Relative Magnitudes of Geomagnetic Daily-Range Measures near the Dip Equator

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Abstract

Different measures of range in use are defined and discussed. Expressions are obtained (a) for correcting the smoothing effects on the various measures of range, and (b) for comparing two measures of range. It is shown how these depend on the time of maximum and therefore to some extent on season. The smoothing corrections when the maximum is at 1100 hrs. L.T. vary from $\frac{1}{2}$ to $15\frac{1}{2}\%$ for measures of range based on averaging over one to 4 hours. The ratio of any two measures of range is found not to be significantly different between H at Huancayo and Z at Ibadan nor between years of solar activity maximum and minimum.

1. Introduction

Many analyses of geomagnetism are based on the daily range, M , of magnetic elements. Investigators of equatorial geomagnetic variations have used different measures of daily range. This has made comparison of their results difficult. Our object is to examine these various measures of range with a view to establishing a method of comparing them quantitatively. This would remove the difficulty in discussing the results of different workers.

2. Measures of Range

Although the word range means maximum minus minimum, instantaneous values should never be used in obtaining range because the geomagnetic field is very variable especially near the dip equator where short duration fluctuations are highly amplified (Onwumechilli 1959, Onwumechilli and Ogbuehi 1962). The following definitions of range have been used by electrojet investigators (arranged in our order of preference)

$$M_4 = \frac{1}{4}(H_{10} + H_{11} + H_{12} + H_{13}) - \frac{1}{4}(H_{00} + H_{01} + H_{22} + H_{23}). \quad (1)$$

$$M_2 = \frac{1}{2}(H_{11} + H_{12}) - \frac{1}{4}(H_{00} + H_{01} + H_{22} + H_{23}). \quad (2)$$

$$M_3 = \frac{1}{3}(H_{10} + H_{11} + H_{12}) - \frac{1}{4}(H_{00} + H_{01} + H_{22} + H_{23}). \quad (3)$$

$$M_1 = H_{\max.} - \frac{1}{4}(H_{00} + H_{01} + H_{22} + H_{23}). \quad (4)$$

$$M_m = H_{\max.} - H_{\min.} \quad (5)$$

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where H_t is usually the average value of the element under study from the hour t to the hour $(t+1)$ local time. M_T is the average of H over T hours less the midnight average of H . It would make little difference if the midnight average is taken as $\frac{1}{2}(H_{00}+H_{23})$. Midnight

Table 1 Monthly values of the departure M_4 , and the ratios, R_{T4} , of other departures to it. Hr is the hour of maximum.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Huancayo H 1954												
M_4	80.5	89.0	91.8	82.5	93.2	52.8	61.7	64.3	100.5	118.7	52.5	66.5
R_{14}	1.158	1.081	1.144	1.133	1.080	1.061	1.122	1.096	1.139	1.147	1.434	1.191
R_{24}	1.065	1.081	1.058	1.091	1.064	1.061	1.041	1.058	1.095	1.072	1.042	1.101
R_{34}	1.096	1.036	1.086	1.048	1.034	1.023	1.068	1.039	1.087	1.086	1.171	1.090
R_{m4}	1.180	1.146	1.155	1.200	1.116	1.136	1.135	1.120	1.154	1.163	1.448	1.278
Hr	10,11	11,12	10	11	11	11,12	10	11	11	11	09,10	11
Huancayo H 1958												
M_4	153.9	186.6	199.8	183.0	157.5	164.0	152.8	148.5	218.8	202.0	132.3	159.5
R_{14}	1.226	1.070	1.106	1.079	1.070	1.111	1.080	1.104	1.058	1.119	1.122	1.182
R_{24}	1.086	1.061	1.081	1.068	1.063	1.059	1.063	1.071	1.053	1.069	1.028	1.069
R_{34}	1.132	1.023	1.020	1.026	1.025	1.046	1.010	1.035	1.023	1.064	1.060	1.094
R_{m4}	1.268	1.082	1.143	1.093	1.079	1.122	1.111	1.111	1.064	1.124	1.172	1.235
Hr	10,11	12	11	11	11	11	12	11	11	11	10	11
Ibadan Z 1958												
M_4	47.5	41.4	40.5	52.2	51.5	36.5	34.3	47.1	47.2	39.5	39.1	36.3
R_{14}	1.168	1.360	1.217	1.088	1.142	1.099	1.160	1.113	1.112	1.385	1.212	1.187
R_{24}	1.149	1.114	1.042	1.042	1.140	1.088	1.090	1.104	1.046	1.023	1.031	1.011
R_{34}	1.059	1.196	1.101	1.056	1.047	1.016	0.977	0.998	1.042	1.144	1.092	1.069
R_{m4}	1.200	1.400	1.472	1.280	1.363	1.288	1.280	1.448	1.390	1.499	1.253	1.289
Hr	12	10	10	11	12	12	12	12	11	10	10	10
Ibadan Z 1964												
M_4	15.5	25.3	29.4	21.3	13.2	14.8	15.3	20.6	22.1	20.5	31.8	28.9
R_{14}	1.368	1.154	1.190	1.127	1.098	1.243	1.144	1.136	1.318	1.556	1.453	1.017
R_{24}	0.974	1.103	1.058	1.075	1.091	1.128	1.118	1.073	1.041	1.029	1.057	1.045
R_{34}	1.103	1.055	1.099	1.094	1.045	1.128	1.105	1.049	1.131	1.205	1.198	1.014
R_{m4}	1.390	1.344	1.347	1.371	1.136	1.297	1.647	1.641	1.511	1.561	1.516	1.073
Hr	09	11	10	10	12	11	11	11	10	10	10	12

