Westward Drift of the Equatorial Component of the Earth's Magnetic Dipole

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Abstract

A theory of free magneto-hydrodynamic oscillation, which gives rise to an S_1^1 type magnetic field, superposing on the fluid dynamo in the earth's core is advanced. It is proved that an S_1^1 field travelling eastwards relative to the core material is possible to exist. If the general westward drift of the core relative to the mantle is taken into account, the drift speed of the S_1^1 field relative to the mantle becomes slow though still westward. The speed thus estimated roughly agrees with the observed velocity for the drift of the equatorial component of the geomagnetic dipole.

1. Introduction

Nagata (1965) recently emphasized a tendency of westward rotation for the equatorial component of the geomagnetic dipole although the rotation speed seems to be much smaller than that of the westward drift of the non-dipole part of the geomagnetic field. He estimated that the angular velocity amounts to only 0.05 degree/yr for the rotation of the equatorial dipole.

According to Nagata, who classified the geomagnetic secular variation into five major categories; i.e. (a) decrease in the moment of the geomagnetic dipole with a rate 0.05%/yr, (b) westward rotation of the equatorial component of the earth's magnetic dipole with an angular speed of 0.05 degree/yr (c) northward shift of the geomagnetic dipole with a speed of 2 km/yr, (d) westward drift of the non-dipole field with an angular speed of 0.2 degree/yr, and (e) growth and decay of the non-dipole field having a rate of 10–100 γ /yr, these features of geomagnetic secular variation seem likely to be interpreted by magneto-hydrodynamic processes taking place within the earth's core in association with a dynamo action which maintains the main portion of the geomagnetic field.

A number of theories have so far been put forward in order to account for the major features of geomagnetic secular variation as reviewed by Nagata (1962, 1965) and Rikitake (1966). Although some of the theories are so crude that discussion was sometimes only qualitative, it has become widely supported that some sort of magneto-hydrodynamic effect would be the main cause of the secular variation. Among the five points cited in the above, however, no adequate theory on (b) has ever been tried.

In his detailed study of the westward drift of the non-dipole field, Yukutake (1962) found that the drift velocity excluding the equatorial component of the dipole amounts to 0.2 degree/yr. He also showed that, if the equatorial component of the dipole is included in the

estimate of the drift velocity, a very small drift velocity amounting to only 0.062 degree/yr is to be obtained. On the basis of the difference in drift velocity between the two fields, he suggested that the origin of the equatorial part of the dipole might be completely different from that of the non-dipole field.

Nagata (1962) examined the westward drift velocity from the Gauss coefficients and their time-derivatives for epoch 1958.5. In Table 1 are reproduced Nagata's result which clearly indicates that the velocity for the equatorial dipole part ($n=1$, $m=1$) is smaller than those for the other non-dipole field.

Table 1. Westward drift velocities in units of degree/yr obtained from the Gauss coefficients and their time-derivatives for 1958.5. (After Nagata)

n	m	Drift velocity
1	1	0.055
2	1	0.26
2	2	0.32
3	1	0.35
3	2	-0.083
3	3	0.14
4	1	0.19
4	2	0.10
4	3	0.17
4	4	0.14

Hide (1966) also summarized the drift velocity of spherical harmonic constituents for various epochs and confirmed the smallness of that of the $n=1$, $m=1$ component.

In the light of the above studies, it appears to the writer that the westward drift velocity of the equatorial component of the geomagnetic dipole is definitely smaller than that of the non-dipole field by a factor 3 or so. As pointed out by Nagata (1965), however, no adequate theory which accounts for the difference in drift velocity between the equatorial dipole and the non-dipole field has been put forward yet.

Rikitake (1955 a, 1955 b, 1956 a, 1956 b) tried to study magneto-hydrodynamic oscillation of the axial dipole type (S_1 field). It transpired, however, that the effect of the Coriolis force due to the earth's rotation is so strong that free oscillation would be prohibited provided a steady or quasi-steady dynamo as suggested by Bullard and Gellman (1954) is supposed to be at work in the core. Nagata and Rikitake (1963) showed, however, the possibility of magneto-hydrodynamic oscillation of the axial quadrupole type (S_2) superimposed on the Bullard-Gellman dynamo, such an oscillation being applied to the interpretation of the northward shift of the dipole though not quite successful.

