Performance Evaluation of Network Models based on Fuzzy Queueing System[†]

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1. Introduction

Many mathematical models have been developed to solve queueing network problems. Most of these assume probability distribution for arrivals and services[8-12]. The conventional methods which are used very widely are based upon probability theory, where the probability is expressed in terms of the statistical information of its components. In some real situations, we may sometimes obtain the statistical information subjectively. Particularly, the average arrival rate and the average service rate of a queueing network model can be obtained from experts. Therefore, these data are subjective and fuzzy. In this case, we will apply to fuzzy numbers rather than crisp numbers for the arrival rate and the service rate of a queueing network model.

Fuzzy set theory is a proper means for modeling uncertainty or imprecision arising from mental phenomena. Its capability of handling uncertainty or imprecision is crucial to all the situations when the available information is not precise and the associated uncertainty can not be ignore[24]. The subjective or preference information can usually (but not always) be represented as a convex fuzzy number[25]. Thus we can use the mathematics of fuzzy calculus to handle these uncertainty and imprecise. The practical merit of fuzzy approach is that there is no need to force analyst to give a probability distribution. And fuzzy approach can get a reasonable solution suitable for all cases ranging from the pessimistic case to the optimistic case.

The problems in modeling with fuzzy queueing system have been analyzed by Prade[1], and Li and Lee[5] through the use of the extension principle. Stanford[21] presented the existence of a unique limiting set for a finite state Markov chain with fuzzy transition probabilities. Buckley[2] considers elementary queueing systems which have with multiple parallel servers with finite or infinite system capacity and calling source, whose arrivals and services are restricted by arbitrary possibility distributions. Negi and Lee's contribution[4] was shown that their approach can utilize the advantages of both the fuzzy and probability approaches to make the model more realistic and less restrictive. Their approach for F/F/1 system is a single value simulation of a fuzzy variable by the use of two random variables. Recently Yoshida[22] constructs a Markov fuzzy process, which represents the transitions of grades of fuzzy sets, with a transition possibility measure and a general state space.

In this paper, we will present the fuzzification of queueing system and the performance evaluation of network models based on the fuzzy set theory. Also, we will show that this performance evaluation is more flexible and realistic than crisp one of conventional method through some numerical examples.

2. Preliminaries

There are various types of fuzzy number to express vagueness[23], the triangular fuzzy number(TFN) among these types is appropriate to express the average arrival rate and the average service rate of fuzzy queueing network models because a TFN is represented by a triplet, *i.e.*, a minimum value, a modal value and a maximum value. Therefore, we use fuzzy numbers which are characterized by α -cuts and TFNs. The reason for using these characterizations is the simplicity and flexibility for the intervals of confidence and the

[†] ファジィ待ち行列システムによるネットワークモデルの性能評価

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triangular shapes. It is well known in fuzzy approach that the interval of confidence associated with monotonic functions can be very convenient.

The TFNs are used extensively in modeling of systems because fuzzy numbers of this type are very simple to treat. In this model, to use a trapezoidal fuzzy number is seldom appropriate to express the average arrival rate and the average service rate of fuzzy queueing network models because the modal value of it has a width.

Let us introduce the notation needed in the rest of the paper. All our fuzzy numbers will be triangular shaped. We will place a tilde over a capital letter if it represents a TFN, so $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{X}$, $\tilde{N}$, $\tilde{N}_q$ are all TFNs. Let $\tilde{A}$ be a TFN with membership function $\mu_{\tilde{A}}(x)$. We represent an interval of confidence of TFN $\tilde{A}$ for all α -cut level, that is, $A(\alpha) = [a_L^{(\alpha)}, a_R^{(\alpha)}]$ for $\alpha \in [0, 1]$. If $\alpha = 1$, $a_L^{(\alpha)}$ is equal to $a_R^{(\alpha)}$. And the TFNs $\tilde{A}$ and $\tilde{P}_n$ are represented by triplet (a_1, a_2, a_3) and $(p_{n,1}, p_{n,2}, p_{n,3})$ respectively. The membership function, $\mu_{\tilde{A}}(x)$ is partially specified by (a_1, a_2, a_3) where :

- (1) $a_1 < a_2 < a_3$;
- (2) $\mu_{\tilde{A}}(x) = 0$ outside (a_1, a_3) and equals one at a_2 ;
- (3) $\mu_{\tilde{A}}(x)$ is continuous and monotonically increasing between 0 and 1 on $[a_1, a_2]$;
- (4) $\mu_{\tilde{A}}(x)$ is continuous and monotonically decreasing from one to zero on $[a_2, a_3]$.

For any TFN $\tilde{A}$, we can write as follows :

- (1) $\tilde{A} \geq c$ if $a_1 \geq c$;
- (2) $\tilde{A} > c$ if $a_1 > c$;
- (3) $\tilde{A} \leq c$ if $a_3 \leq c$;
- (4) $\tilde{A} < c$ if $a_3 < c$,

where c is a crisp real number. The support of $\tilde{A}$ is (a_1, a_3) . Comparisons of two TFNs are assumed to be made by Kaufmann and Gupta[23] methodology using linear ordering of fuzzy numbers. In this paper, addition, subtraction, multiplication and division of fuzzy numbers are used to the intervals of confidence for α -cut level.

