Cost Analysis of Joint Products with the Use of Fuzzy Integral

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There have been many studies about the cost of so-called joint products for the long time, and still it is very hard to determine the appropriate cost allotment to joint products. In the present study we deal with the problem from the viewpoint of macro economic consistency. Several new ideas such as qs-system, set cost, standard units will be introduced, and the Choquet integral with respect to fuzzy measures will be used to describe the present model, and to make an approach to the study about how to determine an appropriate cost assignments to joint products with set costs of composite commodities.

Keywords: Material Flow, Joint Products, Composite Unit, qs-system, Set Cost, Choquet Integral

1 Introduction

As is well known, there are many sorts of production processes with so called joint products. One may easily find many instances in the process industry [Tsukamoto et al.[1]]. The joint production process is the one which produces two or more different products for the same inputs such as raw material, fuel and so on. In this case it would be very difficult to assign the appropriate cost to each of all the joint products. It has been said that it is impossible in principle. In fact this problem have been dealt with from political viewpoints rather than reasonable or scientific views [Okamoto [2]]. To estimate the values of the costs from the market prices in calculating the costs seems to be unnatural, either.

However, once one moves the focus from its individual process into the viewpoint of macro-economic consistency, one will be able to have a resolution of this difficult problem.

In this paper, new concepts such as set cost, set

price, standard unit, etc. are introduced to study the above problem. Furthermore the Choquet integrals with respect to fuzzy measures will be used to develop the present model, which are very proper to describe the economic systems characterized with composite commodities, joint products and set prices.

In what follows we first summarize the notations, mathematical preliminaries, and especially the units which will play an important role in the present study. In the third section we will show how to describe the economic system with joint products. In addition to q-system proposed by Piero Sraffa [3], we will propose qs-system with the standard units as the elementary measures. This qs-system will enable us to investigate the effects to the cost allotment by the introduction of joint products, set prices etc. The analysis will be carried out for both self-replacement and growing economies.

In the final section we will discuss the possibility to extend the present analysis to apply to the economy with the consideration on the costs of fixed capital or waste disposal which in nature should be recognized as a sort of joint products.

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2 Preliminaries

In this section we summarize the basic tools such as notations, standard units and some mathematical preliminaries.

2.1 Notations

In this paper, we mainly follow the terminology and the notations that were used in the book by P. Sraffa [3].

In the case of single-product industries, let $a, b, \dots, k$ denote the names of the commodities produced by separate industries; Γ the set of commodities; $A, B, \dots, K$ the quantities of $a, b, \dots, k$ annually produced; and $A_a, B_a, \dots, K_a$ the quantities of $a, b, \dots, k$ annually used by the industry which produces the commodity a . All these represent known quantities. The unknown are $p_a, p_b, \dots, p_k$, respectively, the values per units of the commodities, which mean prices in this paper. Other notations will be explained later.

2.2 Composite Units

Although usually the natural units are used for expressing the quantities of the commodities, we use the composite standard unit to describe some variables in the present study. The composite standard unit is expressed by the vector with the quantities with respect to the commodities as its elements.

Such coordinate quantities are denoted by $u_a, u_b, \dots, u_k$. Then the composite standard unit is represented by the following vector

$$u = (u_a, u_b, \dots, u_k). \quad (1)$$

It is noted that when we state only "the standard unit" it means each element of the vector, and also that the vector is called simply the composite unit often. Such a composite unit will be used to represent the qs-system as seen in the next section.

2.3 Fuzzy Measures and Choquet Integral

In the present paper we deal with only finite sets, so fuzzy measures are defined as a set function g from the power set of $X = \{x_1, x_2, \dots, x_k\}$ to $[0, \infty)$ with the following two properties:

$$g(\emptyset) = 0 \text{ and}$$

$$g(E_1) \leq g(E_2) \text{ if } E_1 \subset E_2 \subset X.$$

Let $f : X \rightarrow [0, \infty)$ be the following ordered function as:

$$f(x_1) \leq f(x_2) \leq \dots \leq f(x_k).$$

Then the Choquet integral with respect to a fuzzy measure g is defined by

$$(C) \int f dg = \sum_{i=1}^k (f(x_i) - f(x_{i-1})) g(A_i) \quad (2)$$

where

$$A_i = \{x_i, x_{i+1}, \dots, x_k\} \text{ and } f(x_0) = 0.$$

In the sequel the set X will be referred to as the set of commodities, Γ .