value is taken as a base because of the quiescence of the Sq currents then.

The more the hourly values averaged, the more stable is the range, thus M_m and M_1 are more variable than the rest. They also embody two difficulties that can be helped. The hour of maximum or minimum of any element varies from day to day and from place to place. Thus if they are used for comparing many days or many places they do not give the state of the field at any one time. Furthermore M_1 and M_m are always positive by definition and fail to give any indication of the direction of change. Obviously M_4 and M_3 are the least susceptible to disturbance and minor fluctuations.

Two of the advantages of fixing the hours for calculating M_2 , M_3 and M_4 are: (a) they give the state of the field at the same local time everyday and everywhere; and (b) they give the additional information of the direction of change for example their sign at once indicates whether the element is greater near mid-day than midnight and if the daily swing is reversed on opposite sides of the current focus or the equator. Strictly speaking, they are better termed departures or differences or magnitudes.

It is of interest to obtain the relations between the departures, M_T , ($T=1$ to 4) as well as the relation between them and the maximum departure for the day in order to be able to compare the results of different investigators. We define departure ratios

$$R_{T4}=\frac{M_T}{M_4} \tag{ 6 }$$

where $T=1$ to 3.

3. Two ways of computing mean range

Here we compare the mean ranges obtained by the two methods usually used. In method I the hourly values are first averaged for all the days of interest before taking the range or departure. In method II ranges for individual days are averaged.

Mean values of M_T , ($T=1$ to 4) were computed by the two methods using the 5 international quiet days in each month. Altogether 24 monthly means of M_T were obtained for H at Huancayo. The values of M_2 calculated by the two methods did not differ by up to one gamma in any of the 24 months. The same was true of M_3 and M_4 .

On the other hand, in 23 of the 24 months, method II gave values of M_1 and M_m greater by 1 to 7 gammas than their values by method I. The proportional difference will be appre-

Table 2 Annual means of the departure M_4 , and the ratios, R_{T4} , of other departures to it.

	Huancayo H 1954	Huancayo H 1958	Ibadan Z 1958	Ibadan Z 1964
M_4	79.5	171.6	42.7	61.6
R_{14}	1.149 ± 0.027	1.111 ± 0.012	1.187 ± 0.027	1.234 ± 0.044
R_{24}	1.069 ± 0.006	1.066 ± 0.006	1.073 ± 0.014	1.066 ± 0.012
R_{34}	1.072 ± 0.011	1.047 ± 0.007	1.066 ± 0.018	1.101 ± 0.017
R_{m4}	1.186 ± 0.026	1.134 ± 0.016	1.347 ± 0.237	1.403 ± 0.050
Count	12	12	12	12

ciated when the following annual mean values of R_{14} and R_{m4} obtained by method II are compared with their corresponding values by method I in Table 2.