Definition 1 : For TFNs $\tilde{A}_1 = (a_{1,1}, a_{1,2}, a_{1,3})$, $\tilde{A}_2 = (a_{2,1}, a_{2,2}, a_{2,3})$, ..., $\tilde{A}_n = (a_{n,1}, a_{n,2}, a_{n,3})$, we define an average TFN $\tilde{A}$ as follows [26] :

$$\tilde{A} = \left(\frac{a_{1,1} + \dots + a_{n,1}}{n}, \frac{a_{1,2} + \dots + a_{n,2}}{n}, \frac{a_{1,3} + \dots + a_{n,3}}{n} \right). \quad (1)$$

Dubois and Prade[3] and Zimmermann[7] have defined the fuzzifying function and its integration over a crisp interval as follows :

Definition 2 : Let X and Y be universes and $P(Y)$ be the set of all fuzzy sets in Y (power set).

$\tilde{f} : X \rightarrow \tilde{P}(Y)$ is a mapping.

$\tilde{f}$ is a fuzzified function, iff

$$\mu_{\tilde{f}(x)}(y) = \mu_{\tilde{R}}(x, y), \quad \forall (x, y) \in X \times Y,$$

where $\mu_{\tilde{R}}(x, y)$ is the membership function of a fuzzy relation[7].

Definition 3 : Let $\tilde{f}(x)$ be a fuzzified function from $[a, b] \subseteq R$ to R , such that, $\tilde{f}(x)$, $\forall x \in [a, b]$ is a fuzzy number. Let α -level curves $f_{\alpha}^{-}(x)$ and $f_{\alpha}^{+}(x)$ be the curves on $[a, b]$ which satisfy the following estimates for any $x \in [a, b]$:

$$\mu_{\tilde{f}(x)}(f_{\alpha}^{-}(x)) = \mu_{\tilde{f}(x)}(f_{\alpha}^{+}(x)) = \alpha, \\ f_{\alpha}^{-}(x) \leq f_{\alpha}^{+}(x).$$

The integral of $\tilde{f}(x)$ over $[a, b]$ is then defined to be the fuzzy set as follows[7] :

$$\int_a^b \tilde{f}(x) dx = \left\{ \left(\left[\int_a^b f_{\alpha}^{-}(x) dx, \int_a^b f_{\alpha}^{+}(x) dx \right], \alpha \right) \right\}. \quad (2)$$

Sanchez[13] represented solutions of fuzzy equations using extended operations. Buckley and Qu[14] presented necessary and sufficient conditions for $\tilde{A} + \tilde{X} = \tilde{C}$ and $\tilde{A} \cdot \tilde{X} = \tilde{C}$ to have solutions for $\tilde{X}$, when $\tilde{A}$ and $\tilde{C}$ are arbitrary fuzzy subsets of the real numbers.

Theorem 1 : The fuzzy equation $\tilde{A} + \tilde{X} = \tilde{C}$ has a solution $\tilde{X}$ iff $c_1 - a_1 < c_2 - a_2 < c_3 - a_3$. If there is a solution, then $\tilde{X}$ is the TFN $(c_1 - a_1, c_2 - a_2, c_3 - a_3)$.

Proof. Taking α -cuts we obtain $a_L^{(\alpha)} + x_L^{(\alpha)} = c_L^{(\alpha)}$ and $a_R^{(\alpha)} + x_R^{(\alpha)} = c_R^{(\alpha)}$. Then $x_1 < x_2 < x_3$ and $1/(x_2 - x_1) > 0$, $1/(x_2 - x_3) < 0$ iff $c_1 - a_1 < c_2 - a_2 < c_3 - a_3$.

Definition 4 : Using Theorem 1, in order to obtain a solution $\tilde{X}$ for a fuzzy equation $\tilde{A} + \tilde{X} = \tilde{C}$, we define the operator $\ominus$ as follows :

$$\forall \alpha \in [0, 1] \\ \tilde{C} \ominus \tilde{A} = (c_1 - a_1, c_2 - a_2, c_3 - a_3). \quad (3)$$

Theorem 2 : We assume that $\tilde{A} > 0$ and $\tilde{C} \geq 0$.

Then there is a solution $\tilde{X}$ of the fuzzy equation $\tilde{A} \cdot \tilde{X} = \tilde{C}$ iff $a_1 c_2 > a_2 c_1$ and $a_3 c_2 < a_2 c_3$ when $\tilde{A} > 0$, $\tilde{C} \geq 0$.

Proof. Taking α -cuts we obtain $x_L^{(\alpha)} = c_L^{(\alpha)} / a_L^{(\alpha)}$, $x_R^{(\alpha)} = c_R^{(\alpha)} / a_R^{(\alpha)}$ when $\tilde{A} > 0$, $\tilde{C} \geq 0$. We obtain $1/(x_2 - x_1) > 0$ and $1/(x_2 - x_3) < 0$, iff the conditions stated above hold.

Definition 5: We assume that $\tilde{A} > 0$ and $\tilde{C} \geq 0$. Using Theorem 2, in order to obtain a solution $\tilde{X}$ for a fuzzy equation $\tilde{A} \cdot \tilde{X} = C$, we define the operator $\odot$ as follows:

$$\begin{aligned} \forall \alpha \in [0, 1] \\ \tilde{C} \odot \tilde{A} = \left(\frac{c_1}{a_1}, \frac{c_2}{a_2}, \frac{c_3}{a_3} \right), \\ \text{iff } a_1 c_2 > a_2 c_1 \text{ and } a_3 c_2 < a_2 c_3 \\ \text{when } \tilde{A} > 0 \text{ and } \tilde{C} \geq 0. \end{aligned} \quad (4)$$

Theorem 3: We assume that $\tilde{A} > 0$, $\tilde{C} \geq 0$. Then there is a solution $\tilde{X}$ of the fuzzy equation $\tilde{X} \odot \tilde{A} = \tilde{C}$ iff $c_1 a_3 < c_2 a_2 < c_3 a_1$ when $\tilde{A} > 0$ and $\tilde{C} \geq 0$.

