the DNS data. In Cases 3 and 6 with fine grid points, results of both models are in good agreement with the DNS data.

Figure 3 shows profiles of the intensity of velocity fluctuations of GS components. In Cases 1 and 4, $u_{rms}$ of both models is overestimated, while $v_{rms}$ and $w_{rms}$ are underestimated in comparison with the DNS data. In LES, the vortex structure which plays role in the Reynolds stress redistribution near the wall region is represented by the SGS model, when vortex structure cannot be treated as GS components directly. However, isotropic eddy viscosity type SGS models including our model cannot treat the redistribution among Reynolds stress components. Therefore, the distribution to wall-normal and spanwise directions of the fluctuation energy of the generated mainstream direction velocity is not fully reproduced. In case of high grid resolutions, Cases 3 and 6, the agreement between LES results and DNS results is extremely improved. In Figs. 2 and 3, $u_{rms}$ of Case 2 is slightly overestimated compared to Case 3, while other components are practically indistinguishable. This tendency is also observed in Case 5 and Case 6. For the reasonable reproduction of anisotropy of near-wall turbulences, we decide the grid spacing less than 50 and 30 in wall units respectively in streamwise
Uniform stream

\[ C: \text{chord length} \]

Fig. 8 Computational domain and boundary conditions.

and spanwise directions in LES of turbulent flows around the airfoil.

The GS and SGS kinetic energy near the wall are shown in Fig. 4. The profile of GS kinetic energy \( k_{gs} \) should be proportional to \( y^2 \) near the wall. For the SGS kinetic energy \( k_{sgs} \), it is not necessary to behave such as the GS kinetic energy, since it depends on the variation of filter width. However, the behavior of the SGS kinetic energy is in accordance with the GS kinetic energy. As shown in Fig. 4, it is considered that the distribution of the GS and SGS kinetic energy is reasonable. In addition, it should be pointed out that the SGS kinetic energy is sufficiently smaller than the GS kinetic energy due to the grid resolution in our simulation.

In order to represent the effect of the consideration of local non-equilibrium, we show the value of each term of the SGS kinetic energy. The budget of the temporally and spatially averaged SGS kinetic energy is shown in Fig. 5. The root-mean-square (RMS) of the difference of the production and the dissipation is shown in Fig. 6. In Fig. 5, the production and dissipation are dominant in the flow field, and those values seem balanced. However, in Fig. 6, the RMS of the difference of the production and the dissipation is locally and momentarily considerable. These results represent the reason of the consideration of the local non-equilibrium in high Reynolds number flows.

In order to validate our weakly compressible scheme, results for the case of \( Re_\tau = 180 \) at \( M = 0.3 \) are compared with the compressible DNS data by Foysi et al. (2004). In addition, to discuss the compressibility effect by the increase of Mach number, results of \( M = 0.1 \) are also compared. The density variation by different Mach numbers is reproduced: namely, peak values of intensity of density variation are approximately 0.01% and 0.12% of the mean density for \( M = 0.1 \) and 0.3, respectively.

Figure 7 shows the profile of the mean streamwise velocity and the intensity of velocity fluctuations. The cases of \( M = 0.1 \) and 0.3 are practically indistinguishable. This means that the compressibility effect can be ignored in the velocity in the plane channel when the Mach number is under 0.3. In case of \( M = 0.3 \), the results of the mean streamwise velocity and the streamwise intensity \( u_{rms} \) are in good agreement with the DNS data, while the spanwise intensity \( w_{rms} \) is slightly underestimated compared to DNS data. These results represent that our method which is composed of the one-equation SGS model and the weakly compressible scheme can capture the small density variation and reproduce the velocity profiles appropriately as against the DNS data of the compressible flows.

4. Turbulent Flow around NACA0012 Airfoil with Low Mach Numbers

In this section, we perform LES of the incompressible and compressible flows around NACA0012 airfoil. Our results are compared with the LES results of Kato et al. (2007) and the experimental data by Miyazawa et al. (2002). We represent the reliability of our method and the importance of the consideration of the small density variation through the comparison with results of the incompressible and compressible flows.

4.1. Computational Conditions

We used computational conditions corresponding to those in the experiment by Miyazawa et al. (2002): the angle of
attack, $9^\circ$; the Reynolds number based on the chord length $C$ and the mainstream velocity $U_0$, $2 \times 10^5$; the Mach number, $8.75 \times 10^{-2}$.