3 Basic Economic Systems

In this section starting at the description of a primitive economic system, we will construct the fundamental systems, called the qs-systems. The qs-systems are based on q-systems which was proposed by P. Sraffa¹.

3.1 Production Processes

We use the following matrix to describe a single product system in terms of material flows:

$$\begin{bmatrix} A_a & B_a & \cdots & K_a \\ A_b & B_b & \cdots & K_b \\ \vdots & \cdots & \ddots & \vdots \\ A_k & B_k & \cdots & K_k \end{bmatrix} \rightarrow \begin{bmatrix} A \\ B \\ \vdots \\ K \end{bmatrix} \quad (3)$$

Note that this matrix should be read separately every row. For instance, as for the first row, $A_a, B_a, \dots, K_a$ produces A units of commodity a .

A joint production system is expressed by the following matrix

¹ P. Sraffa [3] has proved that "there is always one, and only one, set of multiplier which, if applied to the several equations or industries composing the system, will have the effect of rearranging them in such proportions that the commodity-composition of the aggregate means of production and that of the aggregate product are identical.", and he has also stated that "the rate that applies to the individual commodities is naturally also the rate by which the total product of the Standard system exceeds its aggregate means of production of the system. This ratio is called the Standard ratio."

$$\begin{bmatrix} A_1 & B_1 & \cdots & K_1 \\ A_2 & B_2 & \cdots & K_2 \\ \vdots & \vdots & \ddots & \vdots \\ A_k & B_k & \cdots & K_k \end{bmatrix} \rightarrow \begin{bmatrix} A_{(1)} & B_{(1)} & \cdots & K_{(1)} \\ A_{(2)} & B_{(2)} & \cdots & K_{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ A_{(k)} & B_{(k)} & \cdots & K_{(k)} \end{bmatrix} \quad (4)$$

where, for instance, B_i , denotes the quantity of the commodity b used in the i -th industry as means of production and $B_{(i)}$ stands for the quantity of b produced in the i -th industry, and so on. Note that the system (4) expresses a general case in which every industry produces all kinds of products in a economy. However from a more realistic point of view most of the elements in the right matrix except the diagonal ones will take the value zero.

3.2 q-System

q-system is the system satisfying the following conditions concerning the proportional coefficients.

For the single product case, the system, (3), is a q-system if

$$\sum_{\lambda \in I} A_{\lambda} : \cdots : \sum_{\lambda \in I} K_{\lambda} = A : \cdots : K. \quad (5)$$

For the joint products case, (4) is a q-system when

$$\sum_{i=1}^k A_i : \cdots : \sum_{i=1}^k K_i = \sum_{i=1}^k A_{(i)} : \cdots : \sum_{i=1}^k K_{(i)}. \quad (6)$$

3.3 qs-System

Now we are ready to define qs-system. A qs-system is a q-system in which all the quantities are measured with the standard units. Thus the qs-system for the q-system resulted from the system (3) is characterized by the following conditions

$$\sum_{\lambda \in I} A_{\lambda} = \sum_{\lambda \in I} B_{\lambda} = \cdots = \sum_{\lambda \in I} K_{\lambda}. \quad (7)$$

The conditions for the q-system resulted from the system (4) with joint products to be a qs-system are:

$$\sum_{i=1}^k A_i = \cdots = \sum_{i=1}^k K_i = e (= \text{const}) \quad (8)$$

and

$$\sum_{i=1}^k A_{(i)} = \cdots = \sum_{i=1}^k K_{(i)} = h (= \text{const}). \quad (9)$$