Proof. Taking α -cuts we obtain $x_L^{(\alpha)} = c_L^{(\alpha)} a_L^{(\alpha)}$, $x_R^{(\alpha)} = c_R^{(\alpha)} a_R^{(\alpha)}$ when $\tilde{A} > 0$, $\tilde{C} \geq 0$. We obtain $1/(x_2 - x_1) > 0$ and $1/(x_2 - x_3) < 0$, iff the condition stated above hold.

Theorem 4: We assume that two TFNs, $\tilde{A} > 0$ and $\tilde{C} \geq 0$. When we attempt to obtain a solution $\tilde{X}$ for a fuzzy equation $\tilde{X} \odot \tilde{A} = \tilde{C}$ using Theorem 3, the solution $\tilde{X}$ does not become a TFN. Therefore, we will approximate the solution by a TFN. Then we define the operator $\odot$ as following:

$$\begin{aligned} \tilde{X} = \tilde{A} \odot = (a_3 c_1, a_2 c_2, a_1 c_3), \\ \text{iff } a_3 c_1 < a_2 c_2 < a_1 c_3, \text{ when } \tilde{A} > 0, \tilde{C} \geq 0. \end{aligned} \quad (5)$$

And we also assume that $0 < \tilde{A} < 1$ and $\tilde{C} \geq 0$. Then there is a solution $\tilde{X}$ of the fuzzy equation $\tilde{X} \odot (1 - \tilde{A}) = \tilde{C}$ iff $(1 - a_1) c_1 < (1 - a_2) c_2 < (1 - a_3) c_3$ when $\tilde{A} > 0$ and $\tilde{C} \geq 0$.

Proof. Taking α -cuts we obtain $x_L^{(\alpha)} = (1 - a_L^{(\alpha)}) c_L^{(\alpha)}$, $x_R^{(\alpha)} = (1 - a_R^{(\alpha)}) c_R^{(\alpha)}$ when $0 < \tilde{A} < 1$, $\tilde{C} \geq 0$. We can obtain $1/(x_2 - x_1) > 0$ and $1/(x_2 - x_3) < 0$, iff the condition stated above is held. By Theorem 4, we can obtain a fuzzy number solution $\tilde{X}$ of the fuzzy equation $\tilde{X} \odot (1 - \tilde{A}) = \tilde{C}$ and approximate α -cut solution by a approximate TFN, that is,

$$\begin{aligned} \tilde{X} \doteq (1 - \tilde{A}) \odot \tilde{C} \\ \text{iff } (1 - a_1) c_1 < (1 - a_2) c_2 < (1 - a_3) c_3 \\ \text{when } \tilde{A} > 0 \text{ and } \tilde{C} \geq 0. \end{aligned} \quad (6)$$

The following notations are used throughout this paper:

$\tilde{N}(t)$: number of customers in the system at time t
 $\tilde{A}(t)$: number of customers who arrived in the interval $[0, t]$

$\tilde{N}$: average number of customers in the system

$\tilde{W}$: average waiting time in the system

$\tilde{N}_q$: average number of customers in queue

$\tilde{W}_q$: average waiting time in queue

λ : average arrival rate of customers

μ : average service rate of customers

$\tilde{P}_n$: probability of n customers in the system.

Definition 6: Let $\bar{P}_n$ be the associate value of a TFN $\tilde{P}_n = (p_{n,1}, p_{n,2}, p_{n,3})$, where $p_{n,1} \geq 0$, $p_{n,3} \leq 1$. We define a fuzzy probability as a TFN $\tilde{P}_n$ of which associate value $\bar{P}_n$ is characterized as follows:

$$\sum_{n=0}^{\infty} \bar{P}_n = 1.$$

We use the mode $p_{n,2}$ as the associate value $\bar{P}_n$.

The membership of the fuzzy probability [27] $\tilde{P}_n$ is shown in Fig.1.

Now, we introduce fuzzy set theory into the Little's law [15-20]. The fuzzy number of customers in the system observed up to time t is as follows:

$$\tilde{N}_t = \frac{1}{t} \int_0^t \tilde{N}(\tau) d\tau, \quad (7)$$

Fig.1 Fuzzy probability

where a fuzzified function $\tilde{N}(\tau)$ and a integration of fuzzified function are defined by Definition 2, 3, respectively. Of course, $\tilde{N}_t$ depends on t , but $\tilde{N}_t$ tends to a steady-state $\tilde{N}$ according to increment of t , that is,

$$\tilde{N} = \lim_{t \rightarrow \infty} \tilde{N}_t. \quad (8)$$

The fuzzy arrival rate over the interval $[0, t]$ is as follows:

$$\tilde{\lambda}_t = \frac{\tilde{A}(t)}{t}. \quad (9)$$

For the steady-state, it is defined as

$$\tilde{\lambda} = \lim_{t \rightarrow \infty} \tilde{\lambda}_t, \quad (10)$$

with assuming that the limit exist. The fuzzy average time of the customer delay up to time t is similarly defined as

$$\tilde{W}_t \doteq \frac{1}{\tilde{A}(t)} \int_0^t \tilde{N}(\tau) d\tau, \quad (11)$$

that is, the fuzzy average time spent in the system per a customer up to time t . The steady-state fuzzy average time of the customer delay is defined as

$$\tilde{W} = \lim_{t \rightarrow \infty} \tilde{W}_t, \quad (12)$$

with assuming that the limit exists. Therefore, we have

$$\tilde{W} \doteq \frac{\tilde{N}}{\tilde{\lambda}}. \quad (13)$$

Eq.(13) is represented using α -cut level, as follows :

$$\begin{aligned} \forall \alpha \in [0, 1], \\ W(\alpha) = \left[\frac{N_L^{(\alpha)}}{\lambda_L^{(\alpha)}}, \frac{N_R^{(\alpha)}}{\lambda_R^{(\alpha)}} \right]. \end{aligned} \quad (14)$$