Figure 8 shows the computational domain and outline of boundary conditions. The coordinate is selected as $x$ in the mainstream direction, $z$ in the spanwise direction and $y$ in the direction perpendicular to $x$ and $z$. The coordinate system $X$ and $Y$ have its origin at the leading edge of the airfoil. The boundary-fitted grid of C-type is generated in the $x$-$y$ plane. The size of domain is: the diameter of a half circle of C-type grid is $11C$; $11C$ in the wake side; $0.5C$ in the spanwise direction. The numbers of grid points are: 1600 in the circumferential direction, and 800 on the airfoil surface; 160 to the outward from the surface; 60 in the spanwise direction. The grid width in wall unit
\[
\Delta Y^+ = \frac{u_\tau \Delta Y}{v}, \quad u_\tau = \frac{\tau_w}{\rho},
\]
where $u_\tau$ is the averaged local wall-friction velocity. In the wall-tangential, wall-normal and spanwise directions, the grid widths in wall unit near $X/C = 0.5$ on the suction side are 40, 1.2 and 40, respectively. The CFL number based on the effective sound speed is 0.2.

The uniform stream without disturbance is given at the inflow boundary, where the SGS turbulence $k_{sgs}$ is also zero. Thus, the turbulence develops in the boundary layer around the airfoil. Because the initial condition of $k_{sgs}$ in the flow field is zero, a treatment at the beginning is asked. For the first step of time-marching, the kinetic eddy viscosity evaluated by Kobayashi’s original model (2005), that is $v_C = C_1|\mathbf{F}|^{1/2} \Delta S$, is used for giving the value for the production term in the transport equation of $k_{sgs}$. From the second step of time-marching, Eq. (10) is used. The convective boundary condition is used at the exit. At the top and bottom boundaries, the normal components of the gradients of variables are assumed to be zero. The nonslip boundary condition is given at the airfoil surface. The periodic boundary condition is applied to the spanwise direction. For pressure, the non-reflective boundary condition proposed by Miyazawa et al. (1994) and then improved by Okita and Kajishima (2002) is implemented in the inflow, outflow, top and bottom boundaries to prevent the reflection of pressure waves.

The diffusion terms are discretized by the central finite difference of the 2nd order accuracy. For the convective terms, the QUICK method is applied in the equation of motion and the donor cell method is used for the transport equation of $k_{sgs}$. The QUICK method is employed to reduce the numerical instability due to our grid arrangement in the general curvilinear coordinate system for high Reynolds number flows. The influence of QUICK method was tested, and we confirmed that the influence was small. For the time marching, the Adams-Bashforth method of the 2nd order accuracy is applied to the viscous and convective terms in the equation of motion, and it is applied to the production, dissipation and diffusion terms in the transport equation of $k_{sgs}$.

4.2. Results and Discussion

Hereafter in this section, the ‘average’ denotes the temporally as well as spatially (in the spanwise direction) averaged values.

The instantaneous and cross-sectional view of $\nabla \cdot \mathbf{u}$ around NACA0012 airfoil is shown in Fig. 9. The significant $\nabla \cdot \mathbf{u}$ is confirmed over the suction side near the leading edge, it means that the compressibility effect is not negligible. The density variation of that area is approximately 1% as against the value of top and bottom boundaries.

Figure 10 shows profiles of the averaged pressure coefficient and fluctuation of the pressure coefficient on the airfoil surface. Our LES results are compared with the experimental data by Miyazawa et al. (2002). In order to compare the effect of the SGS model on the flow field, the result of LES for incompressible flows by Kato et al. (2007) is used. The
compressibility effect is discussed through the comparison of results of the incompressible and compressible flows. To investigate the effect of the Mach number, LES of compressible flows at $M = 0.01$ is performed. The pressure coefficient $C_p$ is defined by the freestream pressure $p_0$

$$C_p = \frac{p - p_0}{\frac{1}{2} \rho U_0^2}. \quad (20)$$

First, the dependency of the grid resolution is investigated. We compared the results by changing the number of grid points in the spanwise ($z$) direction: namely, $N_z = 20, 60$ and $100$ for the same width of the computational domain. It was because the resolution in the spanwise direction was most influential for the turbulence statistics. As for the intensity of fluctuation of the pressure coefficient that is the most important issue of our interest, the peak value at the separation region did not depend on the grid resolution within the range of our test. In case of $M = 0$, the averaged pressure coefficient of the present model, in Fig. 10 (a), is in agreement with the experimental data, while the fluctuation of the pressure coefficient, in Fig. 10 (b), is overestimated compared to the experimental data near the leading edge. The same tendency is also shown in the result by Kato et al. (2007). From these results, it is considered that the overestimation of the fluctuation of the pressure coefficient observed near the leading edge is regardless of SGS models. The disregard of the compressibility effect which can be confirmed in Fig. 9 is considered to be the reason of the overestimation. In Fig. 10 (a), the difference between the results of $M = 0, 0.01$ and $0.0875$ is not significant. In Fig. 10 (b), the fluctuation of the pressure coefficient near the leading edge of $M = 0.0875$ shows good agreement with the experimental data, while the other cases are overestimated. In case of each Mach number, positions of the peak in Fig. 10 (b) are observed near $X/C = 0.05$.