Hide (1966) studied magneto-hydrodynamic oscillations of a rotating spherical shell of an incompressible fluid pervaded by a toroidal magnetic field by means of a method similar to that for tidal oscillations in the atmosphere. He suggested that dispersive magneto-hydrodynamic waves derived from the study may account for the observed secular variation

including the westward drift.

This paper aims at extending the theory of magneto-hydrodynamic oscillation superimposed on the dynamo to the field of the equatorial dipole type (S_1^1 field) and also at seeing whether or not a drifting field of that type is possible. If we find a drifting field, direction and speed of the drift are of great interest.

In Section 2 will be outlined the theory of magneto-hydrodynamic oscillation which more or less follows the writer's previous studies on the S_1 and S_2 fields. The theory will reduce to an eigen-value problem solving which a drift velocity of the S_1^1 field will actually be obtained in Section 3. Section 4 will be reserved for discussion about the evaluation of the present theory.

2. Theory

Let us assume a steady or nearly steady dynamo as described by Bullard and Gellman (1954) in the earth's core. When a small disturbance of magnetic field is superimposed on such a system, it is governed by equation

$$\left\{ \frac{\partial}{\partial t} - (4\pi\sigma)^{-1} \nabla^2 \right\} \vec{h} = \text{curl}(\vec{V}_0 \wedge \vec{h}) + \text{curl}(\vec{v} \wedge \vec{H}_0), \quad (1)$$

which is reduced from Maxwell's equations on conditions that displacement current is ignored and that the magnetic permeability is unity in the electromagnetic unit. In (1), t is time and σ electrical conductivity, while $\vec{V}_0$ and $\vec{H}_0$ are the steady velocity and magnetic field of the dynamo in the core. A velocity $\vec{v}$ is set up by the interaction between $\vec{h}$ and $\vec{H}_0$. $\vec{h}$ and $\vec{v}$ are assumed as the first order small quantities, so that the second order quantities like $|\vec{h}| \cdot |\vec{v}|$ are ignored.

The equation of motion, which controls $\vec{v}$, can be written as

$$\rho \frac{\partial \vec{v}}{\partial t} + 2\rho(\vec{\omega}_0 \wedge \vec{v}) = (4\pi)^{-1}(\text{curl} \vec{h} \wedge \vec{H}_0 + \text{curl} \vec{H}_0 \wedge \vec{h}) - \text{grad } p, \quad (2)$$

provided the inertia term is disregarded. $\vec{\omega}_0$, ρ and p denote respectively the rotation vector about the $\theta=0$ axis, density and pressure.

Assuming incompressibility of the core material and taking divergence of both the sides of (2), we obtain

$$\nabla^2 p = (4\pi)^{-1}(\vec{H}_0 \cdot \nabla^2 \vec{h} + \vec{h} \cdot \nabla^2 \vec{H}_0 + 2 \text{curl} \vec{h} \cdot \text{curl} \vec{H}_0) + 2\rho \vec{\omega}_0 \cdot \text{curl} \vec{v}. \quad (3)$$

We are now in a position to rewrite the righthand-side of (3) in a form suitable for the present problem. Since our concern in this paper is time-dependent behaviour of the S_1^1 magnetic field, $\vec{h}$ is assumed to be given by

$$\vec{h} = \vec{h}_c + \vec{h}_s, \quad (4)$$

where

$$\vec{h}_c = \begin{cases} -2s_c P_1^1 \cos \phi \\ -\left(r \frac{\partial s_c}{\partial r} + 2s_c\right) \frac{dP_1^1}{d\theta} \cos \phi \end{cases} \quad \vec{h}_s = \begin{cases} -2s_s P_1^1 \sin \phi \\ -\left(r \frac{\partial s_s}{\partial r} + 2s_s\right) \frac{dP_1^1}{d\theta} \sin \phi \end{cases} \quad (4a)$$

$$\left| \left(r \frac{\partial s_c}{\partial r} + 2s_c \right) \frac{P_1^1}{\sin \theta} \sin \phi, \quad \left| - \left(r \frac{\partial s_s}{\partial r} + 2s_s \right) \frac{P_1^1}{\sin \theta} \cos \phi, \right.$$

in which $P_1^1(\cos \theta)$ is the surface spherical harmonic of degree 1 and order 1. s_c and s_s are generally functions of r and t . S_n^m and T_n^m are in this paper used for indicating poloidal and toroidal fields specified by a spherical function having degree n and order m . General expressions for S_n^m and T_n^m fields can be found elsewhere (i.e. Rikitake (1966)).