By deconvolution of eq.(14) we can compute the $\tilde{N}$ with interval of confidence as follows :

$$\begin{aligned} \forall \alpha \in [0, 1], \\ N(\alpha) = [\lambda_L^{(\alpha)} W_L^{(\alpha)}, \lambda_R^{(\alpha)} W_R^{(\alpha)}]. \end{aligned} \quad (15)$$

Similar formulas exist for $\tilde{N}_q$ and W_q , that is

$$\tilde{W}_q \doteq \frac{\tilde{N}_q}{\tilde{\lambda}}. \quad (16)$$

And we have

$$\begin{aligned} \forall \alpha \in [0, 1], \\ N_q(\alpha) = [\lambda_L^{(\alpha)} W_q^{(\alpha)}, \lambda_R^{(\alpha)} W_q^{(\alpha)}]. \end{aligned} \quad (17)$$

If the number of customers in the system, N , is not fuzzy number but crisp number eqs.(7), (8), (11), (13), and (14) can be as follows :

$$N_t = \frac{1}{t} \int_0^t N(\tau) d\tau, \quad (7)'$$

$$N = \lim_{t \rightarrow \infty} N_t. \quad (8)'$$

$$\tilde{W}_t = \frac{1}{\tilde{A}(t)} \int_0^t N(\tau) d\tau, \quad (11)'$$

$$\tilde{W} = \frac{N}{\tilde{\lambda}}, \quad (13)'$$

$$\forall \alpha \in [0, 1],$$

$$W(\alpha) = [W_L^{(\alpha)}, W_R^{(\alpha)}] = \left[\frac{N}{\lambda_L^{(\alpha)}}, \frac{N}{\lambda_R^{(\alpha)}} \right]. \quad (14)'$$

3. The Fuzzy M/M/1 Queueing System

The fuzzy M/M/1 queueing system consists of a single queueing station with a single server. Customers arrive according to a Poisson process with fuzzy rate $\tilde{\lambda}$, and the fuzzy probability distribution of the service time is exponential with fuzzy mean $1/\tilde{\mu}$ second. From one among of the properties of the Poisson process with fuzzy parameter $\tilde{\lambda}$ [5], we can get :

For every $t \geq 0$ and $\delta \geq 0$,

$$\tilde{P}\{N(t+\delta) - N(t) = 0\} = \sum_{n=0}^{\infty} \tilde{P}_n - \tilde{\lambda} \delta + o(\delta) \quad (18)$$

$$\tilde{P}\{N(t+\delta) - N(t) = 1\} = \tilde{\lambda} \delta + o(\delta) \quad (19)$$

$$\tilde{P}\{N(t+\delta) - N(t) \geq 2\} = o(\delta) \quad (20)$$

where we generically denoted by $o(\delta)$ a function of δ such that

$$\lim_{\delta \rightarrow 0} \frac{o(\delta)}{\delta} = 0.$$

3.1 Fuzzy Markov chain formulation

We could analyze the process $\tilde{N}(t)$ in terms of the discrete-time fuzzy Markov chains. Let us focus attention at the times

$$0, \delta, 2\delta, \dots, k\delta, \dots$$

where δ is a small positive number. We denote that N_k is the number of customers in the system at time $k\delta$. Let $\tilde{P}_{ij}$ denotes the corresponding fuzzy transition probabilities[21],

$$\tilde{P}_{ij} = \tilde{P}\{N_{k+1} = j | N_k = i\}, \quad 1 \leq i, j \leq n.$$

By using eqs.(18) through (20), one can show that

$$\tilde{P}_{00} = \sum_{i=0, j=0}^{\infty} \tilde{P}_{ij} - \tilde{\lambda} \delta + o(\delta) \quad (21)$$

$$\tilde{P}_{ii} = \sum_{i=0, j=0}^{\infty} \tilde{P}_{ij} - \tilde{\lambda} \delta - \tilde{\mu} \delta + o(\delta) \quad i \geq 1 \quad (22)$$

$$\tilde{P}_{i+1,i} = \tilde{\lambda} \delta + o(\delta) \quad i \geq 0 \quad (23)$$

$$\tilde{P}_{i-1,i} = \tilde{\mu} \delta + o(\delta) \quad i \geq 1 \quad (24)$$

$$\tilde{P}_{ij} = o(\delta) \quad i \text{ and } j \neq i, i+1, i-1. \quad (25)$$

The state transition diagram for the fuzzy Markov chain $\{N_k\}$ is shown in Fig. 2, where we have omitted the terms $o(\delta)$.

3.2 Derivation of the stationary distribution

Consider now the steady-state fuzzy probabilities

$$\tilde{P}_n = \lim_{t \rightarrow \infty} \tilde{P}\{N(t) = n\}.$$

Note that during any time interval, the total number of transitions from state n to $n+1$ must differ from the total number of transitions from $n+1$ to n by at most 1. Thus asymptotically, the frequency of transitions from n to $n+1$ is equal to the frequency of transitions from $n+1$ to n . Equivalently, the fuzzy probability that the system is in state n and makes a transition to $n+1$ in the next transition interval is the same as the fuzzy probability that the system is in state $n+1$ and makes a transition to n , i.e.,

$$\tilde{P}_n \tilde{\lambda} \delta + o(\delta) = \tilde{P}_{n+1} \tilde{\mu} \delta + o(\delta). \quad (26)$$