3.4 Illustrative Examples

Let us now use a simple illustrative example to show how to derive a qs-system. Assume an actual system including only three basic industries which produce, respectively, iron, coal and wheat² as expressed in the following way:

$$\begin{aligned} 90\text{iron} + 120\text{coal} + 60\text{wheat} &\rightarrow 180\text{iron}, \\ 50\text{iron} + 125\text{coal} + 150\text{wheat} &\rightarrow 450\text{coal}, \\ 40\text{iron} + 40\text{coal} + 200\text{wheat} &\rightarrow 480\text{wheat}. \end{aligned}$$

The above system can be simply expressed by the matrix as shown in (3) :

$$\begin{bmatrix} 90 & 120 & 60 \\ 50 & 125 & 150 \\ 40 & 40 & 200 \end{bmatrix} \rightarrow \begin{bmatrix} 180 \\ 450 \\ 480 \end{bmatrix} \quad (10)$$

Setting the multipliers³ used for the derivation of q-system as;

$$q_1 = 1, \quad q_2 = 0.6, \quad q_3 = 0.75,$$

this production processes may be transferred into the following q-system

$$\begin{bmatrix} 90 & 120 & 60 \\ 30 & 75 & 90 \\ 30 & 30 & 150 \end{bmatrix} \rightarrow \begin{bmatrix} 180 \\ 270 \\ 360 \end{bmatrix} \quad (11)$$

The total values for the elements of the columns are respectively 150, 225, 300.

So the standard ratio (see the footnote 1) in this system is

$$150 : 225 : 300 = 180 : 270 : 360 = 1 : 1.5 : 2$$

As the composite standard unit as shown in (1), we now have the following:

$$\begin{aligned} u &= (u_{\text{iron}}, u_{\text{coal}}, u_{\text{wheat}}) \\ &= (1 \text{ ton iron}, 1.5 \text{ ton coal}, 2 \text{ quarters wheat}). \end{aligned}$$

Using the above composite unit, we get the qs-sys-

² These commodities themselves may be regarded as some composite goods.

³ As for the detail on such multipliers, one may refer to Chapter 4 The Standard Commodity and Chapter 5 Uniqueness of the Standard System in the P.Sraffa [3]

tem as follows:

$$\begin{bmatrix} 90 & 80 & 30 \\ 30 & 50 & 45 \\ 30 & 20 & 75 \end{bmatrix} \rightarrow \begin{bmatrix} 180 \\ 180 \\ 180 \end{bmatrix} \quad (12)$$

where the quantities of the columns are measured with the standard units as shown in the above.

4 Production Processes with Joint Products

Here we use the qs-system as described in the previous section to consider a simple system of production for subsistence. First, we consider a single product system; as the second case, the qs-system with joint products; finally, the qs-system with set cost.

In order to demonstrate the effects of set cost on the economy, let us start to consider a simple system as:

$$\begin{bmatrix} 90 & 20 \\ 30 & 100 \end{bmatrix} \rightarrow \begin{bmatrix} 120 \\ 120 \end{bmatrix} \quad (13)$$

For simplicity the prices of the first and second commodities are denoted by x and y , respectively.

From the balance equation in money terms, we readily have

$$y = 1.5x.$$

With the use of an arbitrary unit of money, we can set

$$GNP = 120(x+y) = 1000.$$

Then, we obtain as the solution for subsistence;

$$x_1 = \frac{10}{3} \approx 3.33, \quad y_1 = 5$$

where x_1 and y_1 stand for the solutions in the first case.

In the second case of the production system with joint products it is expressed as:

$$\begin{bmatrix} 90 & 20 \\ 30 & 100 \end{bmatrix} \rightarrow \begin{bmatrix} 120 & 10 \\ 0 & 110 \end{bmatrix}. \quad (14)$$

In this case, the first industry produces two different products, and the products in the second industry becomes a bit smaller. Similarly we set

$$y = 3x \text{ and } 120(x+y) = 1000.$$

Then we have the following solution for this second case:

$$x_2 = \frac{1000}{480} \approx 2.09, \quad y_2 = \frac{3000}{480} \approx 6.26.$$

In order to attain at the same *GNP*, the price of the commodity in the second industry should rise somehow. On the other hand, there is cost down in the first industry.