As for the steady magnetic field, we may assume S_1 , T_2 , T_2^{2c} and T_2^{2s} fields which appear in the simplest dynamo studied by Bullard and Gellman (1954). It has been surmised that the intensity of the T_2 field would amount to the order of 100 Gauss, that of the S_1 field to a few Gauss, and also that those of the T_2^{2c} and T_2^{2s} fields are intermediate. It would therefore be justified to neglect the S_1 , T_2^{2c} and T_2^{2s} fields in the following estimation of the terms appearing on the righthand-side of (3). In the case of the previous discussions about the time-dependent behaviour of the S_1 and S_2 fields (Rikitake, 1955 a. 1956 b, Nagata and Rikitake, 1963), however, such a neglect was not possible because the interaction between the disturbance field and the steady S_1 , T_2^{2c} and T_2^{2s} fields was more important rather than the T_2 field for giving rise to motions which in turn excite the fields of the S_1 and S_2 types.

The T_2 field being written as

$$\vec{H}_0 = \begin{cases} 0 \\ 0 \\ a^{-2} T_2 r^2 \frac{dP_2}{d\theta}, \end{cases} \quad (5)$$

where a is the radius of the core, T_2 is assumed to be independent of r for the sake of simplicity. From (4) and (5), it is seen that all the electromagnetic interactions occurring on the righthand-side of (3) involve only $P_2^1 \cos \phi$ and $P_2^1 \sin \phi$.

The simplest dynamo studied by Bullard and Gellman (1954) involves two types of velocities, T_1 and S_2^{2c} . Bullard and Gellman showed, however, that the velocity of the T_1 motion should be larger than that of the S_2^{2c} motion by a factor 10–100 in order to maintain a steady state. In view of this, we may ignore the S_2^{2c} motion in the present discussion. In that case the steady velocity is given by

$$\vec{V}_0 = \begin{cases} 0 \\ 0 \\ a^{-1} V_1 r \frac{dP_1}{d\theta}, \end{cases} \quad (6)$$

where V_1 is assumed to be independent of r . The interaction between the magnetic fields and the electric currents gives rise to S_2^{1c} , S_2^{1s} , T_3^{1c} and T_3^{1s} motions. We select only the first two which set up magnetic fields of the S_1^{1c} and S_1^{1s} types through the interaction with the steady magnetic field. We therefore put

$$\vec{v} = \vec{v}_c + \vec{v}_s, \quad (7)$$

where

$$\vec{v}_c = \begin{cases} -6\xi_c r P_2^1 \cos \phi \\ -\left(r \frac{\partial \xi_c}{\partial r} + 3\xi_c\right) r \frac{dP_2^1}{d\theta} \cos \phi \\ \left(r \frac{\partial \xi_c}{\partial r} + 3\xi_c\right) r \frac{P_2^1}{\sin \theta} \sin \phi, \end{cases} \quad \vec{v}_s = \begin{cases} -6\xi_s r P_2^1 \sin \phi \\ -\left(r \frac{\partial \xi_s}{\partial r} + 3\xi_s\right) r \frac{dP_2^1}{d\theta} \sin \phi \\ -\left(r \frac{\partial \xi_s}{\partial r} + 3\xi_s\right) r \frac{P_2^1}{\sin \theta} \cos \phi. \end{cases} \quad (7a)$$