By taking the limit in this equation as $\delta \rightarrow 0$, we obtain

$$\tilde{P}_n \tilde{\lambda} = \tilde{P}_{n+1} \tilde{\mu}. \quad (27)$$

Eq.(27) can be also written using Theorem 2 as

$$\begin{aligned} \tilde{P}_{n+1} &= \tilde{\rho} \tilde{P}_n, \\ \text{iff } \mu_1 P_{n,2} \lambda_2 &> \mu_2 P_{n,1} \lambda_1 \\ \text{and } \mu_3 P_{n,2} \lambda_2 &< \mu_2 P_{n,3} \lambda_3, \end{aligned} \quad (28)$$

where

$$\tilde{\rho} = \tilde{\lambda} \oplus \tilde{\mu}. \quad (29)$$

we assume that $\lambda_1 \mu_2 < \lambda_2 \mu_1$ and $\lambda_2 \mu_3 < \lambda_3 \mu_2$, therefore $\tilde{\rho}$ is a TFN. It follows that

$$\tilde{P} \doteq \tilde{\rho}^n \tilde{P}_0. \quad (30)$$

If $\tilde{\rho} < 1$, the fuzzy probabilities $\tilde{P}_n$ are all positive, so

$$\sum_{n=0}^{\infty} \tilde{P}_n = \sum_{n=0}^{\infty} \tilde{\rho}^n \tilde{P}_0 \doteq \frac{\tilde{P}_0}{1 - \tilde{\rho}}. \quad (31)$$

Combining eqs.(30) and (31), we finally obtain

$$\tilde{P}_n = \tilde{\rho}^n \{ (1 - \tilde{\rho}) \odot \sum_{n=0}^{\infty} \tilde{P}_n \}, \quad n = 1, 2, 3, \dots \quad (32)$$

We can now calculate the fuzzy average number of customers in the system in steady-state :

$$\begin{aligned} \tilde{N} &\doteq \frac{\sum_{n=0}^{\infty} n \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n} = \frac{\sum_{n=0}^{\infty} n \tilde{\rho}^n \{ (1 - \tilde{\rho}) \odot \sum_{n=0}^{\infty} \tilde{P}_n \}}{\sum_{n=0}^{\infty} \tilde{P}_n} \\ &= \frac{\tilde{\rho} \sum_{n=0}^{\infty} \tilde{P}_n}{(1 - \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n}. \end{aligned} \quad (33)$$

The fuzzy average delay per customer is given by the fuzzy Little's law,

$$\tilde{W} = \frac{\tilde{N}}{\tilde{\lambda}} \doteq \frac{\tilde{\rho} \sum_{n=0}^{\infty} \tilde{P}_n}{\tilde{\lambda} (1 - \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n}. \quad (34)$$

If eq.(34) is represented by α -cut level, we have

$$\forall \alpha \in [0, 1], \quad W(\alpha) = \left[\frac{\rho_L^{(\alpha)} c_L^{(\alpha)}}{\lambda_R^{(\alpha)} c_R^{(\alpha)} (1 - \rho_L^{(\alpha)})}, \frac{\rho_R^{(\alpha)} c_R^{(\alpha)}}{\lambda_L^{(\alpha)} c_L^{(\alpha)} (1 - \rho_R^{(\alpha)})} \right].$$

The fuzzy average waiting time in queue $\tilde{W}_q$, is the fuzzy average delay $\tilde{W}$ less the fuzzy average service time $1/\tilde{\mu}$, so

$$\tilde{W}_q \doteq \frac{\tilde{\rho} \sum_{n=0}^{\infty} \tilde{P}_n}{\tilde{\lambda} (1 - \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n} - \frac{1}{\tilde{\mu}}. \quad (35)$$

In general, the approximation becomes worse according to iteration of the multiplication and the division of TFNs. That is, these operations for TFNs in a computational process may give rise to an error which may then be amplified in subsequent operations. It is important to note that approximation is never affected by a triplet, i.e., a_1, a_2 and a_3 .

4. Fuzzy Queueing Network Models

4.1 Fuzzy Open Central Server Model

One of the key performance concepts used in studying a computer system is the bottleneck device or server. The bottleneck of a system is the first server to saturate as the load on the system is increased. We can identify the bottleneck of the system if we know the service demands $\tilde{D}_k$, for $k =$

Fig.2 Discrete-time Markov chain with fuzzy transition probabilities for the M/M/1 system.

1, 2, ..., K. The bottleneck device is device j where j is the integer for which $\tilde{D}_j = \tilde{D}_{\max}$, where

$$\tilde{D}_{\max} = \max\{\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_K\}. \quad (36)$$

Now, for an application of the proposed fuzzy queueing model, we consider that the open central server system, shown in Fig. 3. The central server is, of course, the CPU. Lavenberg and Sauer in Lavenberg[12] use the central server queueing model to study memory management of a multiprogramming system. They were trying to optimize the performance of operating system. The open central server model can be used to model a system, such as a transaction processing system in which the system is not limited by the multiprogramming level. As shown in Fig. 3, there are $K-1$ I/O devices, each with its own queue, and each exponentially distributed with average service rate $\tilde{\mu}_k (k=2, 3, \dots, K)$.