Finally, we will look into the case in which there is set cost and/or set price. Set price means the price per such a composite unit as defined by the equation (1). For the same qs-system as shown in (14), we denote the set price by p . More precisely, p is the price of the set commodity under the definition of the composite unit consisting of two commodities.

Assume that the set commodities produced by the first industry are used up as means of production by the second industry. Then the balance equations are given as follows:

$$\begin{aligned} 90x + 20y &= 110x + 10p, \\ 20x + 90y + 10p &= 110y, \\ 110x + 110y + 10p &= 1000. \end{aligned}$$

Furthermore, we assume that the set price is sub-additive⁴ [6]. Then let us set

$$p = 8 < (x_2 + y_2) \approx 8.35.$$

Now we obtain as the solution in the final third case:

$$x_3 = \frac{24}{11} \approx 2.18, \quad y_3 = \frac{68}{11} \approx 6.18.$$

Here let us give the summary of the solutions for the above three cases:

$$x_1 = 3.33, \quad y_1 = 5, \quad (15)$$

$$x_2 = 2.09, \quad y_2 = 6.26, \quad (16)$$

$$x_3 = 2.18, \quad y_3 = 6.18. \quad (17)$$

The above results show that the introduction of set price would make the price for each commodity higher. This is quite understandable because of the sub-additivity of set price. In this case, it is the first

⁴ If super-additivity is assumed, the direction of inequality would be reverse in the above relation. This case would be easier to be dealt with than the case of sub-additivity. In this paper we dare to show only the analysis of the sub-additive case.

industry that has the set price, and the second industry that has the set cost which is equal to the set price.

It should be noted that as shown in the second case we now obtain the solution for the costs assignment to each commodity of the joint products in one sense.

5 Economy with Surplus

Let us consider a more realistic system rather than the production for subsistence. In the economies with surplus new terms such as profit, wage or growth rate will be introduced. We start to express a two-commodity qs-system with a surplus and then it will be generalized to the cases with more than two commodities.

5.1 Two Products

Consider a qs-system of joint production with two commodities as follows:

$$\begin{bmatrix} 90 & 20 \\ 30 & 100 \end{bmatrix} \rightarrow \begin{bmatrix} 130 & 10 \\ 20 & 140 \end{bmatrix} \quad (18)$$

The process of finding the solution consists of the following six steps. First let us define the extended class, Γ^* , of the commodities by

$$\Gamma^* = \Gamma \cup \{n\} = \{a, b, n\}$$

where 'n' denotes an implicit commodity with zero price (i.e. $p_n=0$). This extension is required to enable us to deal with the arbitrary quantity of joint products, and at the same time to make the fuzzy measures satisfy the condition of non-decreasing.

The fuzzy measures on Γ^* are given as follows:

$$g(E) = \begin{cases} p & \text{if } E = \Gamma \\ \sum_{\lambda \in E} p^\lambda & \text{else} \end{cases}$$

where p is the set price of the composite unit for the system. We write the related production functions as follows:

$$f_1 = (90, 20, 10) \quad f_2 = (30, 100, 10)$$

$$f_{(1)} = (130, 10, 0) \quad f_{(2)} = (20, 140, 0)$$

As for the derivations of the above formulations, one may refer to the next section for the general

case. Let r , R , L and w denote the profit rate, growth rate, labor and wage, respectively. Naturally, by definition

$$0 \leq r \leq R$$

and the growth rate in such a system as (18) is

$$R = 0.25 .$$

Let C_i and S_i be the total costs and sales in the i -th industry. Then we may use the choquet integral to describe these two total values for each industry ($i=1,2$) as:

$$C_i = (C) \int f_i dg, \quad S_i = (C) \int f_{(i)} dg.$$

As to C_1 in the above, we show the calculation process of the Choquet integral in the figure 1.