From (4), (5) and (7) together with

$$\vec{\omega}_0 = \begin{cases} \omega \cos \theta \\ -\omega \sin \theta \\ 0 \end{cases}, \quad (8)$$

it is now possible to calculate the righthand-side of (3) as

$$\nabla^2 p = f_c P_2^1 \cos \phi + f_s P_2^1 \sin \phi, \quad (9)$$

where

$$\left. \begin{aligned} f_c &= -(4\pi)^{-1} a^{-2} r^2 T_2 \left(r \frac{\partial^3 S_s}{\partial r^3} + 12 \frac{\partial^2 S_s}{\partial r^2} + \frac{28}{r} \frac{\partial S_s}{\partial r} \right) - 2\rho\omega r \left(r \frac{\partial^2 \xi_s}{\partial r^2} + 6 \frac{\partial \xi_s}{\partial r} \right), \\ f_s &= (4\pi)^{-1} a^{-2} r^2 T_2 \left(r \frac{\partial^3 S_c}{\partial r^3} + 12 \frac{\partial^2 S_c}{\partial r^2} + \frac{28}{r} \frac{\partial S_c}{\partial r} \right) + 2\rho\omega r \left(r \frac{\partial^2 \xi_c}{\partial r^2} + 6 \frac{\partial \xi_c}{\partial r} \right). \end{aligned} \right\} \quad (10)$$

The solution of (9) that remains finite at $r=0$ is readily obtained as

$$p = q_c P_2^2 \cos \phi + q_s P_2^1 \sin \phi, \quad (11)$$

where

$$\left. \begin{aligned} q_c &= a^{-2} r^2 K_c - (4\pi)^{-1} a^{-2} T_2 \left(r^3 \frac{\partial S_s}{\partial r} + 4r^2 S_s - 2r^{-3} \int r^4 S_s dr \right) - 2\rho\omega r^2 \xi_s, \\ q_s &= a^{-2} r^2 K_s + (4\pi)^{-1} a^{-2} T_2 \left(r^3 \frac{\partial S_c}{\partial r} + 4r^2 S_c - 2r^{-3} \int r^4 S_c dr \right) + 2\rho\omega r^2 \xi_c, \end{aligned} \right\} \quad (12)$$

in which K_c and K_s are constants which are to be determined from the boundary conditions.

Since p is obtained, the radial parts of $\vec{v}$ can be obtained from (2) as

$$\left. \begin{aligned} 3\rho \frac{\partial \xi_c}{\partial t} &= a^{-2} K_c - \frac{3}{4\pi} a^{-2} T_2 r^{-5} \int r^4 S_s dr + \rho\omega \xi_s, \\ 3\rho \frac{\partial \xi_s}{\partial t} &= a^{-2} K_s + \frac{3}{4\pi} a^{-2} T_2 r^{-5} \int r^4 S_c dr - \rho\omega \xi_c. \end{aligned} \right\} \quad (13)$$

(13) can be rewritten as

$$\left. \begin{aligned} 3\rho \frac{\partial}{\partial t} \left(r \frac{\partial \xi_c}{\partial r} + 5\xi_c \right) &= 5a^{-2} K_c - \frac{3}{4\pi} a^{-2} T_2 S_s + \rho\omega \left(r \frac{\partial \xi_s}{\partial r} + 5\xi_s \right), \\ 3\rho \frac{\partial}{\partial t} \left(r \frac{\partial \xi_s}{\partial r} + 5\xi_s \right) &= 5a^{-2} K_s + \frac{3}{4\pi} a^{-2} T_2 S_c - \rho\omega \left(r \frac{\partial \xi_c}{\partial r} + 5\xi_c \right). \end{aligned} \right\} \quad (14)$$

Going back to (1), the terms on the righthand-side can be calculated on the basis of (4), (5), (6) and (7). Although S_3^{1c} , S_3^{1s} , T_2^{1c} , T_2^{1s} , T_3^{1c} and T_3^{1s} magnetic fields come out from $\text{curl}(\vec{v} \wedge \vec{H}_0)$, we ignore these fields only picking up S_1^{1c} and S_1^{1s} fields. As the time-constant of the core is extremely large compared to that of the present concern, we may assume

infinite conductivity for the core, so that the second term on the lefthand-side of (1) can be dropped. Equating then the radial parts of both the sides of (1), we are led to

$$\left. \begin{aligned} \frac{\partial s_c}{\partial t} &= a^{-1} V_1 s_s + \frac{27}{5} a^{-2} r^2 T_2 \xi_s, \\ \frac{\partial s_s}{\partial t} &= -a^{-1} V_1 s_c - \frac{27}{5} a^{-2} r^2 T_2 \xi_c. \end{aligned} \right\} \quad (15)$$