The CPU was assumed to have an exponential distribution. The queue discipline is assumed to be first come first serve (FCFS) for the I/O devices. Upon completion of a CPU service, a transaction exits the system with fuzzy probability $\tilde{B}_1$ or enters service at I/O device k with fuzzy probability $\tilde{B}_k (k=2, 3, \dots, K)$. Upon completion of an I/O service, the transaction returns to the CPU queue for another cycle. An incoming arrow leading to the CPU queue is added to indicate the arriving traffic with average rate $\tilde{\lambda}$. The arrival pattern is assumed to have an exponential interarrival time with fuzzy parameter. We assume the system is not overload, so the throughput is also $\tilde{\lambda}$. Therefore, Jackson's theorem[6] can be applied to the system as stated in the following algorithm:

Algorithm 1 (Lavenberg's algorithm)

Consider the open central server system described above. Suppose we are given the average arrival rate $\tilde{\lambda}$, the average service time at each

Fig 3. Open central server system.

device, $\tilde{S}_k (k=2, 3, \dots, K)$, and branching probabilities $\tilde{B}_k (k=2, 3, \dots, K)$. The service demand, $\tilde{D}_k$, is the total amount of service that a customer needs at the k -th device. $\tilde{V}_k$ and $\tilde{\rho}_k$ are visit ratio and utilization at the k -th device, respectively. And N_k is the average number of transactions at the k -th device. Then we use Jackson's theorem to calculate the performance measures of the system as follows:

Step 1 [Calculate the demands, $\tilde{D}_k$]

If the demands are not known, use the Algorithm 2 to calculate

$$\tilde{D}_k, \quad k=1, 2, \dots, K.$$

Step 2 [Calculate device performance]

For $k=1, 2, \dots, K$, calculate

$$\tilde{\rho}_k \doteq \tilde{\lambda} \cdot \tilde{D}_k, \quad \tilde{N}_k \doteq \frac{\tilde{\rho}_k \sum_{n=0}^{\infty} \tilde{P}_n}{(1 - \tilde{\rho}_k) \sum_{n=0}^{\infty} \tilde{P}_n} \quad \text{and} \quad \tilde{W}_k \doteq \frac{\tilde{\lambda} \tilde{D}_k \sum_{n=0}^{\infty} \tilde{P}_n}{\tilde{\lambda} (1 - \tilde{\rho}_k) \sum_{n=0}^{\infty} \tilde{P}_n}.$$

Step 3 [Compute system performance measures]

Set average response time (average time in system) to

$$\tilde{W} = \sum_{k=1}^K \tilde{W}_k.$$

Set average number of transaction in the system to

$$\forall \alpha \in [0, 1],$$

$$N(\alpha) = [\lambda_R^{(\alpha)} W_L^{(\alpha)}, \lambda_L^{(\alpha)} W_R^{(\alpha)}].$$

The bottleneck device is device j , where j is the integer for which $\tilde{D}_j = \tilde{D}_{\max}$, where

$$\tilde{D}_{\max} = \max\{\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_K\}.$$

The maximum possible throughput is given by

$$\tilde{\lambda}_{\max} \doteq \frac{1}{\tilde{D}_{\max}}.$$

Algorithm 2

Consider the open central server system of Fig. 3. This algorithm is constructed with the parameters needed to use Lavenberg's algorithm.

Step 1 Set the visit ratio $\tilde{V}_1$ for the CPU to

$$\tilde{V}_1 \doteq \frac{1}{\tilde{B}_1}.$$

Step 2 For $k=2, 3, \dots, K$ calculate

$$\tilde{V}_k \doteq \tilde{B}_k \cdot \tilde{V}_1.$$

Step 3 [Calculate the demands, $\tilde{D}_k$]

Set

$$\tilde{D}_k \doteq \tilde{V}_k \cdot \tilde{S}_k, \quad k=1, 2, \dots, K.$$

4.2 Fuzzy BCMP Queueing Network Model

In their classic paper Baskett, Chandy, Muntz, and Palacios[11] generalized the Jackson queueing network to allow different classes of customers, each with different service requirements, as well as service time distributions other than exponential. They also allow open, closed, mixed networks of queues. Finally, they allow customers to change classes after completing service at a service center. There are four types of service centers, each with a different queueing discipline. There are four types of service centers are :

Type 1 : The queue discipline is first come first serve, and each customer has the same exponential service requirement. The service rate can be load dependent with $\tilde{\mu}(j)$ representing the service rate when there are j customers at the center.

Type 2 : The queue discipline is processor sharing. That is, each of n customers receives $1/n$ of the service effort. Each class of customer may have a distinct service time distribution.

Type 3 : In this service discipline there is one server for each customer. Each class of customer may have a distinct service time distribution. We will sometimes describe a Type 3 service center as a delay center.

Type 4 : The queue discipline is last come first serve preemptive repeat with a single server. Each class of customer may have a distinct service time distribution.

In our paper of fuzzy BCMP queueing network model, we will consider important special cases. For example, we do not allow customer class switching.

The closed BCMP model for one class is shown in Fig. 4. The N terminals are Type 3 servers (delay centers), the CPU can be considered either a Type 1(exponential) or Type 2(processor-sharing) server and the I/O devices are all Type 1 servers. The main memory of this system is not modeled. That is, there is no queue for requests to enter main computer memory. This means that no limit is imposed on the number of requests that can, simultaneously, reside in the central computer sys-

tem. The mean value analysis (MVA) algorithm for calculating the performance measures of the computer system of Fig. 4.

Fig.4 Closed BCMP computer system.

