LETTERS TO THE EDITORS

Low Latitude Whistler Echoes and an Estimate of Electron Density in the Outer Ionosphere

This letter consists of two parts; the observation of low-latitude whistler echoes, and an estimate of electron density in the outer ionosphere from the whistler dispersions.

Maeda and Kimura (1956) have calculated the ray path of whistler based on Fermat's principle, and concluded that the down-coming point of low-latitude whistler is higher in latitude than the origin point in the other hemisphere (Polar creep). According to this, the whistler echoes may not be observed at low latitudes or else the dispersion ratio of the whistler echo to 1-hop whistler may be greater than 3.

Since 1956, many whistler echoes have been observed in low latitude region. Fig. 1 is an example of whistler echo with the dispersion ratio of 3.8 observed at Hiraiso, Japan at 0402 JST, Feb. 5, 1964. This is just the case expected by Maeda and Kimura (1956). Fig. 2 shows a distribution of dispersion ratio of whistler echoes (3-hops) to short whistlers (1-hop) observed at Hiraiso for the period November, 1963-March, 1964. The range of dispersion ratio lies between 2.5 and 3.8. The maximum occurs at 2.8 and an average dispersion ratio is 3.0, where the experimental error in measuring the dispersion ratio is ± 0.2 .

As has been suggested by Storey (1953), whistler echoes propagate along the geomagnetic line of force. This is consistent with the conclusion obtained by Cartwright (1964) that whistlers are loosely guided by the geomagnetic field even at low latitudes. These results may suggest that field-aligned ionization ducts within which the whistler propagates, exist when whistler echoes are received in low latitude.

Field-aligned ionization irregularities have been detected in the altitude range from 400 km to 1000 km during a spread-F condition by the topside sounder (Calvert et al, 1962, and

Fig. 1. Whistler echo with $D_2/D_1=3.8$ observed at Hiraiso at 0402 JST on February 5, 1964.

Fig. 2. Occurrence distribution of dispersion ratio of whistler echoes (3-hops) to short whistlers (1-hop) observed at Hiraiso, November, 1963-March, 1964.

Sayers et al, 1963). The whistler echoes with the dispersion ratio less than 3 may be explained by a model that after 1-hop whistler is reflected, it propagates obliquely and then it is trapped in the field-aligned ionization duct on inner side of the first hop whistler path. The whistler in this case is a 'mixed path' whistler (Morgan et al., 1959; Helliwell, 1964). The field-aligned sheets of ionization have been observed most commonly near local midnight and local noon at low magnetic latitudes by the topside sounder (Muldrew, 1963), although the whistler echoes at Hiraiso for November, 1963-March, 1964 occurred most often in the early morning around 03-05LT, when f_0F_2 took the lowest value.

The whistler activity in low latitude is usually small during the daytime because of increase of D-layer absorption, and there is no good estimation of day-time electron density in the outer ionosphere. Fortunately, on Jan. 14-16, 1964, whistlers were received all day.

Therefore, this material provides an opportunity to estimate the diurnal variation of electron density in the outer ionosphere.

Fig. 3 shows diurnal variations of whistler dispersion at Hiraiso for January 12-13, 1959 (Obayashi and Nemoto, 1959) and for January 14-16 1964, where vertical rods represent diffuse whistlers and circles pure whistlers. A plateau of the diurnal variation of whistler dispersion between 11-15LT agrees well with that of electron density above 400 km height observed by the topside sounder at the same local time (Bauer et al, 1964). The difference of the diurnal variation of dispersion between 1959 and 1964 in magnitude may indicate a variation of total electron contents along the whistler path due to the variation of solar activity.

The diurnal variation of dispersion has a minimum of $14 \sqrt{S}$ around 04LT and a plateau of $28 \sqrt{S}$ between 11 and 15LT. Using these two values we estimate a diurnal variation of average electron density between 500 km height and the top of whistler path (1540 km).