(14) and (15) provide equations that govern the time-dependent behaviour of the S_1^{1c} and S_1^{1s} magnetic fields and the S_2^{1c} and S_2^{1s} motions superimposed on the steady state of the dynamo together with the boundary conditions that the normal components of velocities vanish and that the magnetic fields are continuous at the core boundary. The boundary conditions are given as

$$\left. \begin{aligned} r \frac{\partial s_c}{\partial r} + 3s_c &= 0, \\ r \frac{\partial s_s}{\partial r} + 3s_s &= 0, \\ \xi_c &= \xi_s = 0. \end{aligned} \right\} \quad (\text{at } r=a) \quad (16)$$

In order to study an S_1^1 magnetic field being propagated along parallel circles, we assume

$$\left. \begin{aligned} s &= (s_c + i s_s) e^{i \alpha t}, \\ \xi &= (\xi_c + i \xi_s) e^{i \alpha t}, \end{aligned} \right\} \quad (17)$$

where $i = \sqrt{-1}$.

In that case (14) and (15) lead to

$$\left. \begin{aligned} \rho(3\alpha + \omega) \left(r \frac{d\xi}{dr} + 5\xi \right) &= -i5a^{-2}K + \frac{3}{4\pi} a^{-2} T_2 s, \\ \alpha s &= -a^{-1} V_1 s - \frac{27}{5} a^{-2} r^2 T_2 \xi, \end{aligned} \right\} \quad (18)$$

where

$$K = K_c + i K_s. \quad (19)$$

Meanwhile, (16) is converted to

$$\left. \begin{aligned} r \frac{ds}{dr} + 3s &= 0, \\ \xi &= 0. \end{aligned} \right\} \quad (\text{at } r=a) \quad (20)$$

It is therefore seen that (18) combined with (20) provide an eigen-equation by which time-factor α should be determined. At this point, we take into account the condition that the time-constant of the present concern is very much larger than the time of the earth's rotation. Such a condition make us assume that $|\omega| \gg \alpha$, so that, dropping 3α from the first equation of (18), we obtain

$$\left. \begin{aligned} r \frac{d\xi}{dr} + 5\xi &= C + \frac{3}{\sqrt{4\pi\rho}} \frac{V_A}{\omega} a^{-2} s, \end{aligned} \right\}$$

$$(\alpha + a^{-1}V_1)s = -\frac{27}{5}\sqrt{4\pi\rho} V_A a^{-2}r^2\dot{\xi}, \quad \left. \vphantom{\frac{27}{5}} \right\} \quad (21)$$

where V_A is the Alfvén velocity defined by

$$V_A = T_2 / \sqrt{4\pi\rho} \quad (22)$$

and

$$C = -i \frac{5a^{-2}K}{\omega}. \quad (23)$$

If s is eliminated from (21), we obtain

$$r \frac{d\dot{\xi}}{dr} + \left(5 + \frac{81}{5} \frac{V_A^2}{a^4\omega\beta} r^2 \right) \dot{\xi} = C, \quad (24)$$

where

$$\beta = \alpha + a^{-1}V_1. \quad (25)$$

Since $\dot{\xi}=0$ at $r=a$, it is then obvious that

$$a \left(\frac{d\dot{\xi}}{dr} \right)_{r=a} = C. \quad (26)$$

On the other hand, the first condition of (20) leads to

$$\left(r \frac{d\dot{\xi}}{dr} + 5\dot{\xi} \right)_{r=a} = 0, \quad (27)$$

which is rewritten as

$$C + 5(\dot{\xi})_{r=a} = 0. \quad (28)$$

As $\dot{\xi}=0$ at $r=a$, (28) leads to $C=0$. However, such a result comes from the oversimplification during the procedure of estimating interactions between the motion and the steady magnetic field. If we retain the steady S_1 field, (15) become