Algorithm MVA

Consider the closed BCMP system of Fig. 4. Suppose the mean think time $\tilde{E}[t] = \tilde{T}$ for each of the N terminals. $\tilde{W}_k[n]$ is the average total time that a customer spends at the k -th service center when there are n customers in the system. $\tilde{L}_k[n]$ is the average number of customers that find at k service center when there are n customers in the system. Each terminal is a Type 3 server. The CPU (device number $N+1$) is either a Type 1 or Type 2 server with service demand $\tilde{D} = \tilde{D}_{N+1}$ given. We are also given the service demands $\tilde{D}_k$, $k=N+2, N+3, \dots, K$ for the I/O devices. Then calculate the performance measures of the system as follows :

Step 1 [Initialize]

For $k=N+1, N+2, \dots, K$, set

$$L_k[0] = (0, 0, 0)$$

Step 2 [Iterate]

For $n=1, 2, \dots, N$, calculate

$$\tilde{W}_k[n] = \tilde{D}_k(1 + \tilde{L}_k[n-1]),$$

$$k=N+1, N+2, \dots, K,$$

$$\tilde{W}[n] = \sum_{k=N+1}^K \tilde{W}_k[n],$$

$$\forall \alpha \in [0, 1],$$

$$\lambda[n](\alpha) = \left[\frac{n}{\tilde{W}[n]_2^{(\alpha)} + T_2^{(\alpha)}}, \frac{n}{\tilde{W}[n]_1^{(\alpha)} + T_1^{(\alpha)}} \right],$$

$$L_k[n](\alpha) = [\lambda[n]_2^{(\alpha)} \tilde{W}_k[n]_1^{(\alpha)},$$

$$\lambda[n]_1^{(\alpha)} \tilde{W}_k[n]_2^{(\alpha)}],$$

$$k=N+1, N+2, \dots, K.$$

Step 3 [Compute performance measures]

Set system throughput to

$$\tilde{\lambda} = \tilde{\lambda}[N].$$

Set job response time (turnaround time) to

$$\forall \alpha \in [0, 1],$$

$$W(\alpha) = \left[\frac{N}{\lambda_2^{(\alpha)}} - T_1^{(\alpha)}, \frac{N}{\lambda_1^{(\alpha)}} - T_2^{(\alpha)} \right].$$

[We have already calculated $\tilde{W} = \tilde{W}[N]$ in Step 2, but we want to show the dependance of $\tilde{W}$ on $\tilde{\lambda}$, N and $\tilde{T}$ explicitly.]

Set server utilizations to

$$\tilde{\rho}_k = \tilde{\lambda} \tilde{D}_k, \quad k = N+1, N+2, \dots, K.$$

We calculated $\tilde{L}_k[N]$ and $\tilde{W}_k[N]$ for each server in the last iteration of Step 2.

Table 1 Server utilization and response time

server k	utilization $\tilde{\rho}_k$	response time $\tilde{W}_k$ (sec.)
1	(0.504, 0.600, 0.672)	(3.629, 7.500, 15.176)
2	(0.162, 0.200, 0.231)	(0.690, 1.250, 2.225)
3	(0.324, 0.400, 0.441)	(1.712, 3.333, 5.844)
4	(0.702, 0.800, 0.861)	(8.413, 20.000, 45.833)

In Step 3, from Table 1 we calculate the average transaction response time

$$\begin{aligned} \tilde{W} &= \tilde{W}_1 + \tilde{W}_2 + \tilde{W}_3 + \tilde{W}_4 \\ &= (14.444, 32.083, 69.128) \text{ seconds,} \end{aligned}$$

and the average number of transactions being processed is

$$\tilde{N} \doteq (3.300, 6.417, 12.443).$$

The bottleneck is the third disk driver, so the TFN of maximum possible throughput, $\tilde{\lambda}_{\max}$ is (0.244, 0.25, 0.256) transactions per second.

In this numerical example, we applied fuzzy set theory to the transaction processing computer system that can be modeled as the open central server model with one CPU and three disks. The proposed fuzzy approach can get the reasonable performance measures suitable for all cases ranging from the minimum value to the maximum value. Also, we can verify the reality and the flexibility because the results, *i.e.*, the performance measures maintain the fuzziness of input information.

5. Numerical Examples

Now, we show the numerical examples to demonstrate the proposed fuzzy open central system and fuzzy BCMP queueing network model.

5.1 Example of Fuzzy Open Central Server Model

We investigate a transaction processing computer system that can be modeled as an open central server model with one CPU and three disks as shown in Fig.3. We find that the TFN of average transaction arrival rate, $\tilde{\lambda}$ is (0.18, 0.2, 0.21) transactions per second with an TFN of average CPU service requirement of (2.8, 3, 3.2) seconds per transaction. The TFNs of average I/O service requirements are (0.9, 1, 1.1), (1.8, 2, 2.1), and (3.9, 4, 4.1) seconds, respectively.

Let us step through Lavenberg's algorithm. In Step 1, we set service demands for each device that is, $\tilde{D}_1 = (2.8, 3, 3.2)$, $\tilde{D}_2 = (0.9, 1, 1.1)$, $\tilde{D}_3 = (1.8, 2, 2.1)$, and $\tilde{D}_4 = (3.9, 4, 4.1)$ seconds.

In Step 2, we compute average number of transactions, utilizations and average response time for each device that is, $\tilde{N}_1 = (0.762, 1.5, 2.732)$, $\tilde{N}_2 = (0.145, 0.25, 0.402)$, $\tilde{N}_3 = (0.359, 0.667, 1.052)$, and $\tilde{N}_4 = (1.767, 4, 8.259)$ and $\tilde{\rho}_k, \tilde{W}_k$ ($k=1, 2, 3, 4$) as shown in the following Table 1.

5.2 Example of Fuzzy BCMP Queueing Network Model

We have an interactive computer system consisting of 50 active terminals connected to a central computer system as shown in Fig. 4. We find that we can model the system by the queueing model described in Algorithm MVA with three I/O devices. We find that the average think time is (14,15,17) seconds, the mean CPU service demand per interaction (job) is (0.13,0.15,0.18) seconds, and the mean total service demand per interaction on the three I/O devices is (0.01, 0.02, 0.03), (0.03, 0.04, 0.05), and (0.04, 0.05, 0.06) seconds, respectively.

