Relation between the Carbon 14 Production Rate and the Geomagnetic Moment

In order to discuss the secular variation of carbon 14 content in the atmosphere, a knowledge of its relation to the geomagnetic moment is necessarily required. The secular variation of geomagnetic moment is effective on the intensity of primary cosmic rays and on the intensity of secondary neutrons whose capture rate in the atmosphere is equivalent to the production rate of carbon 14. An investigation on the relation between carbon 14 production rate and the geomagnetic moment undertaken by Elsasser, Ney and Winckler (1956) is based on the assumption that the spectrum of primary cosmic rays is inversely proportional to the rigidities of the particles. The aim of this note is to present the relation deduced from the neutron capture rates estimated by Soberman (1956) and Lingenfelter (1963). The curve already cited by Kigoshi and Hasegawa (1966) has been derived from Soberman’s result. An aspect of the solar cycle variation is further obtained from Lingenfelter’s data.

The carbon 14 production rate averaged all over the earth’s surface is

\[ \bar{Q} = \frac{1}{2\pi} \int_0^{\pi/2} Q_1 \cos \lambda d\lambda \left( \frac{\cos \lambda}{\cos \lambda} \right) \text{ (cm}^2\text{s}^{-1}) \]  

(1)

where \( Q_1 \) is the carbon 14 production rate in a unit column of atmosphere at latitude \( \lambda \). The latitude effect of the carbon 14 production rate arises from the latitude effect of cosmic rays. The relation between the intensity of primary cosmic rays, \( j(P, t) \), and the carbon 14 production rate, \( Q_1 \), is expressed by

\[ Q_1(t) = \int_{P_c}^{P} K(P) j(P, t) dP \text{ (cm}^2\text{s}^{-1}) \]  

(2)

where \( K(P) \) is the yield function of producing carbon 14. Approximating the geomagnetic field by simple dipole, the cut-off rigidity, \( P_c \), is given by

\[ P_c = 14.8 \frac{M}{M_0} \cos^4 \lambda \text{ (GV)} \]  

(3)

where \( M \) is the dipole moment and \( M_0 \) is the value at present.

The relation \( Q_1(M/M_0 = 1) \) vs. \( \lambda \) is experimentally given by Soberman and by Lingenfelter. Then the relation \( Q_1(M/M_0 = 1) \) vs. \( P_c(M/M_0 = 1) \), implied in Eq. (2) is found. The relation between \( Q \) and \( P \) can be indicated without suffix, for it is now independent of \( \lambda \), and further, when the spectrum of primary cosmic rays, \( j(P, t) \) in the integrand of Eq. (2) is not time dependent, it is also independent of \( M \), and \( Q \) is a function of \( P \) only. Once \( Q \) vs. \( P \) is known, \( Q_1(M/M_0) \) vs. \( \lambda \) can be obtained through \( P_c(M/M_0) \) vs. \( \lambda \) for the parameter \( M/M_0 \). In this process, \( Q \) against cut-off rigidities higher than that of equator is estimated under the assumption that \( Q \) is a function of \( P \) with simple exponent, and that there is no solar cycle variation of \( Q \) at rigidities higher than 100 GV. The error introduced by this treatment is
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Included in $Q$ at the higher values of $M/M_0$. $Q(M/M_0)$ are calculated by Eq. (1). Since only relative variation of carbon 14 production rate against the geomagnetic moment is of interest, $Q$ is divided by $Q_0$, the present production rate. In this way the relation between carbon 14 production rate and the geomagnetic moment is composed in the form of $Q/Q_0$ vs. $M/M_0$. The results are tabulated in Table 1 and illustrated in Fig. 1.

The measurements cited by Soberman and Lingenfelter to obtain the values of $Q_2$ were performed mainly on the continent of the North America and on the sea around. If the geomagnetic anomaly is taken into account, the relation $P_2$ vs. $A$ can no longer be expressed by simple dipole approximation. $Q-W$ and $K-K$ in Table 1 and Fig. 1 are the values of $Q/Q_0$, when the values of $P_2$ vs. $A$ by Quenby-Webber (1959) and Kondo-Kodama (1965) are introduced instead of the dipole approximation. As for the values of $Q_2$, for either case of $Q-W$ and $K-K$, that of Lingenfelter's solar maximum are used. The differences between the