$$\left. \begin{aligned} \frac{\partial s_c}{\partial t} &= a^{-1}V_1 s_s + \frac{9}{5} S_1 \left(r \frac{\partial \dot{\xi}_c}{\partial r} + 5\dot{\xi}_c \right) + \frac{27}{5} a^{-2} r^2 T_2 \dot{\xi}_s, \\ \frac{\partial s_s}{\partial t} &= -a^{-1}V_1 s_c + \frac{9}{5} S_1 \left(r \frac{\partial \dot{\xi}_s}{\partial r} + 5\dot{\xi}_s \right) - \frac{27}{5} a^{-2} r^2 T_2 \dot{\xi}_c, \end{aligned} \right\} \quad (29)$$

so that the second equation of (21) reduces to

$$(\alpha + a^{-1}V_1)s = \frac{9}{5} S_1 \left(r \frac{d\dot{\xi}}{dr} + 5\dot{\xi} \right) - \frac{27}{5} \sqrt{4\pi\rho} V_A a^{-2} r^2 \dot{\xi}.$$

Eliminating s , an equation

$$(1-A)r \frac{d\dot{\xi}}{dr} + \left[5(1-A) + \frac{81}{5} \frac{V_A^2}{a^4\omega\beta} r^2 \right] \dot{\xi} = C \quad (30)$$

is obtained in place of (24), where

$$A = \frac{27}{5} \frac{V_A a^{-2}}{\sqrt{4\pi\rho} \omega\beta} S_1. \quad (31)$$

It is therefore seen that the boundary conditions as given by (20) result in

$$\left. \begin{aligned} a \left(\frac{d\xi}{dr} \right)_{r=a} &= \frac{C}{1-A}, \\ \frac{9}{5} \frac{S_1}{\beta} \left[\left(r^2 \frac{d^2\xi}{dr^2} \right)_{r=a} + \frac{9C}{1-A} \right] - \frac{27}{5} \frac{C}{1-A} &= 0, \end{aligned} \right\} \quad (32)$$

which indicates that C remains finite on the condition that $(d^2\xi/dr^2)_{r=a}$ is finite.

In the following, therefore, we may assume $C \neq 0$ even if the steady S_1 field is ignored for the sake of simplicity.

The solution of (24) is known to be given by

$$\xi = C r^{-5} e^{-\frac{B}{2} r^2} \varphi(r) + k r^{-5} e^{-\frac{B}{2} r^2}, \quad (33)$$

in which k is a free constant and $\varphi(r)$ is defined by

$$\varphi(r) = \int r^4 e^{\frac{B}{2} r^2} dr, \quad (34)$$

while we write as

$$B = \frac{81}{5} \frac{V_A^2}{a^4 \omega \beta}. \quad (35)$$

The boundary conditions that $\xi=0$ and $r \frac{ds}{dr} + 3s=0$ (i.e. $r \frac{d\xi}{dr} + 5\xi=0$ as given by (27)) at $r=a$ result in the following simultaneous equations:

$$\left. \begin{aligned} C a^{-5} e^{-\frac{B}{2} a^2} \varphi(a) + k a^{-5} e^{-\frac{B}{2} a^2} &= -\frac{C}{5}, \\ C \left[1 - a^{-5} e^{-\frac{B}{2} a^2} (5 + B a^2) \varphi(a) \right] - k a^{-5} (5 + B a^2) e^{-\frac{B}{2} a^2} &= C. \end{aligned} \right\} \quad (36)$$

Eliminating k from the above two equations, we obtain

$$5 + B a^2 = 0 \quad (37)$$

or

$$5 + \frac{81}{5} \frac{V_A^2}{a^2 \omega \beta} = 0, \quad (38)$$

which reduces to

$$\alpha = -\frac{81}{25} \frac{V_A^2}{\omega a^2} - a^{-1} V_1. \quad (39)$$

3. Drift of the S_1^1 magnetic field

Assuming $T_2=100$ Gauss, $V_1=0.02$ cm/sec and $\rho=10$ g/cm³, which are generally accepted for the core, α is calculated from (39) as

$$\alpha = -0.85 \times 10^{-10} \text{ sec}^{-1}, \quad (31)$$

$a=3.5 \times 10^8$ cm and $\omega=7.5 \times 10^{-5} \text{ sec}^{-1}$ being taken in the estimation.