The balance equations are

$$(1+r)(C) \int f_i dg + L_i w = (C) \int f_{(i)} dg. \quad (19)$$

Thus we get the equations;

$$(80x + 10y + 10p)(1+r) + L_1 w = 120x + 10p$$

Fig. 1 Extended Commodity Set and Choquet Integral

$$(10x + 80y + 20p)(1 + r) + L_2w = 20p + 120y$$

After a simple calculation, we obtain

$$x = \frac{18}{301}p + \frac{0.32}{301}(32L_1 + 11L_2),$$

$$y = \frac{25}{301}p + \frac{0.32}{301}(11L_1 + 32L_2).$$

5.2 More Than Two Products

As a more general case, here we consider a qs-system with set costs and joint products. Let us assume that the set cost is introduced for only one composite commodity. For the qs-system, (4), we set for each i

$$\alpha_i = \min\{A_i, B_i, \dots, K_i\},$$

$$\beta_i = \min\{A_{(i)}, B_{(i)}, \dots, K_{(i)}\},$$

$$\gamma = \min\left\{\sum_{i=1}^k \alpha_i, \sum_{i=1}^k \beta_i\right\} \quad (20)$$

where γ is just the quantities measured with the composite units used up by the system. For the supply part, there should be $2k$ numbers

$$0 \leq \epsilon_1, \epsilon_2, \dots, \epsilon_k \leq 1; 0 \leq \delta_1, \delta_2, \dots, \delta_k \leq 1$$

satisfying

$$\sum_{i=1}^k \epsilon_i \alpha_i = \sum_{i=1}^k \delta_i \beta_i = \gamma > 0. \quad (21)$$

In the above, $\epsilon_i \alpha_i$ and $\delta_i \beta_i$ are the quantities of composite units respectively used up and produced out by the i -th industry. In the example as mentioned in the section 5.1, we have

$$\alpha_1 = 20, \alpha_2 = 30; \beta_1 = 10, \beta_2 = 20$$

$$\epsilon_1 = \frac{1}{2}, \epsilon_2 = \frac{2}{3}; \delta_1 = \delta_2 = 1$$

The important relations concerning our system are studied in the following six steps as:

Step 1. Extended Universal Set

$$\Gamma^* = \Gamma \cup \{n\} = \{a, b, \dots, k, n\} \quad (22)$$

where n denotes a implicit commodity in the system without singleton price, i.e. ($p_n = 0$).

Step 2. Fuzzy Measures on Γ^* :

$$g(E) = \begin{cases} p & \text{if } E = \Gamma \\ \sum_{\lambda \in E} p_\lambda & \text{else} \end{cases} \quad (23)$$

where p is the set price of composite goods and E a subset of Γ^* .

Step 3. Production Functions:

Define the production functions in terms of materials, f_i and $f_{(i)}$, with respective to each process as follows: For $\tau \in \Gamma^*$,

$$f_i(\tau) = \begin{cases} (1 - \epsilon_i) \alpha_i & \text{if } \tau = n \\ \tau_i & \text{else.} \end{cases}$$

$$f_{(i)}(\tau) = \begin{cases} (1 - \delta_i) \beta_i & \text{if } \tau = n \\ \tau_{(i)} & \text{else.} \end{cases}$$

where τ_i is the quantity of i -th commodity used as means of production and $\tau_{(i)}$ stands for the quantity of i -th commodity produced in the i -th industry.

The above two functions can be expressed by the following vectors:

$$f_i = (A_i, B_i, \dots, K_i, (1 - \epsilon_i) \alpha_i),$$

$$f_{(i)} = (A_{(i)}, B_{(i)}, \dots, K_{(i)}, (1 - \delta_i) \beta_i).$$

Step 4. Total Cost and Sales by Choquet Integrals:

For the i -th process, denote C_i , L_i , and S_i , the cost, labor and quantities of products used and produced annually. By means of Choquet integral, we have

$$C_i = (C) \int f_i dg, \quad (24)$$

$$S_i = (C) \int f_{(i)} dg \quad (25)$$

for $i = 1, 2, \dots, k$.

Step 5. Balance Equations:

The input-output equation for the i -th process is written as

$$(1+r)C_i + L_iw = S_i \quad (26)$$

where w and r denote respectively the wage and the profit rate in the system.