The whistler dispersion is given by

Fig. 3. Diurnal variation of the whistler dispersion observed at Hiraiso, January 14-16, 1964, and January 12-13, 1959.

$$D = \frac{1}{2c} \int \frac{f_o}{\sqrt{f_H}} ds \quad (1)$$

where $f_o^2 = \frac{Ne^2}{\epsilon_0 \pi m}$ denotes the electron plasma frequency, $f_H = \frac{\mu_0 H e}{2\pi m}$ the electron gyrofrequency, and ds the path element along the geomagnetic field. The geomagnetic line of force and the geomagnetic field (centered dipole) are given by

$$r = r_e \cdot \cos^2 \theta \quad (2)$$

and

$$H = H_0 \cdot \left(\frac{r_0}{r} \right)^3 \cdot (1 + 3 \sin^2 \theta)^{\frac{1}{2}} \quad (3)$$

respectively, where r is the geocentric distance, r_e the distance between the top of the geomagnetic line of force and the earth's center, θ the geomagnetic latitude, and $H_0 = 0.3$ gauss the geomagnetic field strength on the earth's surface at the geomagnetic equator. Substituting (2) and (3) into (1), and assuming that the electron density is uniform in the integral interval, we have

$$D = \sqrt{\frac{eN}{2}} \int \frac{1}{\sqrt{H}} ds = \left(\frac{e \cdot r_e^5}{2H_0 r_0^3} \right)^{\frac{1}{2}} \cdot \sqrt{N} \int \cos^4 \theta (1 + 3 \sin^2 \theta)^{\frac{1}{2}} \cdot d\theta \quad (4)$$

For a geomagnetic line of force starting at Hiraiso (geomag. lat. $26^\circ 2' N$),

$$D = 2.25 \times 10^{-1} \sqrt{N} \int \cos^4 \theta (1 - 0.15 \cos 2\theta) d\theta = 2.25 \times 10^{-1} \sqrt{N} \int \varphi(\theta) d\theta \quad (5)$$

The dispersion is conveniently divided into two parts as $D = D_i + D_o$, where D_i is a contribution from the ionosphere between 100 km and 500 km height and D_o is that from the outer ionosphere between 500 km and 1540 km height. Geomagnetic latitudes at 100 km and at 500 km height on the geomagnetic line of force passing through Hiraiso are $\theta_1 = 25^\circ 17'$

and $\theta_2 = 21^\circ 17'$ respectively. Thus

$$D = 2.25 \times 10^{-1} \left\{ (\sqrt{N_{in}} + \sqrt{N_{is}}) \int_{\theta_2}^{\theta_1} \varphi(\theta) d\theta + 2\sqrt{N_0} \int_0^{\theta_2} \varphi(\theta) d\theta \right\} \quad (6)$$

where N_{in} , N_{is} and N_0 are the average electron density of the northern ionosphere, southern ionosphere, and outer ionosphere, respectively.

We adopt the parabolic distribution of electron density for 100-400 km height and the exponential distribution of electron density derived by Otsu et al (1963) for 400-500 km height. Therefore an average electron density of ionosphere in each hemisphere is expressed as

$$N_i = \frac{1}{400} \left[N_m \int_0^{300} \frac{2Z}{Z_m} \left(1 - \frac{Z}{2Z_m} \right) dZ + N_{400} \int_{6770}^{7910} \exp \left\{ -0.002(r - 6770) \right\} dr \right] \quad (7),$$

where $N_m = 1.24 \times 10^4 (f_o F_2)^2 / \text{cm}^3$ is the maximum electron density of the F -layer, $f_o F_2$ measured in Mc/s, $Z_m = h_p F_2 - 100$ km, Z an altitude measured from 100 km height, and N_{400} an electron density at 400 km height calculated by the parabolic distribution. Ionospheric data used are shown in Table 1, which have been obtained at Kokubunji (mag. lat. $25.^\circ 4$ N, mag. long. $205.^\circ 4$) and Townsville (Mag. lat. $28.^\circ 5$ S, Mag. long. $218.^\circ 8$).