In Fig. 10, a flow separation region is confirmed on the suction side, and it is qualitatively in agreement with the experimental result by Miyazawa et al. (2002). To examine the separation region quantitatively, Fig. 11 shows the profile of the averaged wall friction coefficient $C_f$ on the suction side of each Mach number. The averaged wall friction coefficient

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig10.png}
\caption{Comparison of pressure profiles obtained from the incompressible and compressible flows.}
\label{fig10}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig11.png}
\caption{Averaged friction coefficients on the suction side of the airfoil of each Mach number case.}
\label{fig11}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig12.png}
\caption{Averaged fluctuation of the pressure coefficient on the suction side of the airfoil}
\label{fig12}
\end{figure}

Fig. 13  Visualized instantaneous vortical structures near the leading edge: Iso-contours of $Q = 300$ colored by $|\omega_z|$.

Fig. 14  Profiles of $k_{gr}$ and $k_{sg}$ near the leading edge.

Fig. 15  Profiles of the production and dissipation of the $k_{gr}$ transport equation near the leading edge.

$C_f$ is given by the following equation.

$$C_f = \frac{\tau_w}{\rho U_0^2/2},$$

$$\tau_w = \mu \left( \frac{\partial u_b}{\partial n} \right)_{n=0},$$

where $u_b$ is the velocity component along the airfoil surface and $n = 0$ is on the surface of the airfoil. According to the experiment by Miyazawa et al. (2002), the reattachment point exists near $X/C = 0.05$. In our LES results, the separation points are $X/C = 0.019$ for all cases, while the reattachment points of $M = 0, 0.01$ and 0.0875 are $X/C = 0.054, 0.052$ and 0.051, respectively. The reattachment point moves upstream as the Mach number increases. Figure 12 shows the profiles of the kinetic energy $(\bar{\rho} |\bar{u}|^2)/2$ of $M = 0$ and 0.0875 at each reattachment point. In Fig. 12, the kinetic energy slightly increases with the Mach number, and it is considered to put the reattachment point ahead.

To visualize vortical structures near the leading edge in cases of $M = 0$ and 0.0875, Fig. 13 shows the instantaneous iso-surface of second invariant of velocity gradient $Q$ colored by the magnitude of the spanwise vorticity $|\omega_z|$. In case
of $M = 0$, stronger spanwise vortices are observed near the reattachment region $X/C = 0.05$ compared to the case of $M = 0.0875$. As mentioned above, the peak value of pressure fluctuation is in agreement with the reattachment point. In Fig. 13, the separated shear layer is reattached near the reattachment region. From these results, it is considered that the peak value of pressure fluctuation near the leading edge is related to the vortical structure, that is affected by the Mach number, near the reattachment region.

In order to demonstrate the effect of non-equilibrium feature in the SGS turbulence, Fig. 14 shows the profiles of GS and SGS components of kinetic energy at typical cross sections before and after the turbulence transition. At $X/C = 0.02$ before the transition, the SGS component is quite small even though the GS velocity fluctuation is evident. On the other hand, at $X/C = 0.10$ after the transition, $k_{sgs}$ is increased especially near the solid wall. In Fig. 15 (a), the values of production and dissipation of $k_{sgs}$ are not balanced. But, the values of production and dissipation in the developed turbulent boundary layer, shown in Fig. 15 (b), are balanced. Thus the non-equilibrium in $k_{sgs}$ is evident and the streamwise development of $k_{sgs}$ in between is reasonably represented by introducing the $k_{sgs}$ transport equation.

In this section, we represent that our method can reproduce appropriately the surface pressure and separation region around the airfoil. The consideration of the small density variation even in low Mach number flows should be considered for the appropriate reproduction of the pressure fluctuation.

5. Conclusions

The numerical method, which is composed of the new one-equation subgrid scale (SGS) model and the weakly compressible scheme, for turbulent flows at low Mach number range was proposed. In our one-equation SGS model, the coherent structure function was utilized for the energy transfer from the resolved turbulence to SGS portion. Any parameters such as the friction velocity on the wall or distance from the wall were not necessary. Our scheme was able to deal with the density variation directly in low Mach number flows without any artificial compressibility parameters used in the preconditioned method.

The test cases by turbulent flows in the plane channel and turbulent flows around NACA0012 airfoil at low Mach number demonstrated the reliability of our method. In our one-equation SGS model, the grid spacing less than 50 and 30 in wall units respectively in streamwise and spanwise directions is required for the reasonable reproduction of anisotropy of near-wall turbulences. Our LES results showed that in case of turbulent flows around NACA0012 airfoil, the influence of the small density variation on the flow field was significant compared to the flows in the plane channel. The consideration of the density variation even in low Mach number flows was essential to reproduce the pressure fluctuation and vortical structure appropriately. It means that our method is more effective to estimate the noise source than the incompressible method. The future work is to construct the sound source model and to reproduce the acoustic field.

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