The aggregate input-output equation is given by the summation of Eq.(26) over i :

$$T_c(1+r) + Lw = T_s \quad (27)$$

where

$$T_c = \sum_{i=1}^k C_i, \quad L = \sum_{i=1}^k L_i, \quad T_s = \sum_{i=1}^k S_i. \quad (28)$$

More concretely, using e , h as defined by (8) and

(9) in the section 3.3, we have

$$T_c = (e - \gamma)(p_a + p_b + \dots + p_k) + \gamma p, \quad (29)$$

$$T_s = (h - \gamma)(p_a + p_b + \dots + p_k) + \gamma p, \quad (30)$$

Step 6. Some Relations among the Variables:

Substituting the above T_c and T_s into Eq.(27) gives

$$\{(e - \gamma) \sum_{i \in I} p_i + \gamma p\} (1 + r) + Lw = (h - \gamma) \sum_{i \in I} p_i + \gamma p$$

Thereby we have

$$\{(h - e) - r(e - \gamma)\} \sum_{i \in I} p_i = r\gamma p + Lw. \quad (31)$$

Since

$$h - e = \frac{h - e}{e} e \geq re > r(e - \gamma)$$

we have

$$(h - e) - r(e - \gamma) > 0.$$

Therefore

$$\sum_{i \in I} p_i = \frac{r\gamma p + Lw}{(h - e) - r(e - \gamma)} \quad (32)$$

It should be noted that if we only want to know the costs for the industries in the system without consideration of relations among the set costs, labor wages and productivity, we can use the equations given in the Appendix B to obtain the similar solutions as shown in the above.

5.3 qs-System with Net National Products

Let us denote the net national products, \$ NTP \$, defined as

$$NTP = T_s - T_c = (h - e) \sum_{i \in I} p_i. \quad (33)$$

Then, by Eqs.(32) and (33),

$$r\gamma p = \frac{(h - e) - r(e - \gamma)}{h - e} NTP - Lw. \quad (34)$$

Let us now discuss the possible cases according to the values of the coefficient of the set price. P.Sraffa [3] has given the the following relations between the profit r , wage w and the growth rate of GNP R :

$$r = R(1 - w) \quad (35)$$

$$R = \frac{h - e}{e}. \quad (36)$$

Case 1. When $r = 0$, we readily have $w = 1$.

Case 2. When $r \neq 0$, the p can be derived from (34) because the net national products, aggregate labor L , the wage w and the profit rate r by means of (34) are decided outside our system. That is,

$$p = \frac{(h - e) - r(e - \gamma)}{r\gamma(h - e)} NTP - \frac{Lw}{r\gamma}. \quad (37)$$

Thus we obtain the singleton prices of all the commodities in the system expressed by (24) to (26). Therefore, the problem of finding the costs concerning joint products can be solved successfully.

By (35) and (36) the equation (32) may be written in terms of set price and wage as follows:

$$\sum_{i \in I} p_i = \frac{r\gamma p + Lw}{Rwe + r\gamma} \quad (38)$$

5.4 Additivity of Set Price

When $r\gamma \neq 0$, by $0 < r \leq R$, we have

$$(h - e) - r(e - \gamma) = Re - r(e - \gamma) = e(R - r) + r\gamma \geq r\gamma.$$

Then

$$\frac{r\gamma}{(h - e) - r(e - \gamma)} \leq 1.$$

This means that, by (32),

$$\sum_{i \in I} p_i \leq p + \frac{Lw}{r\gamma}. \quad (39)$$

What will happen when the all the industries can earn the maximum of profit? We will consider the condition under which each industry attains at the highest rate of the profit. When this is the case, $r = R$ and $Lw = 0$. From the following two equations,

$$\begin{aligned} (h - e) - r(e - \gamma) &= e(R - r) + r\gamma \\ r &= R \end{aligned}$$

we have

$$(h - e) - r(e - \gamma) = r\gamma \quad (40)$$

Thus from $Lw = 0$, (32) and (40), we get

$$p = p_a + p_b + \dots + p_k. \quad (41)$$

One may find that only in the this case the set price becomes additive.