<table>
<thead>
<tr>
<th>$M/M_0$</th>
<th>1952-54</th>
<th>1953-54</th>
<th>1957-1958</th>
</tr>
</thead>
<tbody>
<tr>
<td>dipole</td>
<td>1.87</td>
<td>2.17</td>
<td>1.83</td>
</tr>
<tr>
<td>$Q-W$</td>
<td>1.781</td>
<td>2.007</td>
<td>1.810</td>
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<tr>
<td>$K-K$</td>
<td>1.673</td>
<td>1.877</td>
<td>1.721</td>
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<tr>
<td>relative difference</td>
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<tr>
<td>10</td>
<td>0.029</td>
<td>0.029</td>
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</tr>
</tbody>
</table>

Table 1 $Q/Q_0$ vs. $M/M_0$

Fig. 1 The relation between the production rate of carbon 14 and the geomagnetic dipole moment. $Q$ is the average value all over the earth's surface. $Q_0$ and $M_0$ correspond to the present value.
values deduced from the dipole approximation and that of $Q \cdot W$ or $K \cdot K$ amount only a few percent.

Relation $Q_1$ vs. $P_1$ defined by Eq. (2) is affected by the spectrum change of primary cosmic rays which follows after the solar activity. Remarks may be needed that the time variation of $Q$ should not be compared between the values of Soberman and Lingenfelter, for there are some differences in obtaining their neutron capture rate. Comparing the results for the case of Lingenfelter’s maximum and that of minimum, slight difference is seen as will be referred later on. Since the time constant of the carbon circulation in atmosphere and sea is relatively long compared to eleven years (Craig, 1957), median of both values may be applicable for the related studies. The difference would, however, gives a measure for the study of carbon content in regard to long period variations of the solar activity. Concerning the time variation of production rate, a certain additional effect of the solar cosmic rays is expected. However, according to the work by Lingenfelter and Flamm (1964), it is not significant as long as $Q$ is concerned. Because, although the magnitude of solar proton event is not small, the duration is short. Moreover, since the rigidities of the solar protons are comparatively low, most of the neutrons are produced in the higher layers of the atmosphere, and escape outward not being captured to produce carbon 14.

As an extreme case, if the geomagnetic moment is as small as zero, $\bar{Q}$ becomes identical with $Q_1$ of geomagnetic pole. The values of $\bar{Q}$ at $M=0$ derived from Soberman and Lingenfelter’s maximum and minimum are 1.87, 1.83 and 2.17 times the present values, respectively.

Elsasser et al (1956) expressed their result by $\Delta Q = \Delta M / M_{0.52}$. This expression is in coincidence with the curves of Soberman and Lingenfelter within 5% errors as shown in Fig. 1 in the range of $M / M_0 = 0.7$ to 3. The three curves in Fig. 1 can be approximated by the same expression with different values of exponent: i.e., 0.57 for Soberman and 0.58, 0.56 for the maximum and the minimum, respectively, for the widest ranges of $M / M_0$. Within 5% deviation, the corresponding ranges of $M / M_0$ are 0.7–10, 0.7–10 and 0.5–10, respectively.

A fraction of carbon 14 caused by cosmic rays of rigidities higher than the cut-off rigidity at equator is also estimated. $\bar{Q}$ in Eq. (1) represents the production rate of carbon 14 by whole cosmic rays, and that contributed from cosmic rays of rigidities higher than equator’s cut-off, $\bar{Q}$, is identical with $Q_{z_{eq}}$. The ratios, $Q' / Q$ are 0.34, 0.38 and 0.48 for the values of Soberman, Lingenfelter’s minimum and maximum, respectively. It means about 40 to 50 percent of carbon 14 production rate are originated from cosmic rays of rigidities higher than 15 GV.

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References


Kigoshi, K. and Hasegawa, H., Solar variation of atmospheric radiocarbon concentration and its depen-
Kondo, I. and Kodama, M., Geographic distribution of vertical cosmic ray threshold rigidities, Proc. 9th
Lingenfelter, R.E. and Flamm, E.T., Production of carbon 14 by solar protons. J. Atoms. Sci., 21, 134-
140, 1964.
Quenby, J.J. and Webber, W.R., Cosmic ray cut-off rigidities and the earth's magnetic field, Phil. Mag.,
41, 90-113, 1959.

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