Values of R_{14} and R_{m4} obtained by Method II, using Huancayo H .		
	1954	1958
R_{14}	1.19 ± 0.03	1.14 ± 0.01
R_{m4}	1.27 ± 0.04	1.18 ± 0.02

Z at Ibadan in January, April, and June 1958 was also tested as above. The results were found to be similar to those of H at Huancayo.

It may therefore be concluded that the mean values of the fixed hour departures, M_2 , M_3 , and M_4 for international quiet days are the same whichever method of computation is used. On the other hand, method II usually gives a slightly higher average of M_1 and M_m respectively than method I. This represents a further advantage in using the fixed hour departures.

All the values given in succeeding sections are obtained by method I.

4. Theoretical approximation

We approximate the portion of the daily variation curve near the maximum to the cosine function of time

$$H = H_0 \cos wt \quad (7)$$

where w is an angle, H_0 is maximum H , and t is time measured from the position of maximum H . The mean value of H from

$$\begin{aligned} t = t_0 \quad \text{to} \quad t = t_0 + T \quad \text{is} \\ M_T(t_0) = \frac{1}{T} \int_{t_0}^{t_0+T} H(t) dt \\ = -\frac{H_0}{wT} [\sin w(t_0 + T) - \sin wt_0]. \end{aligned} \quad (8)$$

This enables us to correct the departure $M_T(t_0)$ for smoothing. The smoothing correction is

$$C_T(t_0) = \frac{H_0}{M_T(t_0)} = \frac{wT}{\sin w(t_0 + T) - \sin wt_0}. \quad (9)$$

In this expression, one should really write t_{0T} since t_0 is different for each M_T . It follows that the departure ratios, Equation (6) are given by

$$R_{T4} = \frac{C_4}{C_T} = \frac{4}{T} \frac{[\sin w(t_{0T} + T) - \sin wt_{0T}]}{[\sin w(t_{04} + 4) - \sin wt_{04}]}. \quad (10)$$

Because the initial time, t_0 for M_2 is always one hour greater than the initial time for M_4 ,

$$R_{24} = \sec w. \quad (11)$$

For any given hour of maximum it is possible to express R_{T4} in terms of R_{24} . Such relationships permit a test of the usefulness of the approximation before application.

5. Numerical values of the corrections

Tables 1 and 2 show numerical values calculated from H at Huancayo and Z at Ibadan for international quiet days during years of sunspot maximum 1958 and minimum 1954 and 1964. There is no striking difference between years of high and low solar activity nor between H and Z except those differences arising from the dependence of the ratio on time of maximum. For example Fig. 1 shows some seasonal effect on R_{14} and R_{m4} for Z at Ibadan in 1958 arising from the seasonal variation of the time of maximum. No such variation is obvious in the ratios $R_T(H)$ where the time of maximum is more uniform.

Table 3 shows the average of R_{T4} according to hours of maximum. As the hour of maximum advances from 1000 to 1200, R_{14} , R_{34} , and R_{m4} decrease but R_{24} increases. To test the usefulness of the theoretical approximation we use the mean R_{24} in Table 3 to calculate R_{14} and R_{34} , assuming that in the case of M_1 the maximum is at the beginning of the interval.

Figure 1. Variations of the departure ratios, R_{14} and R_{m4} , for Z at Ibadan in 1958 arising from the variation in the time of maximum.

Table 3 Mean values of departure ratios, R_{T4} , when maximum occurs at 1000, 1100, and 1200 hours local time respectively.

Hour	1000	1100	1200	All
R_{14}	1.261 ± 0.038	1.125 ± 0.009	1.105 ± 0.016	1.170 ± 0.016
R_{24}	1.047 ± 0.006	1.076 ± 0.006	1.092 ± 0.013	1.068 ± 0.005
R_{34}	1.118 ± 0.014	1.058 ± 0.005	1.021 ± 0.008	1.072 ± 0.007
R_{m4}	1.360 ± 0.040	1.230 ± 0.037	1.220 ± 0.042	1.267 ± 0.023
Count	13	20	9	48

The results are as follows.