Table 1. Ionospheric data

	$f_o F_2$	$h_p F_2$	N_m / cm^3	N_{400} / cm^3
1964, Jan 16, 13h, 150° EMT, T	7.7 Mc/s	320 km	7.35×10^5	6.38×10^5
Jan. 16, 12h 135° EMT, K	6.4 Mc/s	290 km	5.08×10^5	3.38×10^5
Jan. 15, 05h, 150° EMT, T	3.2 Mc/s	310 km	1.27×10^5	1.04×10^5
Jan. 15, 04h, 135° EMT, K	2.3 Mc/s	300 km	6.56×10^4	4.92×10^4

Then we can calculate an average electron density in the outer ionosphere as

$$N_0 = \left\{ \frac{D - D_i}{4.50 \times 10^{-1} \int_0^{\theta_2} \varphi(\theta) d\theta} \right\}^2 \quad (8),$$

using D_i obtained from the first term of (6). Results of the computation are given in Table 2.

Table 2. An estimation of a average verage electron density in the outer ionosphere obtained from whistler

	D	D_i	D_0	N_0 / cm^3
Noon on Jan. 16	28	15.2	12.8	8.37×10^3
04LT on Jan. 15	14	6.0	8.0	3.32×10^3

The values of N_0 agree well with the electron density between 800 and 1200 km height measured by the incoherent scatter technique (Bowles et al, 1963).

By comparing a ratio of N_0 at noon to N_0 at 04LT with that of the ionosphere, a degree of diurnal variation of N_0 relative to the ionosphere is inferred. Relations among the ratios

are as follows;

$$\frac{N_o(12LT)}{N_o(04LT)} = 2.5, \quad \frac{N_m(12LT)}{N_m(04LT)} = 3.1 \times \frac{N_o(12LT)}{N_o(04LT)} \quad \text{and} \quad \frac{N_{in}(12LT)}{N_{in}(04LT)} = 3.0 \times \frac{N_o(12LT)}{N_o(04LT)}$$

for the northern ionosphere (Kokubunji), and

$$\frac{N_m(13LT)}{N_m(05LT)} = 2.3 \times \frac{N_o(12LT)}{N_o(04LT)} \quad \text{and} \quad \frac{N_{is}(13LT)}{N_{is}(05LT)} = 2.3 \times \frac{N_o(12LT)}{N_o(04LT)}$$

for the southern ionosphere (Townsville).

It is estimated that the diurnal variation of average electron density between 500 km and 1540 km height is about one-third of that of electron density of the northern (winter) ionosphere and about an half of that of electron density of the southern (summer) ionosphere.

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References

- Bauer, S.J. and Blumle, L.J., Mean diurnal variation of the topside ionosphere at mid-latitudes, *J. Geophys. Res.*, **69**, 3613, 1964.
- Bowles, K.L. et al., Profiles of electron density over the magnetic equator obtained using the incoherent scatter technique, *NBS Tech. Note* 169, 1963.
- Calvert, W., VanZandt, T.E., Knecht, R.W. and Goe, G.B., Evidence for field-aligned ionization irregularities between 400 and 1000 km above the earth's surface, *Proc. International Conference on the Ionosphere*, 324, 1962.
- Cartwright, D.C., Rocket observations of very low frequency radio noise at night, *Planet. Space Sci.*, **12**, 11, 1964.
- Helliwell, R.A., Whistlers and related ionospheric phenomena, *Stanford Univ. Press*, in press, 1964.
- Maeda, K. and Kimura, I., A theoretical investigation on the propagation path of the whistling atmospherics, *Rept. Ionos. Res. Japan* **10**, 105, 1956.
- Morgan, M.G., Curtis, H.W., and Johnson, W.C., Path combinations in whistler echoes, *Proc. IRE*, **47**(2), 328, 1959.
- Muldrew, K.B., Radio propagation along magnetic field-aligned sheets of ionization observed by the Alouette topside sounder, *J. Geophys. Res.*, **68**, 5355, 1963.
- Obayashi, T. and Nemoto, C., Private communication, 1959.
- Outsu, J., Iwai, A. and Tanaka, Y., Annual variations of whistler occurrence rate in middle and low latitudes since July, 1957, *Bull. Res. Inst. Atmosph. Nagoya Univ.*, **13**, 11, 1963.
- Sayers, J., Rothwell, P. and Wager, J.H., Field aligned strata in the ionization above the ionospheric F_2 layer, *Nature*, **198**, 230, 1963.
- Storey, L.R.O., An investigation of whistling atmospherics, *Phil. Trans. Roy. Soc. A.*, **246**, 113, 1953.

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