The magnetic field that is defined by the combination of s_c and s_s as given by the first

equation of (17) is written as

$$\tilde{h}_1^1 = e^{i\alpha t} \begin{cases} -2(s_c \cos \phi + i s_s \sin \phi) P_1^1 \\ - \left(r \frac{d}{dr} + 2 \right) (s_c \cos \phi + i s_s \sin \phi) \frac{dP_1^1}{d\theta} \\ \left(r \frac{d}{dr} + 2 \right) (s_c \sin \phi - i s_s \cos \phi) \frac{P_1^1}{\sin \theta}, \end{cases} \quad (32)$$

which, assuming further

$$s_c = s_s = s_1^1, \quad (33)$$

becomes

$$\tilde{h}_1^1 = \begin{cases} -2s_1^1 P_1^1 e^{i(\phi+\alpha t)} \\ - \left(r \frac{ds_1^1}{dr} + 2s_1^1 \right) \frac{dP_1^1}{d\theta} e^{i(\phi+\alpha t)} \\ - \left(r \frac{ds_1^1}{dr} + 2s_1^1 \right) \frac{\partial}{\partial \phi} \left[\frac{P_1^1}{\sin \theta} e^{i(\phi+\alpha t)} \right]. \end{cases} \quad (34)$$

It is therefore seen that the S_1^1 field moves around the core. Since α is negative, the motion of the field relative to the core material is eastward. If we take the westward drift of the core as revealed by the drift of the non-dipole field into account, the speed of the motion of the S_1^1 field as seen from a man fixed to the mantle is given by $\mathcal{Q} + \alpha$, where $\mathcal{Q}$ is the angular velocity of the westward drift of the non-dipole field. Adopting $\mathcal{Q} = 0.2^\circ/\text{yr}$ as widely accepted, $\mathcal{Q} + \alpha$ is estimated as

$$\mathcal{Q} + \alpha = 0.05^\circ/\text{yr}. \quad (35)$$

It is interesting that a crude theory in the above leads to a value of the westward drift of the S_1^1 field having a velocity which has been generally accepted. Even if we assume somewhat different values for T_2 and V_1 , it is unlikely that the conclusion that the S_1^1 drift velocity is one order of magnitude smaller than that of the non-dipole field would drastically be changed.

4. Discussion and conclusion

It is interesting to note that the theory advanced in the foregoing sections results in a westward drift of the S_1^1 field having a velocity which approximately agrees with the observed one. Owing to the mathematical complexity, however, the theory is developed only for $n=1$ and $m=1$. It is not known, therefore, whether or not a theory similar to the present one for different n and m leads to a drifting field.

We have seen in Table 1 that the S_3^2 field is drifting eastwards relative to the mantle. According to the summary of drift velocities by Hide (1966), the S_3^2 field seems to drift eastwards for all the epochs after the beginning of this century. If it is so, it would be extremely interesting to see whether we arrive at an eastward drift of the S_3^2 field by extending the theory to such a case.

Nothing is practically known about the drift of the S_1^1 field during historical and geological times. Archaeomagnetic studies by Aitken and Weaver (1965) and Kawai (1964) indicate

a westward drift of the virtual geomagnetic pole exceeding 270° in longitude since around 1300 A.D. Even an eastward drift of the pole during a period from 1000 A.D. to 1300 A.D. was reported by them. However, most of such drift motion of the virtual pole seems to be explained by taking the westward drift of the non-dipole field into account as has been discussed by Yukutake (1962). According to the present theory, changes in the slip velocity of the core relative to the mantle, the intensity of the T_2 field and the velocity of the T_1 motion affect the drift velocity. If it is assumed that these elements took different magnitudes some time in the past, possibility of a drastic change in the drift speed of the S_1^1 field cannot be ruled out.

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