5.5 Productivity of qs-Systems

Since the productivity is a term of referring to the ratio of output to inputs, we define the productivity l of qs-system as the ratio of the aggregate product to the aggregate cost. That is,

$$\begin{aligned} l &= \frac{T_s}{T_c} = (1+r) + \frac{Lw}{T_c} \\ &= (1+r) + \frac{Lw}{(e-\gamma)(\sum_{\lambda \in I} p\lambda) + \gamma p} \\ &= (1+r) + \frac{Lw}{\gamma} \{ \frac{NTP}{Re\gamma} + p \}^{-1} \end{aligned} \quad (42)$$

where the last equation is derived from Eqs.(33) and (36).

Therefore, it follows from the last equation (42) that the productivity in a qs-system is in direct proportion to the rate of profit and to the labor wage, and it is in inverse proportion to the set price of standard units. Thus, the productivity of the system will increase when the set price decreases, and vice versa.

6 Concluding Remarks

With the use of qs-system with set prices, we have done some study and obtained some results such as the relations between set cost and labor, or between the set cost and net national products. In our familiar systems of joint products such as wool and mutton, or wheat and straw, we know that there are realistic joint products. In fact, even for the simplest system which produces only one product, we should consider the durable instruments of production as part of the annual intake of the production processes.

Similarly, we should also consider the waste disposal as an additional product with the possibly minus prices. All of these mean that we should recognize almost all of the industries as the production systems with joint products.

The analysis of the cases where we have to consider the set prices for the arbitrary subsets and also the development for the case in which the set price is super-additive still remain to be studied.

Acknowledgements

The authors are grateful to the editor and the anonymous referees for their valuable comments and suggestions.

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Appendix

A Labor and Profit

From (27), we have

$$T_s - T_c = rT_c + W$$

where W stands for the aggregate wages, that is, it represents Lw in (27). Because P. Sraffa used R to denote the maximum profit rate, from (36), we have

$$r \frac{T_c}{T_s - T_c} + \frac{W}{T_s - T_c} = 1. \quad (44)$$

Since $T_s - T_c = 1$ in the system by Sraffa [3], it means that the wage w in [3] should be the percents

of W in the net national product, i.e.

$$w = \frac{W}{T_s - T_c}. \quad (45)$$

Under these considerations, we should rearrange our systems with dividing equations by net national products. Also we should consider L_2 as the labor proportions of the aggregate labor which entered the second industry. Then we can continue the study for qs-system in a more convenient way such as for the relations among the maximum profit rate, profit rate and wage in Sraffer's system that follows (35).

This appendix tells us that if we assume

$$L = \text{Net National Products} = 1$$

then we have to change all the equations by the multiplier:

$$\frac{1}{T_s - T_c}$$

before we transfer the original systems to q-system and then to qs-systems.

B Balance Equations of Joint Production System

As for the unknown variables $p_1, p_2, \dots, p_k$ and p , by (24) to (26), (33) and (34) we can get the following equations as:

$$\begin{aligned} & (A_1 - \epsilon_1 \alpha_1) + \dots + (K_1 - \epsilon_1 \alpha_1) + \epsilon_1 \alpha_1 p \\ &= (A_{(1)} - \delta_1 \beta_1) + \dots + (K_1 - \delta_1 \beta_1) + \delta_1 \beta_1 p \\ & \vdots \quad \vdots \quad \vdots \quad \vdots \\ & (A_k - \epsilon_k \alpha_k) + \dots + (K_k - \epsilon_k \alpha_k) + \epsilon_k \alpha_k p \\ &= (A_{(k)} - \delta_k \beta_k) p_1 + \dots + (K_{(k)} - \delta_k \beta_k) p_k + \delta_k \beta_k p \end{aligned}$$

and

$$p_1 + p_2 + \dots + p_k = \frac{NTP}{h-e}.$$

(2001年 7 月 31 日 受付)

(2002年 5 月 12 日 採録)

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