	1000		1100		1200	
	R_{14}	R_{34}	R_{14}	R_{34}	R_{14}	R_{34}
Calculated	1.27	1.12	1.16	1.10	1.09	1.03
Observed	1.26	1.12	1.13	1.06	1.11	1.02
Error Limits	± 0.12	± 0.05	± 0.04	± 0.02	± 0.05	± 0.02

The error limits indicate the range within which the difference between the calculated and observed ratios must lie so that the difference will not be statistically significant at 5% level. Thus only one of the six calculated ratios is significantly different from its observed mean at 5% level. We may therefore say with that much confidence that the theoretical approximation is useful. w depends on time of maximum but from the overall average of $R_{24}=1.068$ from Huancayo H and Ibadan Z we infer from Equation (11) that $w=20.55^\circ$, approximately $\frac{\pi}{9}$. The difference between this and the value for 1100 is within error limits.

Assuming the maximum is at 1100 hrs. L.T. and taking $w=\frac{\pi}{9}$ we proceed to calculate the smoothing corrections C_T , from Equation (9) to be applied to the departures, M_T , Equations (1) to (4)

$$C_1 = \frac{1}{2}w \operatorname{cosec} \frac{1}{2}w = 1.005 \quad (12)$$

$$C_1 = w \operatorname{cosec} w = 1.020 \quad (13)$$

$$C_2 = w \operatorname{cosec} w \sec w = 1.085 \quad (14)$$

$$C_3 = \frac{3w}{\sin 2w + \sin w} = 1.063 \quad (15)$$

$$C_4 = w \operatorname{cosec} w \sec^2 w = 1.155 \quad (16)$$

Equation (13) applies when the maximum is at one end of the hour interval and Equation (12) when the hourly mean is centred on the time of maximum. The midnight period used as zero level is usually flat and needs no smoothing correction. C_1 applies to the $H_{\max}$ in M_m but the theory does not cover the hour of minimum during which the shape may not be approximately a cosine function. These smoothing corrections should be specifically determined for each investigation using Equation (11), but if that is not possible for any reason the numerical values in Equations (12) to (16) may be taken as order of magnitude values.

6. Conclusion

Having studied the various measures of range used in equatorial geomagnetism we find that:

1. The fixed hour departures, M_2 , M_8 , and M_4 , are not affected by the method of computing mean for international quiet days. The measures involving maximum hourly mean, M_1 , and M_m , are affected by the method of computing mean because the time of maximum varies from day to day.

2. The annual mean ratios of all the other measures of range to M_4 change monotonically as the hour of maximum changes from 1000 to 1200 L.T. and this is not merely because of the definition of M_4 for it is also true of the ratio of M_m to M_1 .
3. The expressions developed for calculating smoothing correction factors, C_T , are appropriate except in the case of the orthodox measure of range M_m . The difficulties with M_m are that the time of minimum changes and also the magnetogram in the region of minimum is not like the mathematical representation.
4. The smoothing correction factors lead to expressions for converting one measure of range into another. The conversion ratios have been found suitable except when the orthodox measure of range M_m is involved.
5. The annual means of the conversion ratios for H at Huancayo are not noticeably different from those for Z at Ibadan nor are they different for years of high and low solar activity.

It is clear from the above that the measures of range based on fixed hour departures are more regular and tractable than the orthodox measure of range. Because of this and two other advantages mentioned in section 2 we prefer the use of the fixed hour departures to the orthodox measure of range. Furthermore, since the departures can now be corrected for smoothing we prefer the most stable M_4 to the others.

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References

- Onwumechilli, C.A. The Relation between H - and Z -Variations near the equatorial electrojet. *J. Atmosph. Terr. Phys.* **16**, 274-282, 1959.
- Onwumechilli, A., and Ogbuehi, P.O. Fluctuations in the geomagnetic horizontal field. *J. Atmosph. Terr. Phys.* **42**, 173-190, 1962.