Let us step through MVA algorithm.

In Step 1, we initialize to $\tilde{L}_{s1}[0] = \tilde{L}_{s2}[0] = \tilde{L}_{s3}$

Table 2 Calculations for Example 1

terminal n	server k	throughput $\tilde{\lambda}_{k[n]}$	response time $\tilde{W}_{k[n]}(\text{sec.})$	number of customers $\tilde{L}_{k[n]}$
1	51	(0.0577, 0.0655, 0.0704)	(0.1300, 0.1500, 0.1800)	(0.0092, 0.0098, 0.0104)
	52		(0.0100, 0.0200, 0.0300)	(0.0007, 0.0013, 0.0017)
	53		(0.0300, 0.0400, 0.0500)	(0.0021, 0.0026, 0.0029)
	54		(0.0400, 0.0500, 0.0600)	(0.0028, 0.0033, 0.0035)
2	51	(0.1155, 0.1310, 0.1407)	(0.1312, 0.1515, 0.1819)	(0.0185, 0.0199, 0.0210)
	52		(0.0100, 0.0200, 0.0301)	(0.0014, 0.0026, 0.0035)
	53		(0.0301, 0.0401, 0.0501)	(0.0042, 0.0053, 0.0056)
	54		(0.0401, 0.0502, 0.0602)	(0.0056, 0.0066, 0.0070)
⋮	⋮	⋮	⋮	⋮
	⋮		⋮	⋮
	⋮		⋮	⋮
	⋮		⋮	⋮
50	51	(2.8541, 3.2449, 3.4913)	(0.2314, 0.2825, 0.3558)	(0.8077, 0.9165, 1.0156)
	52		(0.0104, 0.0214, 0.0327)	(0.0361, 0.0693, 0.0935)
	53		(0.1167, 0.1487, 0.1658)	(0.1167, 0.1487, 0.1658)
	54		(0.1618, 0.1928, 0.2056)	(0.1618, 0.1928, 0.2056)

$$[0] = \tilde{L}_{54}[0] = (0, 0, 0).$$

In Step 2, we calculate average response time and average number of interaction for each device, a part of which are shown in Table 2.

In Step 3, we find that system throughput $\tilde{\lambda} = (2.8541, 3.2448, 3.4913)$ interaction per second, response time $\tilde{W} = (0.3215, 0.4090, 0.5187)$ seconds, utilization of CPU $\tilde{\rho}_{\text{CPU}} = (0.3710, 0.4867, 0.6284)$, and the average number of interaction in the central computer system $\tilde{L} = (1.1223, 1.3273, 1.4805)$.

In this numerical example, we applied fuzzy set theory to the interactive computer system that can be modeled as the closed single class BCMP model with 50 active terminals and three disks. The practical merit of this approach is that there is no need to force designer to give a probability distribution. And fuzzy approach can get a reasonable solution suitable for all cases ranging from the pessimistic case to the optimistic case.

When the crisp data of the conventional BCMP queueing model are the modal values of the above input data, the performance measures of this model get the modal values of the above results. Therefore, we can verify the reality and the flexibility because the results *i.e.* the performance measures maintain the fuzziness of input information.

6. Conclusion

In this paper, a fuzzy M/M/1 system that is applied the fuzzy Little's law was proposed. The performance measures for fuzzy open central server

network model and fuzzy closed single class BCMP queueing network model were analyzed by Lavenberg's algorithm and MVA algorithm, respectively. Therefore we can get the more realistic performance measures when some data of queueing network models are ambiguous.

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Performance Evaluation of Network Models based on Fuzzy Queueing System

by

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Abstract :

The purpose of this paper is to combine the ability of fuzzy set to represent more realistic situations with the well-established traditional queueing network model problem. We are forced to employ subjective probabilities when there is no information about a model or some parameters of a model are vague. The information and data are very fuzzy, because they are frequently very little, and may be sometimes obtained from experts subjectively. We propose a fuzzy M/M/1 queueing system which is derived by fuzzification of ordinary M/M/1 queueing system. We also apply fuzzy set theory to the open central server network model with the fuzzy queues and the closed single class BCMP network model in which each node consists of fuzzy queue. Thus, we represent the characteristic and performance of the open central server network model and the closed single class BCMP network model based on the proposed fuzzy queueing system.

Keywords : Fuzzy Set Theory, M/M/1 Queueing Model, Markov Chains, Open Central Server Model, Closed Single Class BCMP Model

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ファジィ待ち行列システムによるネットワークモデルの性能評価

コンピュータシステムの待ち行列モデルによる性能評価では、従来、確率過程に基づいて、到着時間やサービス時間などを確定値として取り扱ってきた。しかし、現実には、これらの値を確定値で表現するよりはむしろ、「だいたいこれぐらい」というようなあいまいな値で表現するほうが適切な場合が多い。

本論文ではこのあいまいさを表現するために三角型ファジィ数を導入する。まず、ファジィ方程式の解法を示し、次いで従来のM/M/1待ち行列をファジィ化して、コンピュータシステムの待ち行列ネットワークモデル、すなわちここでは解放型セントラル・サーバ・モデルと閉鎖型単一クラスBCMPモデルに適用する。これにより、従来のコンピュータシステムの待ち行列モデルより实际的で柔軟なコンピュータシステムモデルの性能評価を行うことが可能になる。さらに、数値例を用いて提案する手法の有効性を確認する。

キーワード：ファジィ理論、M/M/1待ち行列モデル、マルコフ連鎖、解放型セントラル・サーバ・モデル、閉鎖型BCMPネットワークモデル